The Higgs sector

Recall the scalar potential (at tree-level):

$$
\begin{aligned}
V &= \left(|\mu|^2 + m_{H_u}^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) \\
&\quad + \left[b\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \text{h.c.}\right] \\
&\quad + \frac{1}{8}\left(g^2 + g'^2\right)\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 \\
&\quad + \frac{1}{2} g^2 \left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 \qquad (1)
\end{aligned}
$$

Minimize $\rightarrow$ physical Higgs masses and the h^0-H^0 mixing angle α:

$$m_{A^0}^2 = 2b/\sin 2\beta \qquad (2)$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 \qquad (3)$$

$$m_{h^0,H^0}^2 = \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4 m_Z^2 m_{A^0}^2 \cos^2 2\beta}\right) \qquad (4)$$

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_{A^0}^2 + m_Z^2}{m_{H^0}^2 - m_{h^0}^2} \qquad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_{A^0}^2 - m_Z^2}{m_{H^0}^2 - m_{h^0}^2} \qquad (5)$$

How many free parameters are there?

- b term
- $(|\mu|^2 + m_{H_u}^2)$
- $(|\mu|^2 + m_{H_d}^2)$

One combination determines $v_u^2 + v_d^2 = v^2$ – already known from the Z mass.
That leaves two free parameter combinations:

usually chosen to be m_{A^0} and $\tan\beta$.

This only works at tree level. Once radiative corrections are included, other SUSY parameters enter into the Higgs sector. E.g.:

$$\Delta(m_{h^0}^2) \simeq \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) \qquad (6)$$

Higgs couplings to SM fermions

Recall we had

$$\mathcal{L} = -y_t \bar{u}_3 Q H_u + y_b \bar{d}_3 Q H_d + y_\tau \bar{e}_3 L H_d \quad (7)$$

$$= -y_t \left(\bar{t} t H_u^0 - \bar{t} b H_u^+\right) + y_b \left(\bar{b} t H_d^- - \bar{b} b H_d^0\right) + y_\tau \left(\bar{\tau} \nu_\tau H_d^- - \bar{\tau} \tau H_d^0\right) \quad (8)$$

Using $v_u = \langle H_u^0 \rangle v \sin\beta$ and $v_d = \langle H_d^0 \rangle = v \cos\beta$ and $m_W = gv/\sqrt{2}$, we can solve for the Yukawa couplings in terms of the fermion masses:

$$y_t = \frac{g m_t}{\sqrt{2} m_W \sin\beta} \qquad y_b = \frac{g m_b}{\sqrt{2} m_W \cos\beta} \qquad y_\tau = \frac{g m_\tau}{\sqrt{2} m_W \cos\beta} \quad (9)$$

If $\tan\beta \gg 1$ then y_b and y_τ get enhanced.

Couplings of the Higgs mass eigenstates:

$$g_{h^0 \bar{t} t} = \frac{g m_t}{2 m_W} \frac{\cos\alpha}{\sin\beta} \qquad g_{h^0 \bar{b} b} = -\frac{g m_b}{2 m_W} \frac{\sin\alpha}{\cos\beta} \quad (10)$$

$$g_{H^0 \bar{t} t} = \frac{g m_t}{2 m_W} \frac{\sin\alpha}{\sin\beta} \qquad g_{H^0 \bar{b} b} = \frac{g m_b}{2 m_W} \frac{\cos\alpha}{\cos\beta} \quad (11)$$

$$g_{A^0 \bar{t} t} = \frac{i g m_t}{2 m_W} \cot\beta \gamma^5 \qquad g_{A^0 \bar{b} b} = \frac{i g m_b}{2 m_W} \tan\beta \gamma^5 \quad (12)$$

$$g_{H^+ \bar{t}_R b_L} = \frac{g m_t}{\sqrt{2} m_W} \cot\beta \qquad g_{H^+ \bar{t}_L b_R} = \frac{g m_b}{\sqrt{2} m_W} \tan\beta \quad (13)$$

Decoupling limit

An interesting limit occurs for $m_{A^0} \gg m_Z$: the so-called decoupling limit.
In the decoupling limit:

- $m_{A^0} \simeq m_{H^0} \simeq m_{H^\pm} \gg m_Z$
- m_{h^0} saturates its upper bound; $m_{h^0} \simeq m_Z |\cos 2\beta|$ at tree level
- $\alpha \simeq \beta - \pi/2$:
    - A^0, H^0, $H^\pm$ live together in one linear combination of H_u and H_d
    - h^0 lives in the other linear combination of H_u and H_d, which also contains the Goldstone bosons G^0, $G^\pm$ and the vev $v = \sqrt{v_u^2 + v_d^2}$
    - The couplings of h^0 become the same as the couplings of the Standard Model Higgs

Then the Higgs sector looks like:

- One light SM-like Higgs h^0
- A multiplet A^0, H^0, $H^\pm$ of heavy Higgses that don't affect the low-energy physics very much.

Need high-precision measurements of the h^0 couplings to distinguish it from the SM!

The details:

$\cos(\beta-\alpha)$ goes to zero in the limit $m_{A^0} \gg m_Z$:

$$\cos(\beta-\alpha) \simeq \frac{1}{2}\sin 4\beta \frac{m_Z^2}{m_{A^0}^2} \tag{14}$$

The h^0 couplings can be rewritten in a useful form in terms of $\cos(\beta-\alpha)$:

$$g_{h^0\bar{t}t} = \frac{g m_t}{2 m_W}\left[\sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\right] \tag{15}$$

$$g_{h^0\bar{b}b} = \frac{g m_b}{2 m_W}\left[\sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\right] \tag{16}$$

$$g_{h^0W^+W^-} = g m_W \sin(\beta-\alpha) \tag{17}$$

$$g_{h^0ZZ} = \frac{g m_Z}{\cos\theta_W}\sin(\beta-\alpha) \tag{18}$$

These all approach their SM values in the limit $\cos(\beta-\alpha) \to 0$.

Neutralinos and Charginos

The Higgsinos and the electroweak gauginos mix with each other because of the effects of electroweak symmetry breaking.

- Neutral Higgsinos $\widetilde{H}_u^0$, $\widetilde{H}_d^0$ and neutral gauginos $\widetilde{B}$, $\widetilde{W}^0$ mix:

  form four neutral mass eigenstates called neutralinos, $\widetilde{N}_{1,2,3,4}$ or $\widetilde{\chi}^0_{1,2,3,4}$.

  - Convention: $m_{\widetilde{N}_1} < m_{\widetilde{N}_2} < m_{\widetilde{N}_3} < m_{\widetilde{N}_4}$
  - The lightest neutralino $\widetilde{N}_1$ is usually assumed to be the LSP.

- Charged Higgsinos $\widetilde{H}_u^+$, $\widetilde{H}_d^-$ and winos $\widetilde{W}^+$, $\widetilde{W}^-$ mix:

  form two charged mass eigenstates called charginos, $\widetilde{C}_{1,2}$ or $\widetilde{\chi}^\pm_{1,2}$.

  - Convention: $m_{\widetilde{C}_1} < m_{\widetilde{C}_2}$

Neutralino mass terms:

$$\mathcal{L} = -\frac{1}{2}\left(\psi^0\right)^T \mathbf{M}_{\widetilde{N}}\psi^0 + \text{h.c.} \tag{19}$$

where $\psi^0 = (\widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$.

The mass matrix is:

$$\mathbf{M}_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix} \tag{20}$$

Abbreviations: $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, $s_W = \sin\theta_W$, $c_W = \cos\theta_W$

Danger: Some people use a different sign convention for μ. Looking for the $-\mu$ in the neutralino mass matrix tells you which convention they're using.

The neutralino mass matrix is diagonalized by a unitary matrix $\mathbf{N}$: $\widetilde{N}_i = \mathbf{N}_{ij}\psi^0_j$:

$$\mathbf{M}^{diag}_{\widetilde{N}} = \mathbf{N}^* \mathbf{M}_{\widetilde{N}} \mathbf{N}^{-1} \tag{21}$$

where $\mathbf{M}^{diag}_{\widetilde{N}}$ is defined to have positive real entries along the diagonal:

$$\mathbf{M}^{diag}_{\widetilde{N}} = \begin{pmatrix} m_{\widetilde{N}_1} & & & \\ & m_{\widetilde{N}_2} & & \\ & & m_{\widetilde{N}_3} & \\ & & & m_{\widetilde{N}_4} \end{pmatrix} \tag{22}$$

$m_{\widetilde{N}_i}$ are the absolute values of the eigenvalues of $\mathbf{M}_{\widetilde{N}}$
or equivalently, $m^2_{\widetilde{N}_i}$ are the eigenvalues of $\mathbf{M}^\dagger_{\widetilde{N}}\mathbf{M}_{\widetilde{N}}$
Caution: some people use a convention in which $m_{\widetilde{N}_i}$ can be negative or even complex, to get a simpler mixing matrix $\mathbf{N}$.

$\mathbf{M}_{\widetilde{N}}$ has 4 free parameters: M_1, M_2, μ, and $\tan\beta$.
If gaugino mass unification holds, so that $M_1 \simeq 0.5 M_2$ due to the RGEs, this reduces to 3.

If $m_Z \ll |\mu \pm M_1|, |\mu \pm M_2|$ then the gaugino-Higgsino mixing is small.
Then (assuming also $M_1 < M_2 < |\mu|$)

$$\widetilde{N}_1 \approx \widetilde{B}, \qquad m_{\widetilde{N}_1} \approx M_1 + \cdots \tag{23}$$

$$\widetilde{N}_2 \approx \widetilde{W}^0, \qquad m_{\widetilde{N}_2} \approx M_2 + \cdots \tag{24}$$

$$\widetilde{N}_3, N_4 \approx \frac{1}{\sqrt{2}}\left(\widetilde{H}^0_u \pm \widetilde{H}^0_d\right), \qquad m_{\widetilde{N}_3}, m_{\widetilde{N}_4} \approx |\mu| + \cdots \tag{25}$$

Chargino mass terms:

$$\mathcal{L} = -\frac{1}{2}\left(\psi^{\pm}\right)^T \mathbf{M}_{\widetilde{C}} \psi^{\pm} + \text{h.c.} \tag{26}$$

where $\psi^{\pm} = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$.
The mass matrix is (in 2×2 block form):

$$\mathbf{M}_{\widetilde{C}} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix}, \qquad \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} s_\beta m_W \\ \sqrt{2} c_\beta m_W & \mu \end{pmatrix} \tag{27}$$

The chargino mass matrix is diagonalized by *two* unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$, one acting on the $+$ charged states and one acting on the $-$ charged states:

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix} \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix} \tag{28}$$

$\mathbf{U}$ and $\mathbf{V}$ diagonalize the mass matrix:

$$\mathbf{U}^* \mathbf{X} \mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix} \tag{29}$$

$m^2_{\widetilde{C}_i}$ are the eigenvalues of $\mathbf{X}^\dagger \mathbf{X}$.

If $m_Z \ll |\mu \pm M_2|$ then the gaugino-Higgsino mixing is small.
Then (assuming also $M_2 < |\mu|$)

$$\widetilde{C}_1^{\pm} \approx \widetilde{W}^{\pm}, \qquad m_{C_1} \approx M_2 \tag{30}$$

$$\widetilde{C}_2^+ \approx \widetilde{H}_u^+, \quad \widetilde{C}_2^- \approx \widetilde{H}_d^-, \qquad m_{C_2} \approx |\mu| \tag{31}$$

In practice the neutralino and chargino masses and mixing angles are best computed numerically.

Squarks and sleptons

In principle, all of the states with the same electric charge, R-parity, and colour can mix with each other:

$(\widetilde{u}_L, \widetilde{c}_L, \widetilde{t}_L, \widetilde{u}_R, \widetilde{c}_R, \widetilde{t}_R) \to$ 6×6 mass-squared matrix
$(\widetilde{d}_L, \widetilde{s}_L, \widetilde{b}_L, \widetilde{d}_R, \widetilde{s}_R, \widetilde{b}_R) \to$ 6×6 mass-squared matrix
$(\widetilde{e}_L, \widetilde{\mu}_L, \widetilde{\tau}_L, \widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R) \to$ 6×6 mass-squared matrix
$(\widetilde{\nu}_e, \widetilde{\nu}_\mu, \widetilde{\nu}_\tau) \to$ 3×3 mass-squared matrix

Fortunately, for flavour-blind soft parameters, the inter-generation mixing is very small.

3rd family (t, b, τ) can have very different masses than first 2 families because of Yukawa and a-term contribution in RGEs.

Can also have substantial L-R mixing – see below

1st and 2nd generation sfermions end up in nearly degenerate, unmixed pairs:
$(\widetilde{e}_R, \widetilde{\mu}_R)$, $(\widetilde{\nu}_e, \widetilde{\nu}_\mu)$, $(\widetilde{u}_R, \widetilde{c}_R)$, $(\widetilde{d}_R, \widetilde{s}_R)$, $(\widetilde{u}_L, \widetilde{c}_L)$, $(\widetilde{d}_L, \widetilde{s}_L)$

A second look at RGE solution for first two families (for universal m_0^2):

$$m^2_{Q_1} = m^2_{Q_2} = m_0^2 + K_3 + K_2 + \frac{1}{36}K_1, \tag{32}$$

$$m^2_{\bar{u}_1} = m^2_{\bar{u}_2} = m_0^2 + K_3 \qquad + \frac{4}{9}K_1, \tag{33}$$

$$m^2_{\bar{d}_1} = m^2_{\bar{d}_2} = m_0^2 + K_3 \qquad + \frac{1}{9}K_1, \tag{34}$$

$$m^2_{L_1} = m^2_{L_2} = m_0^2 \qquad + K_2 + \frac{1}{4}K_1, \tag{35}$$

$$m^2_{\bar{e}_1} = m^2_{\bar{e}_2} = m_0^2 \qquad + K_1. \tag{36}$$

$K_3 \gg K_2 \gg K_1$; come from SU(3), SU(2), U(1) gaugino contributions

Squarks generically heavier than sleptons.

Also a "hyperfine splitting" due D-term (sfermion)^2(Higgs)^2 contribution:

$$\Delta_\phi = \left(T_3^\phi - Q_{EM}^\phi \sin^2\theta_W \right) \cos 2\beta m_Z^2 \tag{37}$$

Splits members of Q, L by small amount: $\widetilde{u}_L$-$\widetilde{d}_L$, $\widetilde{e}_L$-$\widetilde{\nu}_e$

Within a single family, $\widetilde{q}_L$-$\widetilde{q}_R$ mixing can only arise through involvement of a Higgs coupling y_q or a_q. Only important for 3rd family! E.g. for the stops:

$$\mathcal{L} = -\left(\widetilde{t}_L^*, \widetilde{t}_R^*\right) \mathbf{m}^2_{\widetilde{\mathbf{t}}} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix} \tag{38}$$

where

$$\mathbf{m}^2_{\widetilde{\mathbf{t}}} = \begin{pmatrix} m^2_{Q_3} + m_t^2 + \Delta_u & v(a_t \sin\beta - \mu y_t \cos\beta) \\ v(a_t \sin\beta - \mu y_t \cos\beta) & m^2_{\bar{u}_3} + m_t^2 + \Delta_{\bar{u}} \end{pmatrix} \tag{39}$$

Diagonalize mass-squared matrix → mass eigenstates ($m^2_{\widetilde{t}_1} < m^2_{\widetilde{t}_2}$, $0 \leq \theta_{\widetilde{t}} \leq \pi$):

$$\begin{pmatrix} \widetilde{t}_1 \\ \widetilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\widetilde{t}} & \sin\theta_{\widetilde{t}} \\ -\sin\theta_{\widetilde{t}} & \cos\theta_{\widetilde{t}} \end{pmatrix} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix} \tag{40}$$

RG effects tend to make $m^2_{\bar{u}_3} < m^2_{Q_3}$, and both of these lighter than first 2 families. Mixing splits $m^2_{\widetilde{t}_1}$ and $m^2_{\widetilde{t}_2}$ even more.

Often find $\widetilde{t}_1$ to be the lightest squark in the model!

Analogous mixing for sbottoms $\widetilde{b}_1, \widetilde{b}_2$ and staus $\widetilde{\tau}_1, \widetilde{\tau}_2$.

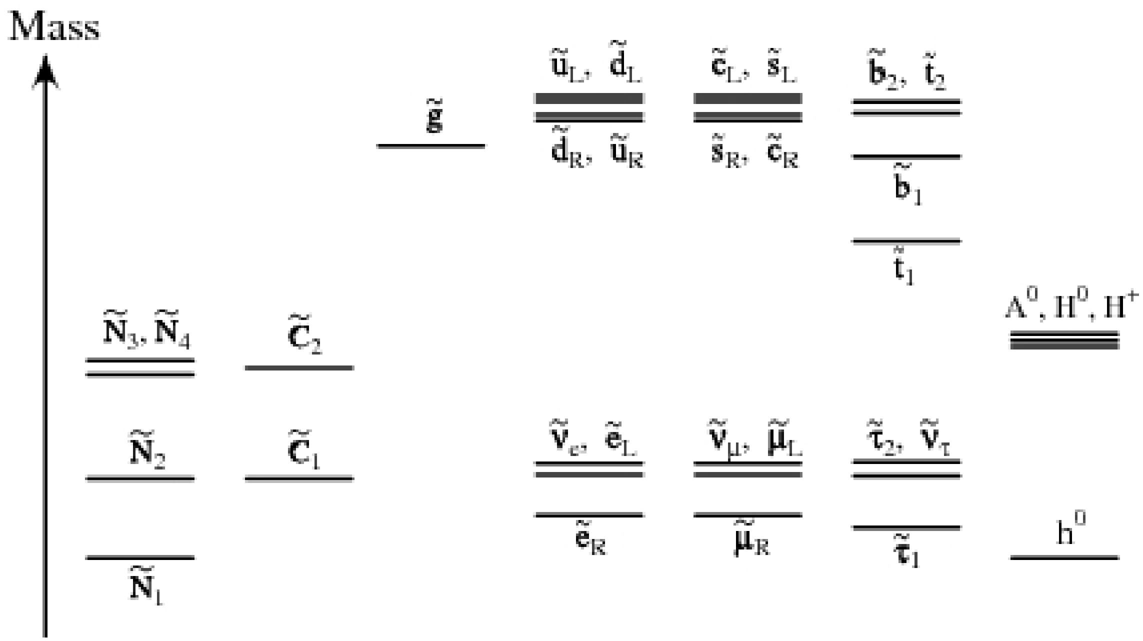
y_b, y_τ and a_b, a_τ in off-diagonal terms: mixing more important at large $\tan\beta$.

$\widetilde{b}_1$, $\widetilde{\tau}_1$ can be significantly lighter than 1st and 2nd family
Sometimes get $\widetilde{\tau}_1$ as lightest sfermion
– e.g., "stau coannihilation region" for dark matter

Summary: the particle content of the MSSM

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	$+1$	$h^0\ H^0\ A^0\ H^\pm$	$H_u^0\ H_d^0\ H_u^+\ H_d^-$
squarks	0	-1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$ $\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$ $\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$	" " " " $\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$
sleptons	0	-1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$ $\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$ $\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$	" " " " $\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$
neutralinos	1/2	-1	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$
charginos	1/2	-1	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$
gluino	1/2	-1	$\widetilde{g}$	" "
gravitino/ goldstino	3/2	-1	$\widetilde{G}$	" "

A schematic sample SUSY spectrum:
(This may or may not have anything to do with reality)



Some features of this sample spectrum:

- $\widetilde{N}_1$ is LSP
- $\widetilde{t}_1$ and $\widetilde{b}_1$ are the lightest squarks
- $\widetilde{\tau}_1$ is the lightest charged slepton
- Coloured particles are heavier than uncoloured particles