## $e^+e^-$ techniques: Kinematic endpoints

Imagine we produce $\tilde{\ell}_R\tilde{\ell}_R$ pairs in $e^+e^-$ collisions, and they each decay to $\ell\tilde{N}_1$.
How can we measure their masses?
One technique: "kinematic endpoints"
Measure maximum and minimum values of $\ell$ energies $\rightarrow$ extract $m_{\tilde{\ell}_R}$ and $m_{\tilde{N}_1}$.

Here's how it works:
(1) Consider the rest frame of one $\tilde{\ell}$. Energy and momentum conservation:

$$E_\ell + E_{\tilde{N}} = m_{\tilde{\ell}}, \qquad \vec{p}_\ell = -\vec{p}_{\tilde{N}} \tag{1}$$

Neglect the mass of $\ell$. Then $E_\ell = |\vec{p}_\ell|$.
Also have $E_{\tilde{N}} = \sqrt{m^2_{\tilde{N}} + \vec{p}^2_{\tilde{N}}} = \sqrt{m^2_{\tilde{N}} + E^2_\ell}$.
Plug in to energy conservation equation, rearrange, and square both sides:

$$m^2_{\tilde{N}} + E^2_\ell = m^2_{\tilde{\ell}} - 2m_{\tilde{\ell}}E_\ell + E^2_\ell \tag{2}$$

$$\text{or} \qquad E_\ell = |\vec{p}_\ell| = \frac{m^2_{\tilde{\ell}} - m^2_{\tilde{N}}}{2m_{\tilde{\ell}}} \tag{3}$$

(2) Now we'll boost the $\tilde{\ell}$ to the collider centre-of-mass frame.

$$E_{\tilde{\ell}_{R1}} + E_{\tilde{\ell}_{R2}} = \sqrt{s}, \qquad \vec{p}_{\tilde{\ell}_{R1}} = -\vec{p}_{\tilde{\ell}_{R2}} \tag{4}$$

Use the fact that two particles of the same mass $m_{\tilde{\ell}}$ are produced:

$$E_{\tilde{\ell}_{R1}} = \sqrt{m^2_{\tilde{\ell}} + \vec{p}^2_{\tilde{\ell}_{R1}}} = E_{\tilde{\ell}_{R2}} = \frac{\sqrt{s}}{2} = \gamma m_{\tilde{\ell}}, \qquad |\vec{p}_{\tilde{\ell}_{R1}}| = \sqrt{\frac{s}{4} - m^2_{\tilde{\ell}}} = \gamma m_{\tilde{\ell}}|\vec{v}| \tag{5}$$

Compute $E_\ell^{CM}$ in the CM frame by doing the boost:

(cos $\theta^*$ is defined in $\tilde{\ell}$ rest frame)

$$E_\ell^{CM} = \gamma\,(E_\ell + \beta p_{\ell z}) = \gamma\,(E_\ell + |\vec{p}_\ell| \cos\theta^*) = E_\ell\,(\gamma + \gamma|\vec{v}| \cos\theta^*) \tag{6}$$

From before we have

$$E_{\tilde{\ell}} = \gamma m_{\tilde{\ell}} = \frac{\sqrt{s}}{2} \qquad \rightarrow \qquad \gamma = \frac{\sqrt{s}}{2m_{\tilde{\ell}}} \tag{7}$$

$$|\vec{p}_{\tilde{\ell}}| = \gamma|\vec{v}|m_{\tilde{\ell}} = \sqrt{E_{\tilde{\ell}}^2 - m_{\tilde{\ell}}^2} \qquad \rightarrow \qquad \gamma|\vec{v}| = \frac{\sqrt{s - 4m_{\tilde{\ell}}^2}}{2m_{\tilde{\ell}}} \tag{8}$$

Put it all together:

$$E_\ell^{CM} = \frac{m_{\tilde{\ell}}^2 - m_{\tilde{N}}^2}{4m_{\tilde{\ell}}^2}\left(\sqrt{s} + \sqrt{s + 4m_{\tilde{\ell}}^2} \cos\theta^*\right) \tag{9}$$

Maximum (minimum) lepton energy corresponds to $\cos\theta^* = 1$ $(-1)$

$\sqrt{s}$ is known: collider CM energy.

Measure $E_\ell^{max}$ and $E_\ell^{min}$ from lepton kinematic distributions.

Solve for $m_{\tilde{\ell}}$ and $m_{\tilde{N}}$! A little algebra gives:

$$m_{\tilde{\ell}}^2 = \frac{s}{4}\left[1 - \left(\frac{E^{max} - E^{min}}{E^{max} + E^{min}}\right)^2\right] \qquad m_{\tilde{N}}^2 = m_{\tilde{\ell}}^2\left[1 - \frac{2(E^{max} + E^{min})}{\sqrt{s}}\right] \tag{10}$$

Need to isolate data sample with only $\tilde{\ell}_R\tilde{\ell}_R$ pair production:

can use $e^+e^-$ beam polarization to suppress $\tilde{\ell}_L\tilde{\ell}_L$ and $W^+W^-$ background

In practise, things are a little more complicated.

Example: $e^+e^- \to \widetilde{\mu}^+_{L,R}\widetilde{\mu}^-_{L,R}$ with $m_{\widetilde{\mu}_R} = 178$ GeV, $m_{\widetilde{\mu}_L} = 287$ GeV

Note the muon energy edges at about 65 and 220 GeV.

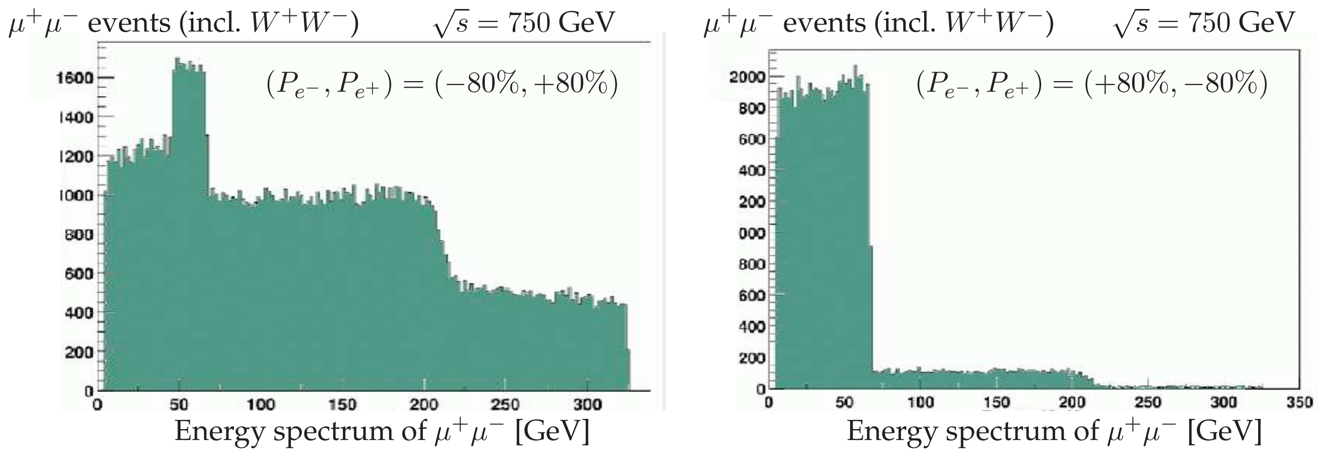



Figure 3.4: Energy spectrum of muons from $\tilde{\mu}_{L,R}$ decays into $\mu\tilde{\chi}^0_1$ final states, including the $W^+W^-$ background decaying into $\mu\nu$ final states in the scenario S3, cf. table 3.1, for two combinations of beam polarizations for $\sqrt{s} = 750$ GeV and $\mathcal{L}_{\rm int} = 500$ fb$^{-1}$ [87].

from hep-ph/0507011

These plots also demonstrate effect of beam polarization:
RH $e^-$ and LH $e^+$ eliminate t-channel $W^+W^-$ production (a large background).
Beam pol also changes the strength of the $Z^*$ contribution:

different effect on $\widetilde{\mu}_L$ and $\widetilde{\mu}_R$ pair production

Eyeballing the endpoints:

$\widetilde{\mu}_L$: $E^{max} \approx 220$ GeV, $E^{min} \approx 65$ GeV (note drop: pol dep $\to \widetilde{\mu}_L$)

$\widetilde{\mu}_R$: $E^{max} \approx 65$ GeV, $E^{min}$ not visible!

Solve: get $m_{\widetilde{\mu}_L}$ and $m_{\widetilde{N}}$ from $\widetilde{\mu}_L$ endpoints; plug in $m_{\widetilde{N}}$ to get $m_{\widetilde{\mu}_R}$ from $E^{max}$

$m_{\widetilde{\mu}_L} \approx 282$ GeV (compare input 287 GeV)

$m_{\widetilde{N}_1} \approx 153$ GeV

$m_{\widetilde{\mu}_R} \approx 167$ GeV (compare input 178 GeV)

Why are the lepton energy distributions flat?
Take another look at the formula:

$$E_\ell^{CM} = \frac{m_{\tilde\ell}^2 - m_{\tilde N}^2}{4m_{\tilde\ell}^2}\left(\sqrt{s} + \sqrt{s + 4m_{\tilde\ell}^2}\cos\theta^*\right) \tag{11}$$

We're asking about the differential cross section,

$$\frac{d\sigma}{dE_\ell^{CM}} = \frac{d\sigma}{d\cos\theta^*}\frac{d\cos\theta^*}{dE_\ell^{CM}} \tag{12}$$

- $d\cos\theta^*/dE_\ell^{CM}$ is a constant.
- $d\sigma/d\cos\theta^*$ is the $\tilde\ell$ decay distribution in the $\tilde\ell$ rest frame.

$\tilde\ell$ is a scalar: it can't single out any direction.
$\rightarrow$ uniform decay distribution over the solid angle:

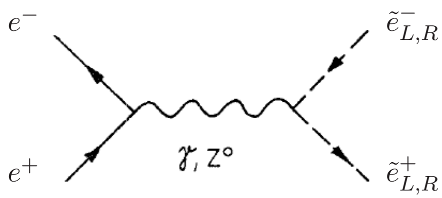
$$\frac{d\sigma}{d\cos\theta^* d\phi^*} = \text{const} \tag{13}$$

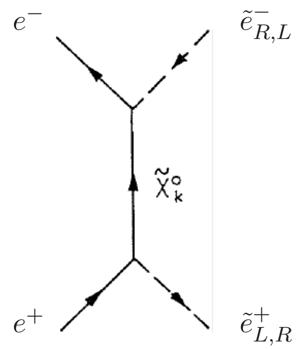
Integrating over the $\phi^*$ angle gives us what we want to know:
$d\sigma/dE_\ell^{CM}$ is flat (with endpoints).

## $e^+e^-$ techniques: Beam polarization

As a test of supersymmetry, we want to check whether the selectrons carry the same chiral quantum numbers as the electrons, and whether the electron-selectron-gaugino Yukawa couplings are related to the electron gauge couplings as predicted by SUSY.

Study the production processes $e^+e^- \to \widetilde{e}^+_{L,R}\widetilde{e}^-_{L,R}$.

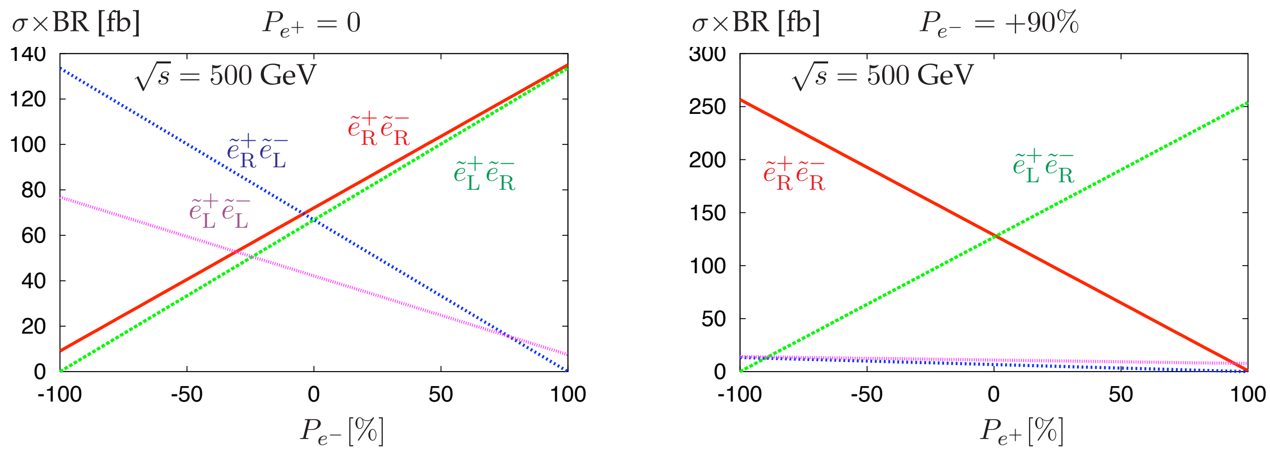



- s-channel: Can produce $\widetilde{e}^+_L\widetilde{e}^-_L$ and $\widetilde{e}^+_R\widetilde{e}^-_R$ through $\gamma$, $Z$ couplings.
- t-channel: Can produce all 4 combinations: $\widetilde{e}^+_L\widetilde{e}^-_L$, $\widetilde{e}^+_R\widetilde{e}^-_R$, $\widetilde{e}^+_L\widetilde{e}^-_R$, and $\widetilde{e}^+_R\widetilde{e}^-_L$.

Signal rates depend on:
- $\widetilde{e}_L$ and $\widetilde{e}_R$ masses
- Selectron gauge couplings and $e\widetilde{e}\widetilde{N}$ Yukawa couplings
- masses and compositions of all 4 neutralinos exchanged in the t-channel

(1) Check whether $e_L$ couples to $\widetilde{e}_L$ and a neutralino, and $e_R$ couples to $\widetilde{e}_R$ and a neutralino. Only the t-channel process is useful here.
Collide $\bar{e}_R e_L \to \widetilde{e}^+_R \widetilde{e}^-_L$, $\bar{e}_L e_R \to \widetilde{e}^+_L \widetilde{e}^-_R$. (Note $\bar{e}_R$ is a left-handed positron.)




Plots assume $m_{\widetilde{e}_L} \approx m_{\widetilde{e}_R}$, decay mode $\widetilde{e}_{L,R} \to e\widetilde{N}_1$ from hep-ph/0507011

Potentially achievable polarization at ILC: $e^-$ up to 90%, $e^+$ up to 60% (if it will be included in the design).

Linear dependence of cross sections with polarization is just due to dialling the luminosity of the relevant polarization component of the beam

(2) Check whether $e\tilde{e}\tilde{N}$ Yukawa couplings are the same as the $ee\gamma, eeZ$ gauge couplings.

Want to separately measure:

- $e_L \tilde{e}_L \tilde{N}_i$ coupling
- $e_R \tilde{e}_R \tilde{N}_i$ coupling

For this we must assume that the neutralino masses and mixing parameters have already been measured!

Technique: measure $\tilde{e}\tilde{e}$ production cross sections from polarized initial beams.

- If only one beam is polarized:
  - Measure $\bar{e}e_R \to \tilde{e}_R\tilde{e}_R, \tilde{e}_L^+\tilde{e}_R^-, \tilde{e}_L\tilde{e}_L$ via s-channel $\gamma, Z$ and t-channel $\tilde{N}_i$
  - Measure $\bar{e}e_L \to \tilde{e}_R\tilde{e}_R, \tilde{e}_R^+\tilde{e}_L^-, \tilde{e}_L\tilde{e}_L$ via s-channel $\gamma, Z$ and t-channel $\tilde{N}_i$

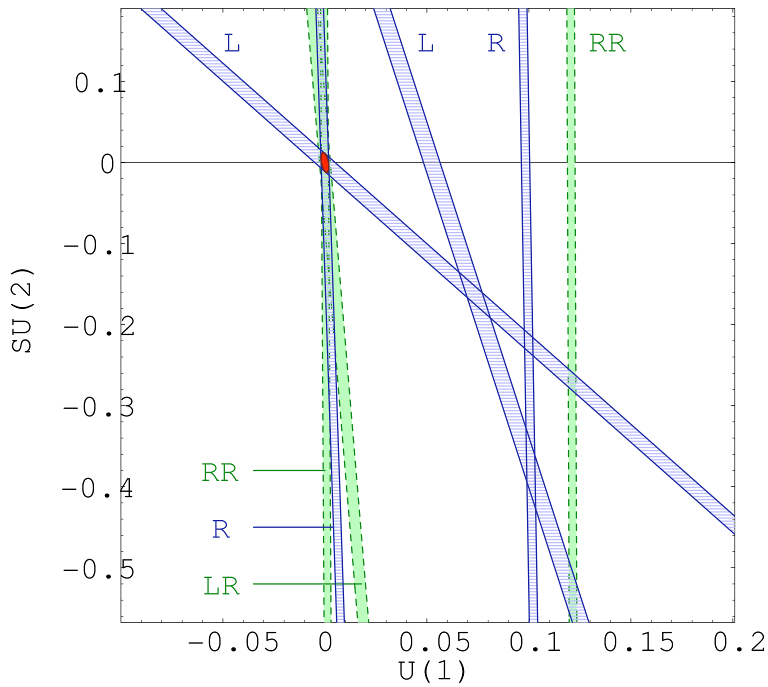
Can separate $\tilde{e}_L$ from $\tilde{e}_R$ if their decay modes can be distinguished,

e.g., if $\tilde{e}_L \to e\tilde{N}_2$ is open, while $\tilde{e}_R \to e\tilde{N}_1$ only.

- If both beams are polarized:
  - Measure $\bar{e}_R e_R \to \widetilde{e}_R\widetilde{e}_R, \widetilde{e}_L\widetilde{e}_L$ via s-channel $\gamma, Z$ and t-channel $\widetilde{N}_i$
  - Measure $\bar{e}_L e_L \to \widetilde{e}_R\widetilde{e}_R, \widetilde{e}_L\widetilde{e}_L$ via s-channel $\gamma, Z$ and t-channel $\widetilde{N}_i$
  - Measure $\bar{e}_R e_L \to \widetilde{e}_R^+\widetilde{e}_L^-$ via t-channel $\widetilde{N}_i$

Can extract separate couplings even if final-state $\widetilde{e}_R$ and $\widetilde{e}_L$ cannot be distinguished easily (e.g., close in mass, same decay modes)

Need both beams polarized in this case to get a unique solution:




from hep-ph/0507011

Plot of deviations of Yukawas from SM gauge couplings (for 90% $e^-$ pol, 60% $e^+$ pol)

## Kinematic endpoints at LHC

At the LHC: proton-proton collider

- $\sqrt{s}$ not known; varies event-by-event
- $v$ of pair-produced particles not known $\rightarrow$ can't use boost to get $E$
- Boost of CM along beam direction not known

$e^+e^-$ technique for $\widetilde{\ell}\widetilde{\ell} \rightarrow \ell\widetilde{N}_1\ell\widetilde{N}_1$ reconstruction won't work!

But: LHC can generally produce heavier particles $\rightarrow$ longer decay chains.

More kinematic variables to play with.

Don't know the boost of individual events:
$\rightarrow$ use kinematic invariants, like invariant masses.

Consider the decay chain $\widetilde{N}_2 \rightarrow \widetilde{\ell}_R^{\pm}\ell^{\mp} \rightarrow \widetilde{N}_1\ell^+\ell^-$

First need to select events that contain a $\widetilde{N}_2$ and identify the $\ell^+\ell^-$ coming from the $\widetilde{N}_2$ decay.

Invariant observable: invariant mass of $\ell^+\ell^-$: $M_{\ell\ell}$

How is this related to the SUSY masses?

Momentum and energy conservation in each decay:

$$p_{\widetilde{N}_2} = p_{\ell_1} + p_{\widetilde{\ell}} \qquad\qquad p_{\widetilde{\ell}} = p_{\ell_2} + p_{\widetilde{N}_1} \tag{14}$$

Combine and rearrange:

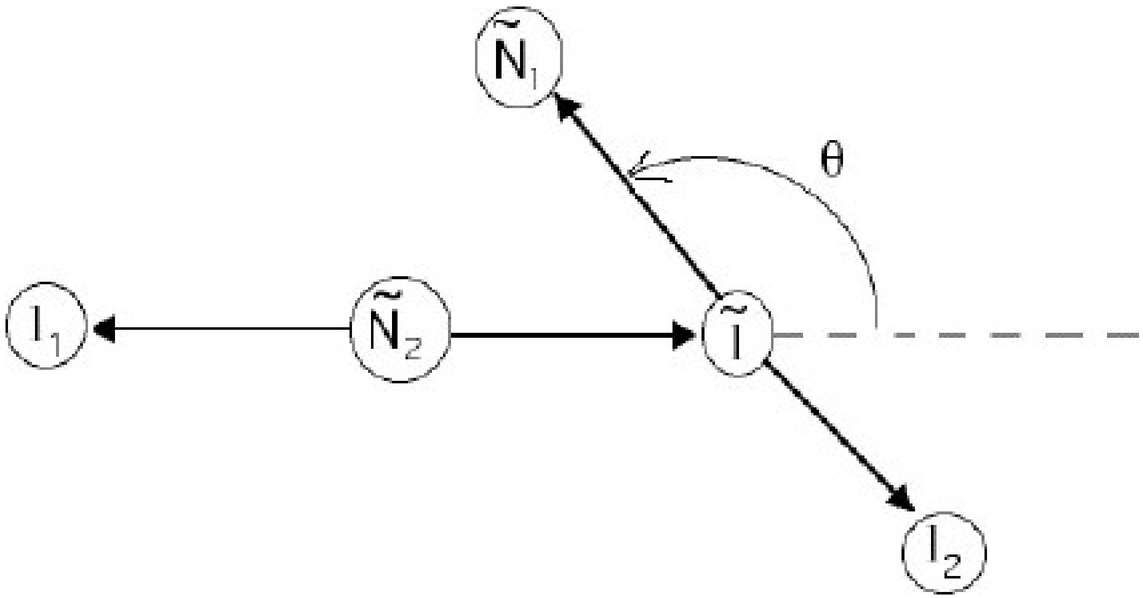
$$M_{\ell\ell}^2 = (p_{\ell_1} + p_{\ell_2})^2 = (p_{\widetilde{N}_2} - p_{\widetilde{N}_1})^2 = m_{\widetilde{N}_2}^2 + m_{\widetilde{N}_1}^2 - 2p_{\widetilde{N}_2} \cdot p_{\widetilde{N}_1} \tag{15}$$

What is this? Let's work in the $\widetilde{N}_2$ rest frame (can do that because we're calculating kinematic invariants!)

$p_{\widetilde{N}_2} \cdot p_{\widetilde{N}_1} = m_{\widetilde{N}_2} E_{\widetilde{N}_1}$ where $E_{\widetilde{N}_1}$ is in the $\widetilde{N}_2$ rest frame

$$M_{\ell\ell}^2 = m_{\widetilde{N}_2}^2 + m_{\widetilde{N}_1}^2 - 2m_{\widetilde{N}_2} E_{\widetilde{N}_1} \tag{16}$$

Now we need to find the kinematic endpoint(s) of $E_{\widetilde{N}_1}$ in the $\widetilde{N}_2$ rest frame in terms of the SUSY masses.
Strategy: Relate the energies to masses and the $\widetilde{\ell}$ decay angle $\theta$




Look at $\widetilde{N}_2$ decay: $m_{\widetilde{N}_2} = E_{\ell_1} + E_{\widetilde{\ell}}$, $\vec{p}_{\ell_1} = -\vec{p}_{\widetilde{\ell}}$

$$E_{\ell_1} = \frac{1}{2m_{\widetilde{N}_2}}\left(m^2_{\widetilde{N}_2} - m^2_{\widetilde{\ell}}\right) \qquad |\vec{p}_{\ell_1}| = E_{\ell_1} \tag{17}$$

$$E_{\widetilde{\ell}} = \frac{1}{2m_{\widetilde{N}_2}}\left(m^2_{\widetilde{N}_2} + m^2_{\widetilde{\ell}}\right) \qquad |\vec{p}_{\widetilde{\ell}}| = |\vec{p}_{\ell_1}| = E_{\ell_1} \tag{18}$$

Now let's do the $\widetilde{\ell}$ decay in the $\widetilde{\ell}$ rest frame (denoted by a star – will need to boost back to the $\widetilde{N}_2$ rest frame at the end!)
Energy & momentum conservation: $m_{\widetilde{\ell}} = E^*_{\ell_2} + E^*_{\widetilde{N}_1}$, $\qquad \vec{p}^{\,*}_{\ell_1} = -\vec{p}^{\,*}_{\widetilde{N}_1}$

$$E^*_{\ell_2} = \frac{1}{2m_{\widetilde{\ell}}}\left(m^2_{\widetilde{\ell}} - m^2_{\widetilde{N}_1}\right) \qquad\qquad |\vec{p}^{\,*}_{\ell_2}| = E^*_{\ell_2} \tag{19}$$

$$E^*_{\widetilde{N}_1} = \frac{1}{2m_{\widetilde{\ell}}}\left(m^2_{\widetilde{\ell}} + m^2_{\widetilde{N}_1}\right) \qquad\qquad |\vec{p}^{\,*}_{\widetilde{N}_1}| = |\vec{p}^{\,*}_{\ell_2}| = E^*_{\ell_2} \tag{20}$$

We have $E^*_{\widetilde{N}_1}$ in the $\widetilde{\ell}$ rest frame; need to boost it to the $\widetilde{N}_2$ rest frame.
Work out the kinematic boost from the $\widetilde{\ell}$ energy and momentum:

$$\gamma = \frac{E_{\widetilde{\ell}}}{m_{\widetilde{\ell}}} = \frac{m^2_{\widetilde{N}_2} + m^2_{\widetilde{\ell}}}{2m_{\widetilde{N}_2} m_{\widetilde{\ell}}}, \qquad\qquad \gamma\beta = \frac{|\vec{p}_{\widetilde{\ell}}|}{m_\ell} = \frac{m^2_{\widetilde{N}_2} - m^2_{\widetilde{\ell}}}{2m_{\widetilde{N}_2} m_{\widetilde{\ell}}} \tag{21}$$

Now do the boost:

$$E_{\widetilde{N}_1} = \gamma\left(E^*_{\widetilde{N}_1} + \beta|\vec{p}^{\,*}_{\widetilde{N}_1}|\cos\theta^*\right) \tag{22}$$

where $\theta^*$ is the angle between the $\widetilde{\ell}$ decay direction and the $\widetilde{\ell}$ boost (in the $\widetilde{\ell}$ rest frame).
Plug in $\gamma$ and $\beta\gamma$:

$$E_{\widetilde{N}_1} = \frac{1}{4m_{\widetilde{N}_2} m_{\widetilde{\ell}}}\left[\left(m^2_{\widetilde{N}_2} + m^2_{\widetilde{\ell}}\right)\left(m^2_{\widetilde{\ell}} + m^2_{\widetilde{N}_1}\right) + \left(m^2_{\widetilde{N}_2} - m^2_{\widetilde{\ell}}\right)\left(m^2_{\widetilde{\ell}} - m^2_{\widetilde{N}_1}\right)\cos\theta^*\right] \tag{23}$$

Remember our original formula for the $\ell\ell$ invariant mass:

$$M_{\ell\ell}^2 = m_{\widetilde{N}_2}^2 + m_{\widetilde{N}_1}^2 - 2m_{\widetilde{N}_2} E_{\widetilde{N}_1} \tag{24}$$

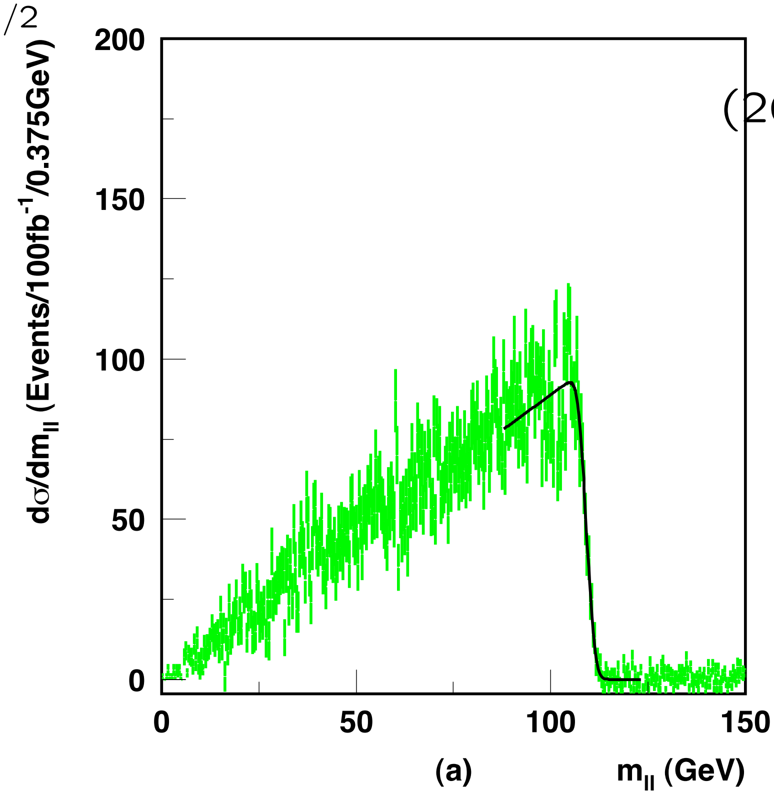
We want a kinematic endpoint: the maximum of $M_{\ell\ell}$.
This corresponds to the minimum of $E_{\widetilde{N}_1}$.
$\rightarrow$ Minimum of $E_{\widetilde{N}_1}$ occurs for $\cos\theta^* = -1$:

$$E_{\widetilde{N}_1}\Big|^{\text{min}} = \frac{1}{2m_{\widetilde{N}_2} m_{\tilde{\ell}}^2}\left(m_{\tilde{\ell}}^4 + m_{\widetilde{N}_2}^2 m_{\widetilde{N}_1}^2\right) \tag{25}$$

Plugging in to $M_{\ell\ell}^2$ formula and simplifying gives

$$M_{\ell\ell}\Big|^{\text{max}} = \left[\frac{\left(m_{\widetilde{N}_2}^2 - m_{\tilde{\ell}}^2\right)\left(m_{\tilde{\ell}}^2 - m_{\widetilde{N}_1}^2\right)}{m_{\tilde{\ell}}^2}\right]^{1/2} \tag{26}$$

One endpoint measurement constrains a combination of three SUSY masses.



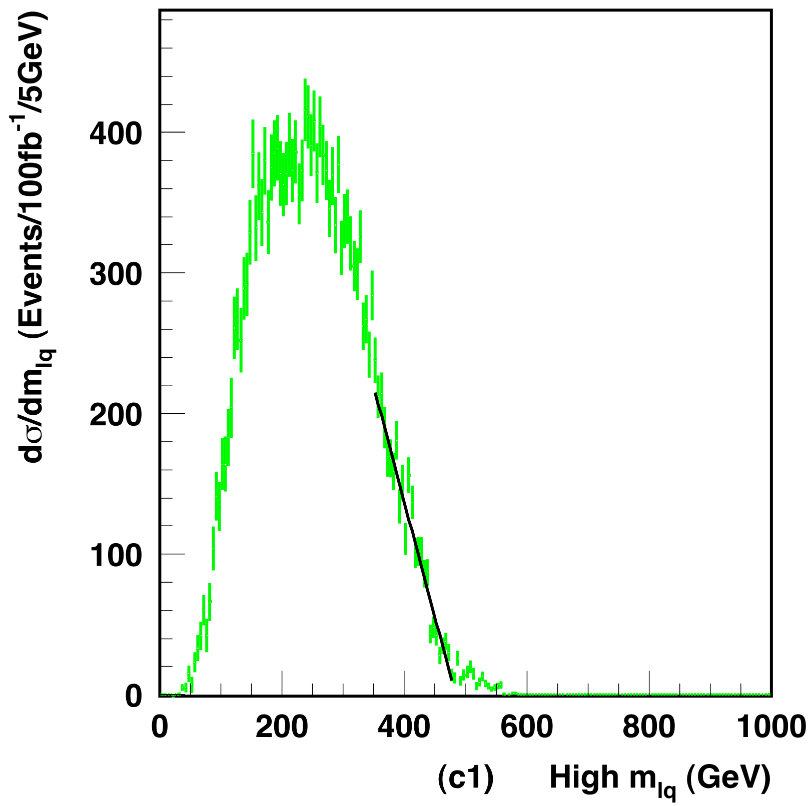

from hep-ph/0211017

Can we do more?
Yes, if we look at longer decay chains.
$\longrightarrow$ more kinematic invariants to play with!

Consider the (longer) decay chain $\tilde{q} \to \tilde{N}_2 q \to \tilde{\ell}^{\pm}\ell^{\mp} q \to \tilde{N}_1 \ell^+ \ell^- q$.
• The invariant mass of $q$ and the first lepton emitted ($\ell_1$) has an endpoint analogous to the $\ell\ell$ endpoint, derived in exactly the same way:

$$M_{q\ell_1}|^{\text{max}} = \left[ \frac{\left(m_{\tilde{q}}^2 - m_{\tilde{N}_2}^2\right)\left(m_{\tilde{N}_2}^2 - m_{\tilde{\ell}}^2\right)}{m_{\tilde{N}_2}^2} \right]^{1/2} \tag{27}$$

How to distinguish $\ell_1$ from $\ell_2$?
$\longrightarrow$ $\ell_1$ likely to have higher energy.
With $M_{q\ell_1}|^{\text{max}}$ and $M_{\ell\ell}|^{\text{max}}$ we have 2 measurements and 4 unknowns.
Not doing better than before... yet.




from hep-ph/0211017

More kinematic invariants:

- How about the invariant mass of $q$ and the two leptons $\ell_1, \ell_2$?

$\widetilde{q} \to \widetilde{N}_2 q \to \widetilde{\ell}^{\pm} \ell^{\mp} q \to \widetilde{N}_1 \ell^+ \ell^- q$

Four-momentum conservation: $p_{\widetilde{q}} = p_q + p_{\ell_1} + p_{\ell_2} + p_{\widetilde{N}_1}$

Get the invariant mass:

$$M_{q\ell\ell}^2 = (p_q + p_{\ell_1} + p_{\ell_2})^2 = (p_{\widetilde{q}} - p_{\widetilde{N}_1})^2 = m_{\widetilde{q}}^2 + m_{\widetilde{N}_1}^2 - 2 p_{\widetilde{q}} \cdot p_{\widetilde{N}_1} \qquad (28)$$

Let's work in the $\widetilde{q}$ rest frame: $p_{\widetilde{q}} \cdot p_{\widetilde{N}_1} = m_{\widetilde{q}} E_{\widetilde{N}_1}$ with $E_{\widetilde{N}_1}$ in the $\widetilde{q}$ rest frame

$$M_{q\ell\ell}^2 = m_{\widetilde{q}}^2 + m_{\widetilde{N}_1}^2 - 2 m_{\widetilde{q}} E_{\widetilde{N}_1} \qquad (29)$$

Want the kinematic endpoint: $M_{q\ell\ell}|^{\text{max}}$

$\to$ find the minimum of $E_{\widetilde{N}_1}$ in the $\widetilde{q}$ rest frame.

This turns out to occur when the 2 leptons are parallel, and back-to-back with the quark.

Two successive Lorentz boosts along the same axis:

(1) $\quad E' = \gamma_1 E + \gamma_1 \beta_1 p \qquad\qquad p' = \gamma_1 p + \gamma_1 \beta_1 E$

Boost $\widetilde{N}_1$ forward to keep the two leptons parallel

(2) $\quad E'' = \gamma_2 E' + \gamma_2 \beta_2 p'$

Boost $\widetilde{N}_1$ backward to put the quark in the forward direction

$E'' = \gamma_2(\gamma_1 E + \gamma_1 \beta_1 p) + \gamma_2 \beta_2 (\gamma_1 p + \gamma_1 \beta_1 E) = \gamma_1 \gamma_2 [(1 - \beta_1 \beta_2) E + (\beta_1 - \beta_2) p]$

The boosts and $E$ and $p$ are:

$$\gamma_1 = \frac{m^2_{\tilde{N}_2} + m^2_{\tilde{\ell}}}{2m_{\tilde{N}_2} m_{\tilde{\ell}}} \qquad \gamma_2 = \frac{m^2_{\tilde{q}} + m^2_{\tilde{N}_2}}{2m_{\tilde{q}} m_{\tilde{N}_2}} \qquad E = \frac{m^2_{\tilde{\ell}} + m^2_{\tilde{N}_1}}{2m_{\tilde{\ell}}} \tag{30}$$

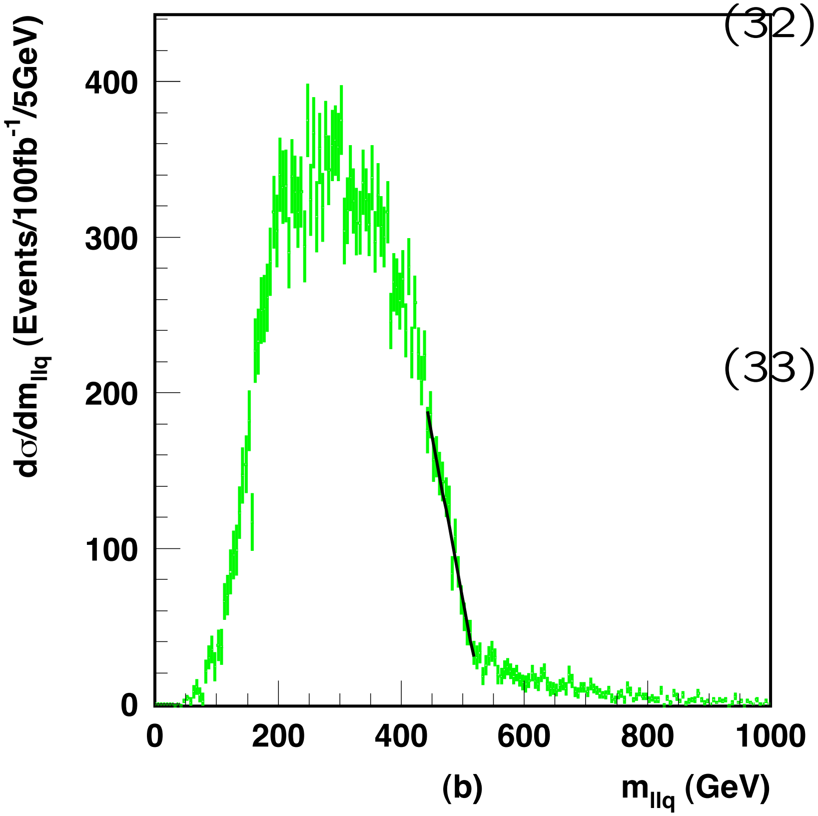
$$\beta_1 = \frac{m^2_{\tilde{N}_2} - m^2_{\tilde{\ell}}}{m^2_{\tilde{N}_2} + m^2_{\tilde{\ell}}} \qquad \beta_2 = \frac{m^2_{\tilde{q}} - m^2_{\tilde{N}_2}}{m^2_{\tilde{q}} + m^2_{\tilde{N}_2}} \qquad p = \frac{m^2_{\tilde{\ell}} - m^2_{\tilde{N}_1}}{2m_{\tilde{\ell}}} \tag{31}$$

Plug in to $E'' = E_{\tilde{N}_1}|^{\rm min}$ in $\tilde{q}$ frame, and crunch through a lot of algebra:

$$E_{\tilde{N}_1}|^{\rm min} = \frac{m^4_{\tilde{N}_2} + m^2_{\tilde{N}_1} m^2_{\tilde{q}}}{2m_{\tilde{q}} m^2_{\tilde{N}_2}} \tag{32}$$

Finally plug into
$M^2_{q\ell\ell}|^{\rm max} = m^2_{\tilde{q}} + m^2_{\tilde{N}_1} - 2m_{\tilde{q}} E_{\tilde{N}_1}|^{\rm min}$:

$$M_{q\ell\ell}|^{\rm max} = \left[ \frac{\left(m^2_{\tilde{q}} - m^2_{\tilde{N}_2}\right)\left(m^2_{\tilde{N}_2} - m^2_{\tilde{N}_1}\right)}{m^2_{\tilde{N}_2}} \right]^{1/2} \tag{33}$$

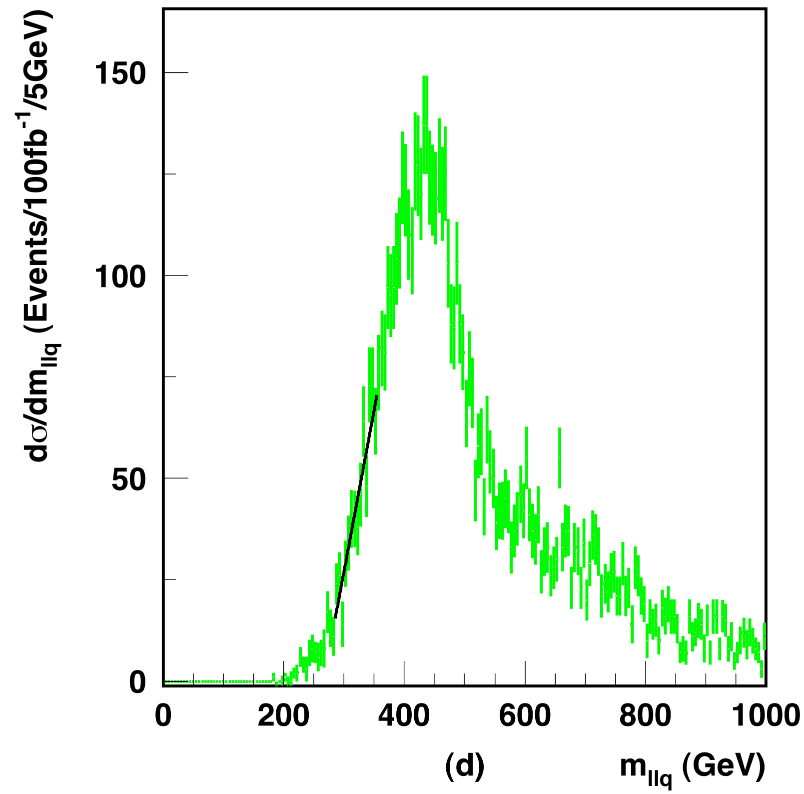



from hep-ph/0211017

So far have 3 measurements and 4 unknowns! Getting better!

- There are also lower kinematic edges.

After putting a cut on $M_{\ell\ell}$, $M_{\ell\ell} > M_{\ell\ell}^{\text{max}}/\sqrt{2}$,
get a complicated formula for a lower kinematic endpoint for $M_{q\ell\ell}$




from hep-ph/0211017

- Can also consider the decay chain $\widetilde{q} \to \widetilde{N}_2 q \to \widetilde{N}_1 h q$ with $h \to b\bar{b}$

Higgs mass can be measured elsewhere.
Then $M_{hq}$ has a threshold (lower kinematic edge)

• Get enough measurables to extract all the masses.
Uncertainties from blurring of the kinematic endpoints by backgrounds, wrong jet/lepton combinations, also gluon radiation off the jet at NLO.

• Statistics are not super: we're only making use of the events right near the kinematic endpoints, and not using the information from the events in the middles of the distributions!

Can we do better?
Each event contains kinematic information; can we use them all?

A new technique: [hep-ph/0410160]
Completely solve the kinematics of each SUSY cascade decay.
Assumptions:

- Selected events are from one particular decay chain
- SUSY particles in the decay chain are on mass shell

Each event gives you the 4-momenta of all the decay products except $\widetilde{N}_1$.

Have to consider a longer decay chain: $\widetilde{g} \to q\widetilde{q} \to qq\widetilde{N}_2 \to qq\ell\widetilde{\ell} \to qq\ell\ell\widetilde{N}_1$.
5 sparticles involved $\to$ 5 mass-shell conditions:

- $m^2_{\widetilde{N}_1} = p^2_{\widetilde{N}_1}$
- $m^2_{\widetilde{\ell}} = (p_{\widetilde{N}_1} + p_{\ell_1})^2$
- $m^2_{\widetilde{N}_2} = (p_{\widetilde{N}_1} + p_{\ell_1} + p_{\ell_2})^2$
- $m^2_{\widetilde{q}} = (p_{\widetilde{N}_1} + p_{\ell_1} + p_{\ell_2} + p_{q_1})^2$
- $m^2_{\widetilde{g}} = (p_{\widetilde{N}_1} + p_{\ell_1} + p_{\ell_2} + p_{q_1} + p_{q_2})^2$

Each $qq\ell\ell\widetilde{N}_1$ event contains 4 unmeasured degrees of freedom, the 4 components of the $\widetilde{N}_1$ 4-momentum.
$\to$ Each event picks out a 4-dimensional hypersurface in a 5-dimensional mass parameter space.
Overlap multiple events in this hyperspace $\to$ find a discrete set of solutions from overlap of different hypersurfaces.