The mass spectrum of the MSSM

The SUSY breaking terms get set at a high scale $\gg$ TeV. $\leftarrow$ input

Gravity mediated: M_P

Gauge mediated: $M_{mess} \gg$ TeV

Use renormalization group equations (RGEs) to determine the parameters of the Lagrangian at the EW scale.

Must "run down" the parameters to the low scale.

SUSY breaking terms serve as boundary conditions at high scale.

Predict mass spectrum, mixing angles, interactions of new particles. $\leftarrow$ output

Gauge couplings: Running is given by the beta functions b_a.

$$\frac{d}{dt} g_a = \frac{1}{16\pi^2} b_a g_a^3 \tag{1}$$

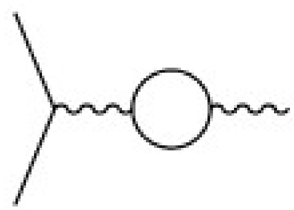
or equivalently

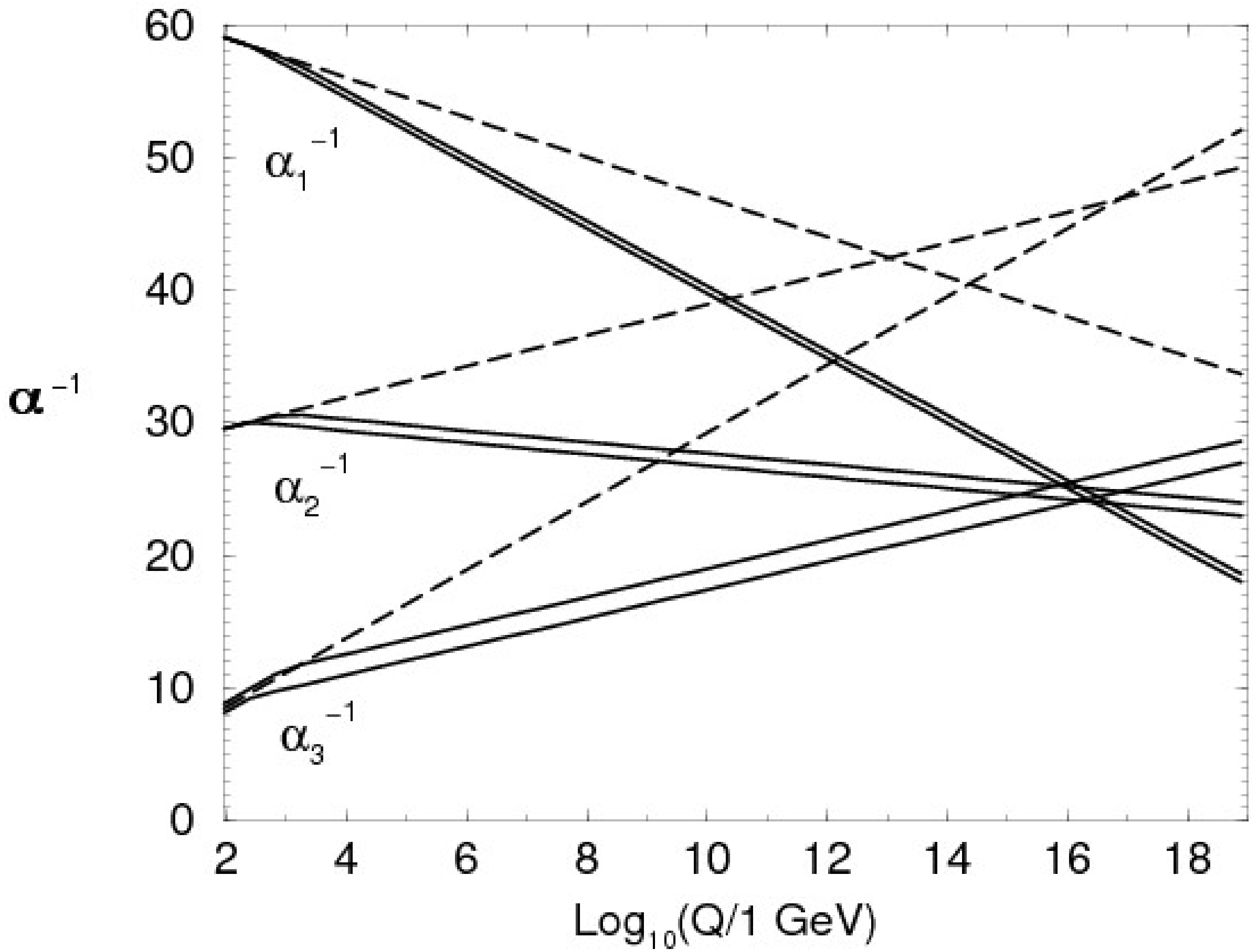
$$\frac{d}{dt} \alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad (a = 1, 2, 3) \tag{2}$$

where

$$t = \ln(Q/Q_0) \qquad b_a^{SM} = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) \qquad b_a^{MSSM} = \left(\frac{33}{5}, 1, -3\right) \tag{3}$$

(Q is the "current" scale; Q_0 is the starting scale)





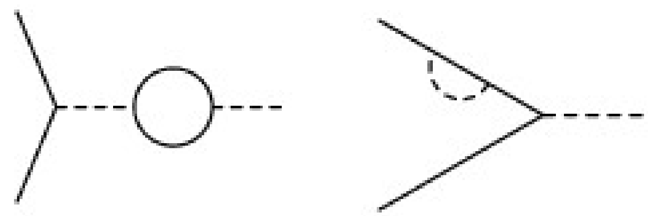
Dashed lines: SM   Solid lines: MSSM

(Bands are the uncertainties in the low-energy values.)

Superpotential parameters:

$$W_{MSSM} = \overline{u}\mathbf{y_u}QH_u - \overline{d}\mathbf{y_d}QH_d - \overline{e}\mathbf{y_e}LH_d + \mu H_u H_d \tag{4}$$

$\mathbf{y_u}$, $\mathbf{y_d}$, $\mathbf{y_e}$ are 3×3 matrices.



These also run with scale.
Approximation: only the 3rd generation Yukawa couplings are significant.

$$\frac{d}{dt}y_t = \frac{y_t}{16\pi^2}\left[6|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right] \tag{5}$$

$$\frac{d}{dt}y_b = \frac{y_b}{16\pi^2}\left[6|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right] \tag{6}$$

$$\frac{d}{dt}y_\tau = \frac{y_\tau}{16\pi^2}\left[4|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5}g_1^2\right] \tag{7}$$

$$\frac{d}{dt}\mu = \frac{\mu}{16\pi^2}\left[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2\right] \tag{8}$$

Beta-functions for each superpotential parameter are proportional to the parameter itself

e.g., if μ starts out zero, it stays zero.

RG corrections to superpotential parameters are logarithmic.

[Actually a consequence of a SUSY nonrenormalization theorem – can prove that the log divergent contributions can be written as wave-function renormalization, with no vertex renormalization.]

Gaugino mass parameters:

1-loop RGEs determined by same b_a^{MSSM} as gauge coupling RGEs:

$$\frac{d}{dt}M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a \qquad \left(b_a = \frac{33}{5}, 1, -3\right) \tag{9}$$

Ratios M_a/g_a^2 are constant (scale independent) up to small 2-loop corrections.

In mSUGRA models the gaugino masses unify:

$$M_a(Q) = \frac{g_a^2(Q)}{g_a^2(Q_0)} m_{1/2} \qquad (a = 1, 2, 3) \tag{10}$$

Q_0 is the input scale, $\sim M_P$.
Gauge couplings unify at $M_U \approx 0.01 M_P$, so in mSUGRA:

$$g_1^2(Q_0) \approx g_2^2(Q_0) \approx g_3^2(Q_0) \qquad \rightarrow \qquad \frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = \frac{m_{1/2}}{g_U^2} \tag{11}$$

This means the low-scale gaugino mass params satisfy unification relations:

$$M_1 = \frac{g_1^2}{g_2^2} M_2 \simeq 0.5 M_2 \tag{12}$$

$$M_3 = \frac{g_3^2}{g_2^2} M_2 \simeq 3.5 M_2 \tag{13}$$

Note that $g_1 = \sqrt{5/3} g'$: GUT normalization.
These relations can be avoided in models in which the gaugino masses do not unify at the GUT scale; e.g. gauge mediated models.

a-terms (analytic soft parameters):

$$\mathcal{L}_{soft}^{MSSM} = -\tilde{\bar{u}}\mathbf{a_u}\tilde{Q}H_u + \tilde{\bar{d}}\mathbf{a_d}\tilde{Q}H_d + \tilde{\bar{e}}\mathbf{a_e}\tilde{L}H_d + \text{h.c.} \tag{14}$$

$\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ are 3×3 matrices – at SUSY breaking scale they are proportional to Yukawa matrices in many models.
Approximation: only the 3rd generation Yukawa couplings are significant.

$$\begin{aligned} 16\pi^2 \frac{d}{dt} a_t &= a_t \left[18|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right] \\ &\quad +2a_b y_b^* y_t + y_t \left[\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1 \right] \end{aligned} \tag{15}$$

$$\begin{aligned} 16\pi^2 \frac{d}{dt} a_b &= a_b \left[18|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right] \\ &\quad +2a_t y_t^* y_b + 2a_\tau y_\tau^* y_b + y_b \left[\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1 \right] \end{aligned} \tag{16}$$

$$\begin{aligned} 16\pi^2 \frac{d}{dt} a_\tau &= a_\tau \left[12|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5}g_1^2 \right] \\ &\quad +6a_b y_b^* y_\tau + y_\tau \left[6g_2^2 M_2 + \frac{18}{5}g_1^2 M_1 \right] \end{aligned} \tag{17}$$

Note that the RGEs for the a-terms are not proportional to the parameters themselves.
These coups violate SUSY: not protected by SUSY nonrenormalization theorem.

b-term:

$$\mathcal{L}_{soft}^{MSSM} = -bH_uH_d + \text{h.c.} \tag{18}$$

b has dimensions of mass-squared.

$$\begin{aligned} 16\pi^2 \frac{d}{dt} b &= b\left[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2\right] \\ &\quad + \mu\left[6a_t y_t^* + 6a_b y_b^* + 2a_\tau y_\tau^* + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1\right] \end{aligned} \tag{19}$$

Note that the RGE for the b-term is not proportional to b itself.
This coup violates SUSY: not protected by SUSY nonrenormalization theorem.

Scalar masses: Higgs sector

$$\mathcal{L}_{soft}^{MSSM} = -m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d \quad (20)$$

The RGEs are

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3X_t - 6g_2^2|M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 \quad (21)$$

$$16\pi^2 \frac{d}{dt} m_{H_d}^2 = 3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 \quad (22)$$

where we define some convenient parameter combinations,

$$X_t = 2|y_t|^2(m_{H_u}^2 + m_{Q_3}^2 + m_{\tilde{u}_3}^2) + 2|a_t|^2 \quad (23)$$

$$X_b = 2|y_b|^2(m_{H_d}^2 + m_{Q_3}^2 + m_{\tilde{d}_3}^2) + 2|a_b|^2 \quad (24)$$

$$X_\tau = 2|y_\tau|^2(m_{H_d}^2 + m_{L_3}^2 + m_{\tilde{e}_3}^2) + 2|a_\tau|^2 \quad (25)$$

Note that X_t, X_b, X_τ are always positive:
In the RGEs they decrease the Higgs masses as you evolve down from the GUT scale.
Because of this you can start with positive m_{H_u} and m_{H_d} at the GUT scale, and wind up with one of them going negative by the time you get to the EW scale.
This is called radiative electroweak symmetry breaking and is usually caused by X_t because y_t is large.

Scalar masses: Squark/slepton sector

$$\mathcal{L}_{soft}^{MSSM} = -\tilde{Q}^\dagger \mathbf{m_Q^2} \tilde{Q} - \tilde{L}^\dagger \mathbf{m_L^2} \tilde{L} - \tilde{\bar{u}} \mathbf{m_{\bar{u}}^2} \tilde{\bar{u}}^\dagger - \tilde{\bar{d}} \mathbf{m_{\bar{d}}^2} \tilde{\bar{d}}^\dagger - \tilde{\bar{e}} \mathbf{m_{\bar{e}}^2} \tilde{\bar{e}}^\dagger \quad (26)$$

$\mathbf{m_Q^2}$, $\mathbf{m_L^2}$, $\mathbf{m_{\bar{u}}^2}$, $\mathbf{m_{\bar{d}}^2}$, and $\mathbf{m_{\bar{e}}^2}$ are 3×3 matrices in flavour space.

In many models, the squark/slepton masses start out universal (same for all 3 generations) and flavor-diagonal (no mixing terms between different generations).

Such SUSY breaking is called flavor-blind. It's nice because it doesn't lead to large FCNC.

The only thing that can change this in the RGE running is the Yukawa couplings.

Neglect all but the 3rd generation Yukawas → scalar masses stay almost diagonal:

$$\mathbf{m_Q^2} = \begin{pmatrix} m_{Q_1}^2 & 0 & 0 \\ 0 & m_{Q_1}^2 & 0 \\ 0 & 0 & m_{Q_3}^2 \end{pmatrix}, \qquad \mathbf{m_{\bar{u}}^2} = \begin{pmatrix} m_{\bar{u}_1}^2 & 0 & 0 \\ 0 & m_{\bar{u}_1}^2 & 0 \\ 0 & 0 & m_{\bar{u}_3}^2 \end{pmatrix}, \qquad \text{etc.} \quad (27)$$

For the first two generations,

$$16\pi^2 \frac{d}{dt} m_\phi^2 = -\sum_{a=1,2,3} 8 g_a^2 C_a^\phi |M_a|^2 \quad (28)$$

where C_a^ϕ are the quadratic Casimir invariants: $C_3^\phi = 4/3$ for colored scalars, $C_2^\phi = 3/4$ for SU(2)-doublet scalars, and $C_1^\phi = 3Y_\phi^2/5$ with Y_ϕ the hypercharge.

Scalar masses: Squark/slepton sector, continued

For the third generation, we don't neglect the Yukawa couplings:

$$16\pi^2 \frac{d}{dt} m^2_{Q_3} = X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 \quad (29)$$

$$16\pi^2 \frac{d}{dt} m^2_{\bar{u}_3} = 2X_t - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2 \quad (30)$$

$$16\pi^2 \frac{d}{dt} m^2_{\bar{d}_3} = 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 \quad (31)$$

$$16\pi^2 \frac{d}{dt} m^2_{L_3} = X_\tau - 6 g_2^2 |M_2|^2 - \frac{3}{5} g_1^2 |M_1|^2 \quad (32)$$

$$16\pi^2 \frac{d}{dt} m^2_{\bar{e}_3} = 2X_\tau - \frac{24}{5} g_1^2 |M_1|^2 \quad (33)$$

The coefficients of $g^2_{1,2,3}$ are just the Casimir invariants mentioned before.

The top squarks get an X_t contribution just like H_u – might they also run negative?

- Coefficient for H_u was 3, here is 1 or 2: smaller negative contribution
- Big contribution from $|M_3|^2$ runs the squark masses upward.

Notice if soft masses all start out the same at GUT scale, squarks become heavier than sleptons at EW scale due to the $|M_3|^2$ contribution to the running.
Similarly $M_3 \simeq 3.5 M_2$ for models with gaugino mass unification.
$\rightarrow$ Expect coloured sparticles to be heavier than non-coloured sparticles.

LHC: Produce heavy coloured particles via QCD; lighter non-coloured particles harder to see – lower rates.
ILC: Produce lighter non-coloured particles via EW interactions; heavy coloured particles beyond kinematic reach.
Complementarity!

Implementation of RGEs

The SUSY RGEs are implemented in an assortment of computer codes.
In the codes, you typically specify some high-scale SUSY parameters, the code churns, and then it spits out the low-scale SUSY spectrum.
What are the codes doing?

High-scale: use a specific SUSY breaking mechanism to limit the number of independent parameters
Low-scale: we measure g_1, g_2, g_3, M_Z at the EW scale: "predicted" in terms of high-scale inputs.

Since these SM parameters are already measured, there are some constraints on the high-scale parameters: not everything is free.

Often the b term is dropped in favour of M_Z.

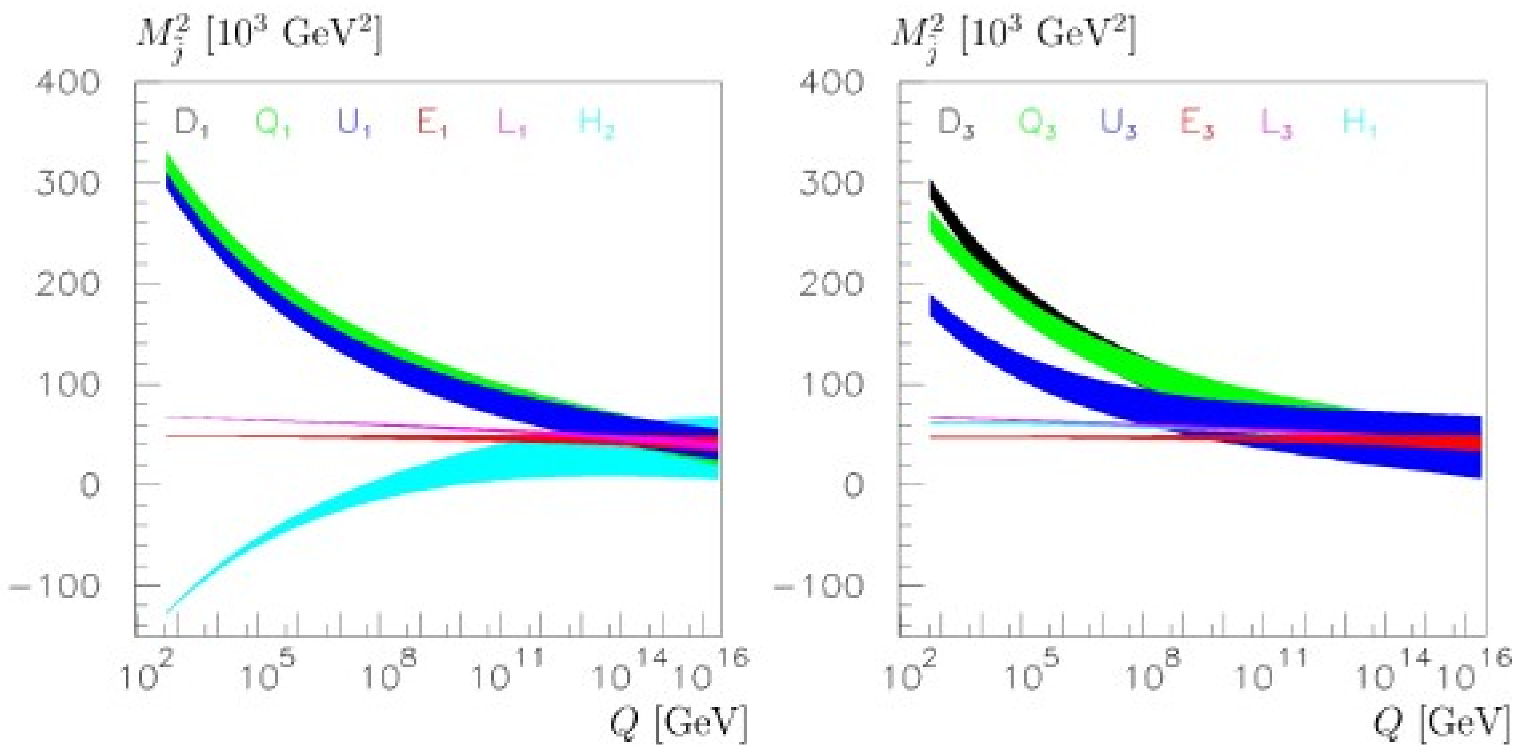
The codes typically run the parameters up and down between the EW and GUT scales 2 or 3 times, tweaking parameter values until the low-energy parameters come out right.

"Einstein's Telescope"

The RGEs will let us extrapolate the high-scale physics based on measurements of the EW scale parameters.

Need high precision: experimental uncertainties can be amplified by the RGE running.

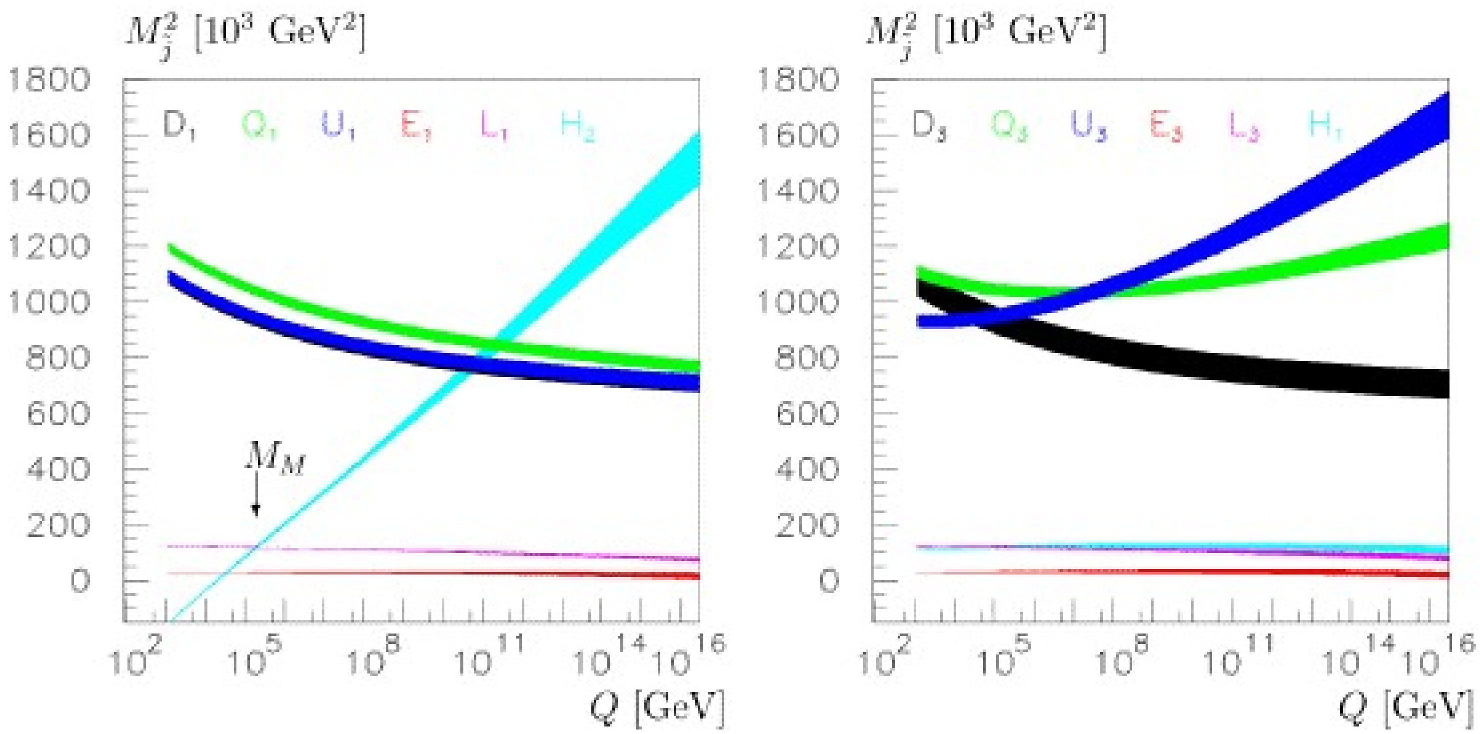
mSUGRA model:



from Blair, Porod & Zerwas, hep-ph/0210058

Run soft-SUSY-breaking parameters up, see if they unify at the high scale!

Contrast GMSB model:



from Blair, Porod & Zerwas, hep-ph/0210058

Soft-SUSY-breaking parameters do not unify:
they are related to beta-functions at the messenger scale M_M.

Hope is to learn about high-scale physics from low-scale SUSY spectrum.

Electroweak symmetry breaking and the Higgs bosons

The MSSM has two Higgs doublets, $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$.

8 d.o.f. $\rightarrow$ 3 longitudinal gauge bosons $(G^0, G^\pm)$ + 5 physical states:

- one CP-odd neutral scalar A^0
- two CP-even neutral scalars h^0, H^0
- a charged Higgs pair $H^\pm$ (2 d.o.f.)

The two Higgs doublets have vacuum expectation values (vevs)

$$\langle H_u^0 \rangle = v_u, \qquad \langle H_d^0 \rangle = v_d \tag{34}$$

(Sometimes these are defined with a $1/\sqrt{2}$ on the right-hand side.)

Sum of squares fixed by Z mass:

$$v_u^2 + v_d^2 = v_{SM}^2 = \frac{2M_Z^2}{(g^2 + g'^2)} \simeq (174 \text{ GeV})^2 \tag{35}$$

Ratio is a free parameter (a key parameter in SUSY!):

$$v_u / v_d \equiv \tan\beta \tag{36}$$

Physical states are defined in terms of H_u, H_d by mixing angles α and β:

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} \sqrt{2}\text{Im}H_u^0 \\ \sqrt{2}\text{Im}H_d^0 \end{pmatrix} \tag{37}$$

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} \tag{38}$$

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \sqrt{2}\text{Re}H_u^0 - v_u \\ \sqrt{2}\text{Re}H_d^0 - v_d \end{pmatrix} \tag{39}$$

All this is true for the most general CP-conserving two Higgs doublet model.

The Higgs masses, the mixing angle α, and the 3-Higgs and 4-Higgs couplings are determined by the scalar potential.
The MSSM contains a constrained two Higgs doublet model:
The Higgs potential is of a special form, determined by SUSY.

At tree level:

$$\begin{aligned} V &= \left(|\mu|^2 + m_{H_u}^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) \\ &+ \left[b\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \text{h.c.}\right] \\ &+ \frac{1}{8}\left(g^2 + g'^2\right)\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 \\ &+ \frac{1}{2} g^2 \left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 \end{aligned} \tag{40}$$

Dimensionful terms: $(|\mu|^2 + m_{H_{u,d}}^2)$, b set the mass-squared scale.

μ terms come from F-terms: SUSY-preserving

$m_{H_{u,d}}^2$ and b terms come directly from soft SUSY breaking

Dimensionless terms: fixed by the gauge couplings g and g'

D-term contributions: SUSY-preserving

We're free to make SU(2) gauge transformations to rotate away a possible vev for one isospin component: Choose $\langle H_u^+ \rangle = 0$.

Then minimizing V gives $\langle H_d^- \rangle = 0$: electromagnetism unbroken!

The potential becomes

$$\begin{aligned} V &= \left(|\mu|^2 + m_{H_u}^2\right)\left|H_u^0\right|^2 + \left(|\mu|^2 + m_{H_d}^2\right)\left|H_d^0\right|^2 - \left(b H_u^0 H_d^0 + \text{h.c.}\right) \\ &+ \frac{1}{8}(g^2 + g'^2)\left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2\right)^2 \end{aligned} \tag{41}$$

Can absorb a phase into H_u and H_d to make b real and positive.

No explicit CP violation (at tree level)

Then the minimum of the potential is where $H_u^0 H_d^0$ is also real and positive

$\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ must have opposite phases

H_u and H_d have opposite weak hypercharges ($\pm 1/2$):

Can use a U(1)$_Y$ transformation to make v_u and v_d both real and positive, with no loss of generality

No spontaneous CP violation either.

To get electroweak symmetry breaking, need one linear combination of H_u^0 and H_d^0 to have a negative mass-squared.
This will happen if

$$b^2 > \left(|\mu|^2 + m_{H_u}^2\right) \left(|\mu^2 + m_{H_d}^2\right) \tag{42}$$

Large negative contribution to $m_{H_u}^2$ from RGE helps, but is not strictly necessary.

The potential also needs to be bounded from below: require

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 \tag{43}$$

In practice the b term is usually not set as a high-scale input – getting successful electroweak symmetry breaking and the correct $M_Z \propto v_u^2 + v_d^2$ is used to fix b.
Also, one combination of $m_{H_u}^2$ and $m_{H_d}^2$ is often traded for $\tan\beta$ in the inputs.

The scalar potential fixes the tree-level Higgs masses:
(all these get modified by radiative corrections)

$$m_{A^0}^2 = 2b/\sin 2\beta \quad (44)$$
$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 \quad (45)$$
$$m_{h^0,H^0}^2 = \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}\right) \quad (46)$$

By convention, h^0 is lighter than H^0.
The mixing angle α is given by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_{A^0}^2 + m_Z^2}{m_{H^0}^2 - m_{h^0}^2} \qquad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_{A^0}^2 - m_Z^2}{m_{H^0}^2 - m_{h^0}^2} \quad (47)$$

The m_W^2 and m_Z^2 factors comes from $g^2 v^2$ and $(g^2 + g'^2)v^2$ terms – it is due to the g^2 and g'^2 in the scalar potential coming from the D-terms
SUSY relates gauge couplings to couplings in the scalar potential!

- A^0, H^0 and $H^\pm$ masses can be arbitrarily large: they grow with $b/\sin 2\beta$.
- h^0 mass is bounded from above!

$$m_{h^0} < |\cos 2\beta| m_Z \leq m_Z \qquad (!!) \quad (48)$$

This is already ruled out by LEP!
The MSSM would be dead if not for the large radiative corrections to m_{h^0}.
The largest correction comes from top and stop loops:

$$\Delta(m_{h^0}^2) \simeq \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) \quad (49)$$

Revised bound (full one-loop plus dominant 2-loop): $m_{h^0} \lesssim 135$ GeV.