Supersymmetric extensions of the Standard Model

(Lecture 1 of 4)

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Outline

Lecture 1: Introducing SUSY

Lecture 2: SUSY Higgs sectors

Lecture 3: Superpartner spectra and detection

Lecture 4: Measuring couplings, spins, and masses

Why supersymmetry?

Two threads of motivation:

- The last spacetime symmetry

- Solution to the hierarchy problem

Supersymmetry as the last spacetime symmetry

The "super symmetry" itself is an extension of the Poincare algebra [translations, rotations, boosts], discovered in the early '70s. Operator Q that implements symmetry transformations:

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle, \qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle$$

Spinors intrinsically complex $\rightarrow$ $Q^\dagger$ must also be a symmetry generator.

Q, $Q^\dagger$ are fermionic operators: carry spin angular momentum $\frac{1}{2}$. Spacetime is involved!

$$\{Q, Q^\dagger\} = P^\mu$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$$

Q and $Q^\dagger$ carry spinor indices – Lorentz structure is ok.

Irreducible representations of SUSY algebra are called supermultiplets—contain both bosonic and fermionic states that transform into each other under the supersymmetry.

Very beautiful, linked to spacetime—it "must" be true. (well...)

Supersymmetry and the hierarchy problem

Quick recap: the Higgs mechanism in the Standard Model.
Electroweak symmetry is broken by a single scalar Higgs doublet.

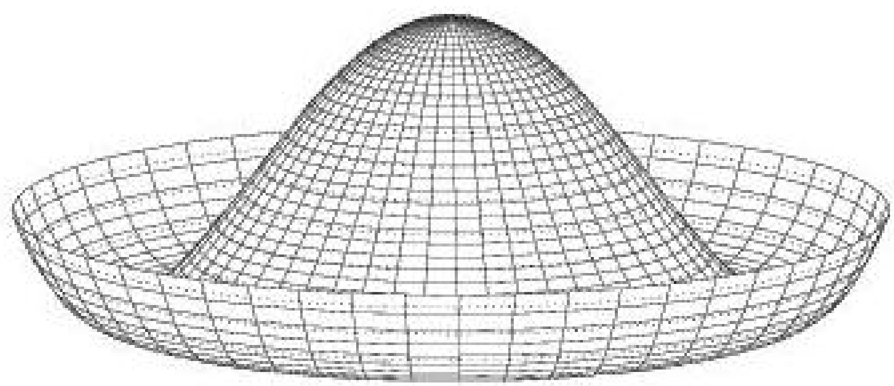
$$H = \begin{pmatrix} G^+ \\ (h+v)/\sqrt{2} + iG^0/\sqrt{2} \end{pmatrix}$$

- G^+ and G^0 are the Goldstone bosons (eaten by W^+ and Z).
- v is the SM Higgs vacuum expectation value (vev), $v = 2m_W/g \simeq 246$ GeV.
- h is the SM Higgs field, a physical particle.

Electroweak symmetry breaking comes from the Higgs potential:

$$V = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

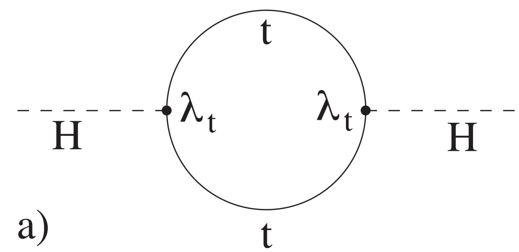
where $\lambda \sim \mathcal{O}(1)$
and $\mu^2 \sim -\mathcal{O}(M^2_{\mathrm{EW}})$



The Hierarchy Problem

The Higgs mass-squared parameter μ^2 gets quantum corrections that depend quadratically on the high-scale cutoff of the theory.

Calculate radiative corrections from, e.g., a top quark loop.



For internal momentum p, large compared to m_t, external h momentum:

$$\begin{aligned} \text{Diagram} &= \int \frac{d^4p}{(2\pi)^4}(-)N_c \text{Tr}\left[i\lambda_t \frac{i}{\not{p}} i\lambda_t \frac{i}{\not{p}} \right] \\ &= -N_c \lambda_t^2 \int \frac{d^4p}{(2\pi)^4} \text{Tr}\left[\frac{1}{p^2}\right] \qquad \text{Tr}[1] = 4 \\ &= -\frac{4N_c\lambda_t^2}{(2\pi)^4} \int \frac{d^4p}{p^2} \end{aligned}$$

Dimensional analysis: integral diverges like p^2_{max}.

Momentum cutoff Λ:

$$\text{Diagram} \sim -\frac{4N_c\lambda_t^2}{(2\pi)^4}\Lambda^2$$

Full calculation gives

$$\Delta\mu^2 = \frac{N_c\lambda_t^2}{16\pi^2}\left[-2\Lambda^2 + 6m_t^2 \ln(\Lambda/m_t) + \cdots\right]$$

We measure $\mu^2 \sim -\mathcal{O}(M_{\rm EW}^2) \sim -10^4\ \text{GeV}^2$.
Nature sets μ_0^2 at the cutoff scale Λ.
If $\Lambda = M_{\rm Pl} = \frac{1}{\sqrt{8\pi G_N}} \sim 10^{18}$ GeV, then $\Delta\mu^2 \sim -10^{35}\ \text{GeV}^2$!

- Not an inconsistency in the theory.
- But it is an implausibly huge top-down coincidence that μ_0^2 and $\Delta\mu^2$ cancel to 31 decimal places!

and not just at one loop – must cancel two-, three-, four-, ... loop contributions

Looks horrible; there "must" be a physics reason why $|\mu^2| \ll M_{\rm Pl}^2$!

Solutions to the hierarchy problem

How low must the cutoff scale Λ be for the cancellation to be "natural"? Want $|\Delta\mu^2| \sim 10^4$ GeV^2 $\longrightarrow \Lambda \sim 1$ TeV!
The fine-tuning argument tells us to expect New Physics that solves the hierarchy problem to appear around 1 TeV!

(plus or minus an order of magnitude...)

So what is the New Physics?
There are three main approaches in BSM physics:

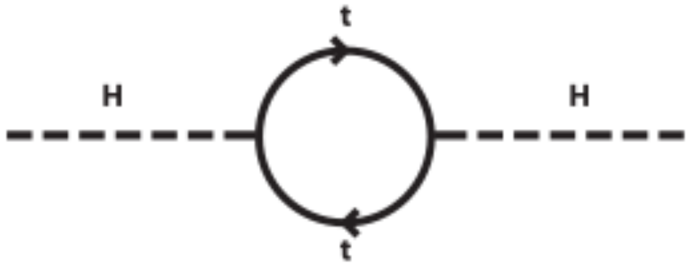
1. Use supersymmetry
- MSSM and extensions

2. Lower the fundamental scale of gravity to $\sim$TeV
- Large extra dimensions
- Warped extra dimensions (Randall-Sundrum)

3. Make the Higgs composite
- Technicolor and its variants
- Warped extra dimensions reinterpreted via AdS/CFT

Supersymmetry as a solution to the hierarchy problem

$\Delta\mu^2$ from a fermion loop is negative.



$\Delta\mu^2$ from a boson loop is positive.

$$\text{2nd diagram} = \int \frac{d^4p}{(2\pi)^4} i\lambda \frac{i}{p^2} = -\frac{\lambda}{(2\pi)^4} \int \frac{d^4p}{p^2}$$

If we could arrange for $\lambda = -4N_c\lambda_t^2$ *exactly*, then our problem would be solved. (can get the N_c if scalar is also a color triplet.)

Have to do this for Higgs μ^2 correction diagrams involving all fermions, W and Z bosons, and the Higgs itself.

Need to impose a symmetry relating fermions to bosons.

This is how Supersymmetry solves the hierarchy problem:

- Each SM fermion gets a boson partner (sfermion)

- Each SM boson gets a fermion partner (-ino)

The relevant couplings for the $\Delta\mu^2$ cancellation are forced to be identical by the (super-) symmetry ← this is a key point

Straightforward to show that it works at one loop.
More difficult to check the two-, three-, ... loops (but it works!).

It's easier to understand the cancellation from a symmetry point of view.

Fermion masses don't have a hierarchy problem:
e.g., fermion self-energy diagram with a gauge boson loop gives

$$\Delta m_f \sim \frac{g^2}{16\pi^2} m_f \log\left(\frac{\Lambda^2}{m_f^2}\right)$$

Notice that $\Delta m_f \propto m_f$.
This is a manifestation of chiral symmetry:
In the limit $m_f = 0$ the system has an extra symmetry: the left- and right-handed components of the fermion are separate objects.

In this limit, radiative corrections cannot give $m_f \neq 0$: fermion mass is protected by chiral symmetry.

Scalars have no such symmetry protection (in a non-SUSY theory).
But supersymmetry relates a scalar to a partner fermion:
it links the scalar mass to the fermion mass!
(In unbroken SUSY they are degenerate.)
So the scalar mass is also protected by chiral symmetry – the Λ^2 divergences all cancel and only $\log(\Lambda^2/m^2)$ divergences are left.

References:

S.P. Martin, "A Supersymmetry Primer," hep-ph/9709356
- Nice accessible introduction to supersymmetry algebra and the MSSM

J. Wess and J. Bagger, "Supersymmetry and Supergravity"
- Quite formal little book on supersymmetry algebra, building supersymmetric Lagrangians, and supersymmetry breaking

H. Baer and X. Tata, "Weak Scale Supersymmetry: From Superfields to Scattering Events"
I. Aitchison, "Supersymmetry in Particle Physics"
M. Drees, R.M. Godbole and P. Roy, "Theory and Phenomenology of Sparticles"
- Recent textbooks on supersymmetry and MSSM phemomenology

Recap:

We've seen two motivations for SUSY:

- Mathematical beauty (the only possible extension of the space-time symmetry)
- A solution of the hierarchy problem

Next we need to look at how SUSY is implemented.

Implementing SUSY

We need at least all the observed SM particles.

The Minimal Supersymmetric Standard Model is defined by adding the minimal set of new particles for a working supersymmetric theory that contains the SM.

Each fermion gets a boson (scalar) partner:

$e_L,\ e_R \leftrightarrow \tilde{e}_L,\ \tilde{e}_R$ "selectrons"

$t_L,\ t_R \leftrightarrow \tilde{t}_L,\ \tilde{t}_R$ "top squarks" (or "stops")

and similarly for the rest of the quarks and leptons

The number of degrees of freedom match:

chiral fermion has 2 d.o.f $\leftrightarrow$ complex (charged) scalar has 2 d.o.f.

Each gauge boson gets a fermionic partner:

$W^\pm \leftrightarrow \widetilde{W}^\pm$ "winos"

$Z,\ \gamma \leftrightarrow \tilde{Z},\ \tilde{\gamma}$ "zino", "photino"

(or $W^0,\ B \leftrightarrow \widetilde{W}^0,\ \tilde{B}$ "neutral wino", "bino")

Again the number of degrees of freedom match:

Transverse gauge boson has 2 d.o.f. (polarizations) $\leftrightarrow$ chiral fermion

Building a supersymmetric Lagrangian

In a supersymmetric theory, the Lagrangian must be invariant under supersymmetry transformations.

This can be constructed (tediously):
- free chiral supermultiplet
- interactions of chiral supermultiplets (Yukawa couplings, etc)
- gauge interactions

Invariance under supersymmetry transformations turns out to be a really strict requirement.

The upshot is that the interactions and masses of all particles in a renormalizable, supersymmetric theory are determined just by their gauge transformation properties and by the so-called superpotential W.

The superpotential: (not actually a potential in the usual sense)

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k$$

* M^{ij} is a mass matrix
* y^{ijk} will turn out to be Yukawa coupling matrices
* W is gauge invariant and analytic in the ϕ's
* $\begin{pmatrix} \phi \\ \psi \end{pmatrix}$ is a chiral supermultiplet (change ∂^μ to $\mathcal{D}^\mu$ for gauge ints)

$$\mathcal{L}_{\text{free}} = -\partial^\mu\phi^{*i}\partial_\mu\phi_i - i\psi^{\dagger i}\bar{\sigma}^\mu\partial_\mu\psi_i + F^{*i}F_i$$

where $F_i = -W_i^*$ ("F-terms"), $W^i = \frac{\delta W}{\delta\phi_i} = M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k$

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}W^{ij}\psi_i\psi_j + W^iF_i + \text{c.c.}$$

where $W^{ij} = \frac{\delta^2 W}{\delta\phi_i\delta\phi_j} = M^{ij} + y^{ijk}\phi_k$ (will give Yukawa couplings)

Adding in the gauge interactions gives more Lagrangian pieces:

* $\begin{pmatrix} \lambda^a \\ A^a_\mu \end{pmatrix}$ is a gauge supermultiplet, λ^a is a gaugino

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\lambda^{a\dagger}\bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \frac{1}{2}D^a D^a$$

where $D^a = -g(\phi^* T^a \phi)$ ("D-terms")

$$\mathcal{L}_{\text{gauge int}} = -\sqrt{2}g\left[(\phi^* T^a \psi)\lambda^a + \text{h.c.}\right] + g(\phi^* T^a \phi)D^a$$

summed over all the gauge groups and chiral multiplets, with g being the relevant gauge coupling for each group.

Notice the parts that involve only scalars: (we will use later: Higgs)

$$\begin{aligned} \mathcal{L} &\supset F^{*i}F_i + W^i F_i + W^*_i F^{*i} + \frac{1}{2}D^a D^a + g(\phi^* T^a \phi)D^a \\ &= -W^*_i W_i - \frac{1}{2}\sum_a g_a^2 (\phi^* T^a \phi)^2 \end{aligned}$$

recall $W^i = M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k$

Summary of allowed Lagrangian terms:

Gauge interactions
- Higgs, squark, slepton self-interactions through $-\frac{1}{2}\sum_a g_a^2(\phi^* T^a \phi)^2$
- gaugino interactions through $-\sqrt{2}g\,[(\phi^* T^a \psi)\lambda^a + \text{h.c.}]$

Fermion Yukawa couplings through $-\frac{1}{2}W^{ij}\psi_i\psi_j$ $\quad (W^{ij} = M^{ij} + y^{ijk}\phi_k)$

SM Yukawas: $y_u \bar{u}_R \tilde{\Phi}^\dagger Q + y_d \bar{d}_R \Phi^\dagger L$. SUSY: no conjugate fields allowed: need a second Higgs doublet with opposite hypercharge.

- also show up in squark and slepton interactions through $W_i^* W_i$

A Higgsino mass term called the μ parameter from $-\frac{1}{2}W^{ij}\psi_i\psi_j$

And some problematic fermion-fermion-sfermion Yukawa couplings also from $-\frac{1}{2}W^{ij}\psi_i\psi_j$.

"Problematic"? $\mathcal{L} \supset -\frac{1}{2} y^{ijk} \phi_k \psi_i \psi_j$

Taking $\phi_k \psi_i \psi_j = \tilde{d}^c Q L$ violates lepton number.

Taking $\phi_k \psi_i \psi_j = \tilde{d}^c u^c d^c$ violates baryon number.

These two couplings together allow very fast proton decay:
$uu \to e^+ \bar{d}$ via t-channel down-type squark $\Rightarrow$ $p \to e^+ \pi^0$

Very very bad! Need to forbid at least one of these two couplings.

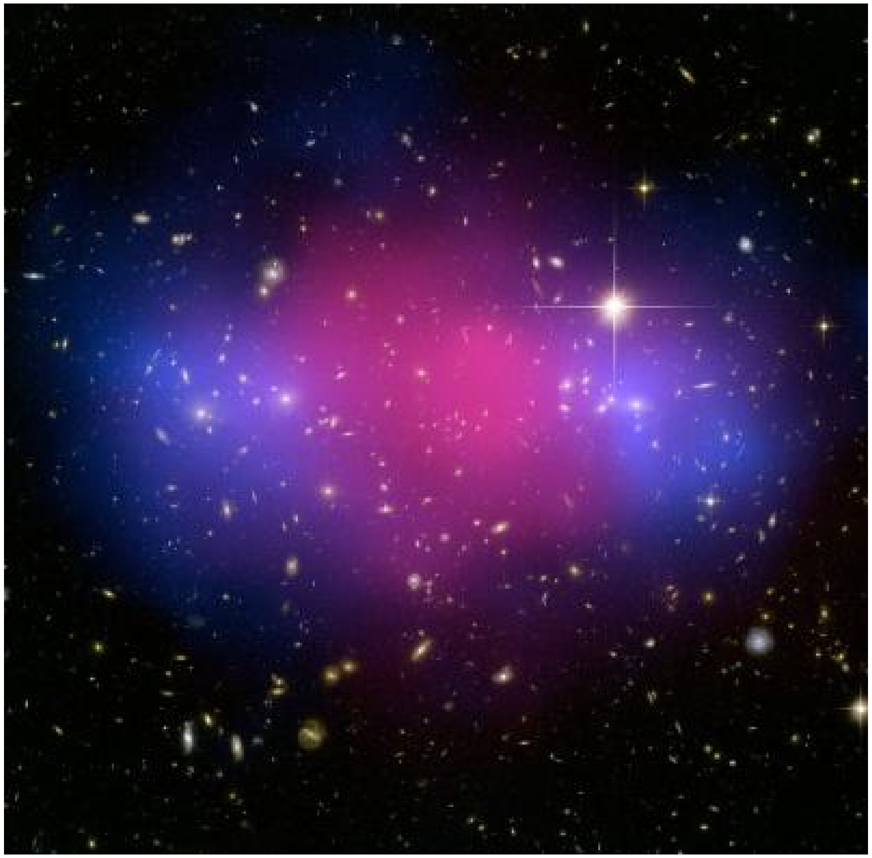
R-parity gets rid of them both: $R = (-1)^{2S+3B+L}$
S = spin, B = baryon number, L = lepton number.

Upshot: familiar SM particles are R-parity even; SUSY partners are R-parity odd.

Conserved R-parity $\to$ lightest R-odd particle (LSP) is stable
$\to$ dark matter candidate!

Why is this good?

We need a particle explanation for dark matter!



Pink – hot gas via x-ray emission

Blue – mass density as reconstructed from gravitational lensing

Particle dark matter: what do we know?

- Needs to be neutral ("dark").
- Needs to be stable (around since early universe).
- Limits on interaction cross section from direct detection searches.
- Thermal production $\leftrightarrow$ EW-strength coupling, 0.1–1 TeV mass.

Note: without thermal production, all bets are off.

- Axions: super-light particles, produced coherently in a "cold" state, search via resonant conversion to photons in a microwave cavity.
- WimpZillas: way too heavy to produce in colliders, number density too low to detect.
- SuperWimps: coupling extremely weak; produced in decay of some other relic particle. Collider: search for parent particle?

Dark matter: direct experimental evidence that we need something new. Not guaranteed to be a new weak-scale particle. Many BSM models provide a dark matter candidate.
(Weakly-Interacting Massive Particle = WIMP)

WIMP needs to be stable $\rightarrow$ some conserved quantum number.
- Lightest particle carrying the conserved quantum number is forced to be stable.

- SUSY: R-parity, a Z_2 parity wanted for proton stability.

- Universal extra dimensions: KK-parity, also an imposed Z_2

- Little Higgs with T-parity: an imposed Z_2 parity motivated to improve EWP consistency.

- Twin Higgs, inert doublet model, singlet scalar dark matter, etc etc... all have a conserved parity and a dark matter candidate.

Z_2 parities:

Particle has quantum number $+1$ or -1 under the parity:

$\phi \to +\phi$ (even) $\qquad\qquad$ $\psi \to -\psi$ (odd)

A Lagrangian invariant under the Z_2 can only contain terms with even powers of odd-charged fields.

This means that interaction vertices must involve only even numbers of odd-charged fields.

– Starting from a Z_2-even initial state, Z_2-odd particles can be produced only in pairs. [SUSY particles must be pair produced.]

– A Z_2-odd particle must decay to an odd number of Z_2-odd particles plus any number of Z_2 even particles.

[SUSY particles decay via a decay chain to the lightest SUSY particle (LSP), which is stable.]

– Two Z_2-odd particles can annihilate into a final state involving only Z_2-even particles.

[Two LSPs in the galactic halo can annihilate to SM particles.]

The particle content of the MSSM

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	$+1$	$H_u^0 \ H_d^0 \ H_u^+ \ H_d^-$	$h^0 \ H^0 \ A^0 \ H^\pm$
squarks	0	-1	$\widetilde{u}_L \ \widetilde{u}_R \ \widetilde{d}_L \ \widetilde{d}_R$ $\widetilde{s}_L \ \widetilde{s}_R \ \widetilde{c}_L \ \widetilde{c}_R$ $\widetilde{t}_L \ \widetilde{t}_R \ \widetilde{b}_L \ \widetilde{b}_R$	(same) (same) $\widetilde{t}_1 \ \widetilde{t}_2 \ \widetilde{b}_1 \ \widetilde{b}_2$
sleptons	0	-1	$\widetilde{e}_L \ \widetilde{e}_R \ \widetilde{\nu}_e$ $\widetilde{\mu}_L \ \widetilde{\mu}_R \ \widetilde{\nu}_\mu$ $\widetilde{\tau}_L \ \widetilde{\tau}_R \ \widetilde{\nu}_\tau$	(same) (same) $\widetilde{\tau}_1 \ \widetilde{\tau}_2 \ \widetilde{\nu}_\tau$
neutralinos	1/2	-1	$\widetilde{B}^0 \ \ \widetilde{W}^0 \ \ \widetilde{H}_u^0 \ \ \widetilde{H}_d^0$	$\widetilde{N}_1 \ \widetilde{N}_2 \ \widetilde{N}_3 \ \widetilde{N}_4$
charginos	1/2	-1	$\widetilde{W}^\pm \ \ \widetilde{H}_u^+ \ \ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm \ \ \widetilde{C}_2^\pm$
gluino	1/2	-1	$\widetilde{g}$	(same)
gravitino/ goldstino	3/2	-1	$\widetilde{G}$	(same)

... plus the usual SM quarks, leptons, and gauge bosons.

MSSM particle content: a few details

In the MSSM we are forced to expand to two Higgs doublets

- Structure of MSSM couplings require a second Higgs to give masses to both up and down type fermions
- Since Higgses now have fermionic partners, anomaly cancellation requires two Higgs doublets with opposite hypercharges

Instead of 1 Higgs boson, get 5 d.o.f.: $h^0, H^0, A^0, H^\pm$

Each Higgs boson gets a fermionic partner:

$$H_u = (H_u^+, H_u^0) \leftrightarrow (\widetilde{H}_u^+, \widetilde{H}_u^0)$$
$$H_d = (H_d^0, H_d^-) \leftrightarrow (\widetilde{H}_d^0, \widetilde{H}_d^-)$$

"Higgsinos"

Again the number of degrees of freedom match:

Complex scalar has 2 d.o.f. $\leftrightarrow$ chiral fermion.

Dealing with the chiral fermions:

– Have 4 neutral chiral fermions: $\widetilde{B}$, $\widetilde{W}^0$, $\widetilde{H}_u^0$, $\widetilde{H}_d^0$. These mix and give four Majorana neutralinos $\widetilde{N}_i$ or $\tilde{\chi}_i^0$.

– Have 4 charged chiral fermions: $\widetilde{W}^\pm$, $\widetilde{H}_u^+$, $\widetilde{H}_d^-$. These pair up (and mix) and give two Dirac charginos $\widetilde{C}_i$ or $\tilde{\chi}_i^\pm$.

If supersymmetry were an exact symmetry, the SUSY particles would have the same masses as their SM partners.

Clearly they are not: SUSY must be broken.
- Not easy to break SUSY without extra model content.
- Almost always SUSY-breaking happens in a "hidden sector" with no SM interactions.
- SUSY-breaking must be communicated to the "visible sector" by some "mediation" mechanism.

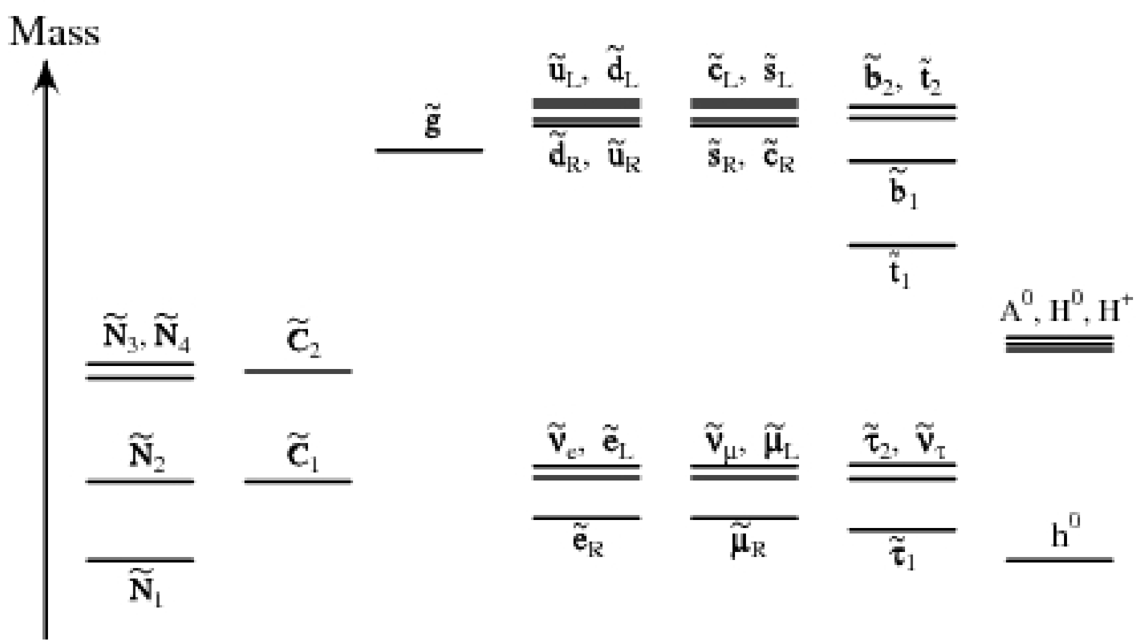
Most general set of SUSY-breaking Lagrangian terms introduces more than 100 new parameters (so much for the beauty of SUSY ...)

Specific SUSY-breaking-mediation models introduce $\mathcal{O}(5\text{--}10)$ new parameters.

Most of the SUSY phenomenology is controlled by the (unknown) SUSY-breaking parameters.

A schematic sample SUSY spectrum:

(This may or may not have anything to do with reality)



from Martin, hep-ph/9709356

Some features:

- $\widetilde{N}_1$ is the LSP
- $\widetilde{t}_1$ and $\widetilde{b}_1$ are the lightest squarks
- $\widetilde{\tau}_1$ is the lightest charged slepton
- Colored particles are heavier than uncolored particles

(More on this in Lecture 3.)

Summary: Today we looked at:

- Motivations to consider supersymmetry
- How to build a supersymmetric Lagrangian
- R-parity and dark matter
- Why SUSY must be broken

Still to come:

Lecture 2:
- SUSY Higgs sectors and their phenomenology

Lecture 3:
- Superpartner spectra in various schemes of SUSY breaking, and how to observe them

Lecture 4:
- Techniques for measuring couplings and spins and testing the supersymmetry coupling relations
- Techniques for measuring masses!