

SUSY phenomenology

Part 1

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Outline

Introducing SUSY

SUSY Higgs sectors

Superpartner spectra

Superpartner production and decays; experimental signals

Measuring couplings, spins, and masses

Why supersymmetry?

Two threads of motivation:

- The last spacetime symmetry
- Solution to the hierarchy problem

Supersymmetry as the last spacetime symmetry

The “super symmetry” itself is an extension of the Poincare algebra [translations, rotations, boosts], discovered in the early '70s. Operator Q that implements symmetry transformations:

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle, \quad Q|\text{Fermion}\rangle = |\text{Boson}\rangle$$

Spinors intrinsically complex $\rightarrow Q^\dagger$ must also be a symmetry generator.

Q, Q^\dagger are fermionic operators: carry spin angular momentum $\frac{1}{2}$. Spacetime is involved!

$$\begin{aligned} \{Q, Q^\dagger\} &= P^\mu \\ \{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0 \\ [P^\mu, Q] &= [P^\mu, Q^\dagger] = 0 \end{aligned}$$

Q and Q^\dagger carry spinor indices – Lorentz structure is ok.

Irreducible representations of SUSY algebra are called **supermultiplets**—contain both bosonic and fermionic states that transform into each other under the supersymmetry.

Very beautiful, linked to spacetime—it “must” be true. (well...)

Supersymmetry and the hierarchy problem

Quick recap: the Higgs mechanism in the Standard Model.
Electroweak symmetry is broken by a **single scalar Higgs doublet**.

$$H = \begin{pmatrix} G^+ \\ (h + v)/\sqrt{2} + iG^0/\sqrt{2} \end{pmatrix}$$

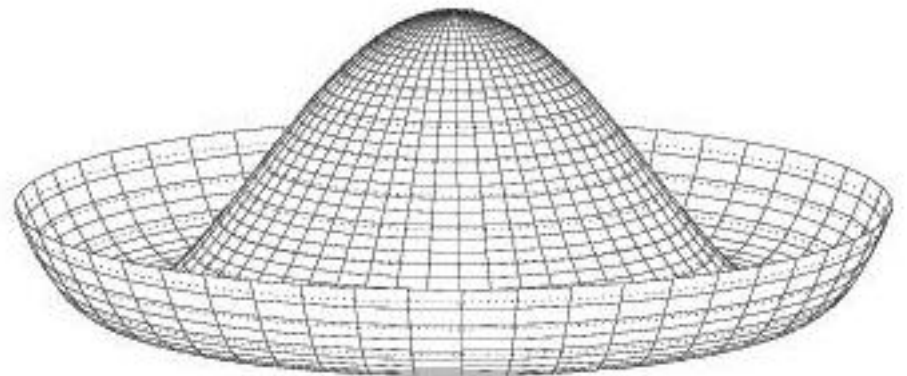
- G^+ and G^0 are the **Goldstone bosons** (eaten by W^+ and Z).
- v is the SM Higgs **vacuum expectation value (vev)**,
 $v = 2m_W/g \simeq 246$ GeV.
- h is the SM Higgs field, a physical particle.

Electroweak symmetry breaking comes from the **Higgs potential**:

$$V = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

where $\lambda \sim \mathcal{O}(1)$

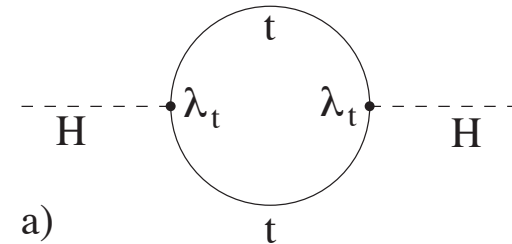
and $\mu^2 \sim -\mathcal{O}(M_{EW}^2)$



The Hierarchy Problem

The Higgs mass-squared parameter μ^2 gets quantum corrections that depend quadratically on the high-scale cutoff of the theory.

Calculate radiative corrections from, e.g., a top quark loop.



For internal momentum p , large compared to m_t and external h momentum:

$$\begin{aligned}
 \text{Diagram} &= \int \frac{d^4 p}{(2\pi)^4} (-) N_c \text{Tr} \left[i\lambda_t \frac{i}{\not{p}} i\lambda_t \frac{i}{\not{p}} \right] \\
 &= -N_c \lambda_t^2 \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[\frac{1}{p^2} \right] \quad \text{Tr} [1] = 4 \\
 &= -\frac{4N_c \lambda_t^2}{(2\pi)^4} \int \frac{d^4 p}{p^2}
 \end{aligned}$$

Dimensional analysis: integral diverges like p_{max}^2 .

Momentum cutoff Λ :

$$\text{Diagram} \sim -\frac{4N_c\lambda_t^2}{(2\pi)^4}\Lambda^2$$

Full calculation gives

$$\Delta\mu^2 = \frac{N_c\lambda_t^2}{16\pi^2} \left[-2\Lambda^2 + 6m_t^2 \ln(\Lambda/m_t) + \dots \right]$$

We measure $\mu^2 \sim -\mathcal{O}(M_{EW}^2) \sim -10^4 \text{ GeV}^2$.

Nature sets μ_0^2 at the cutoff scale Λ .

If $\Lambda = M_{Pl} = \frac{1}{\sqrt{8\pi G_N}} \sim 10^{18} \text{ GeV}$, then $\Delta\mu^2 \sim -10^{35} \text{ GeV}^2$!

- Not an inconsistency in the theory.

- But it is an implausibly huge top-down coincidence that μ_0^2 and $\Delta\mu^2$ cancel to 31 decimal places!

and not just at one loop – must cancel two-, three-, four-, ... loop contributions

Looks horrible; there “must” be a physics reason why $|\mu^2| \ll M_{Pl}^2$!

Solutions to the hierarchy problem

How low must the cutoff scale Λ be for the cancellation to be “natural”? Want $|\Delta\mu^2| \sim 10^4 \text{ GeV}^2 \rightarrow \Lambda \sim 1 \text{ TeV!}$

The fine-tuning argument tells us to expect New Physics that solves the hierarchy problem to appear around 1 TeV!

(plus or minus an order of magnitude...)

So what is the New Physics?

There are three main approaches in BSM physics:

1. Use supersymmetry

- MSSM and extensions

2. Lower the fundamental scale of gravity to $\sim \text{TeV}$

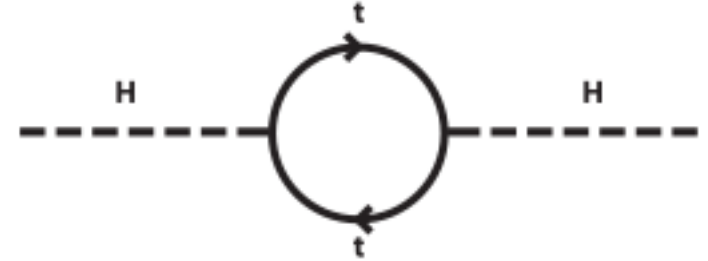
- Large extra dimensions
- Warped extra dimensions (Randall-Sundrum)

3. Make the Higgs composite

- Technicolor and its variants
- Warped extra dimensions reinterpreted via AdS/CFT

Supersymmetry as a solution to the hierarchy problem

$\Delta\mu^2$ from a fermion loop is negative.



$\Delta\mu^2$ from a boson loop is positive.



$$\begin{aligned} \text{2nd diagram} &= \int \frac{d^4p}{(2\pi)^4} i\lambda \frac{i}{p^2} \\ &= -\frac{\lambda}{(2\pi)^4} \int \frac{d^4p}{p^2} \end{aligned}$$

If we could arrange for $\lambda = -4N_c\lambda_t^2$ exactly, then our problem would be solved. (can get the N_c if scalar is also a color triplet.)

Have to do this for Higgs μ^2 correction diagrams involving all fermions, W and Z bosons, and the Higgs itself.

Need to impose a [symmetry](#) relating fermions to bosons.

This is how Supersymmetry solves the hierarchy problem:

- Each SM fermion gets a boson partner (sfermion)
- Each SM boson gets a fermion partner (-ino)

The relevant couplings for the $\Delta\mu^2$ cancellation are forced to be identical by the (super-) symmetry ← this is a key point

Straightforward to show that it works at one loop.

More difficult to check the two-, three-, ... loops (but it works!).

It's easier to understand the cancellation from a symmetry point of view.

Fermion masses don't have a hierarchy problem:
e.g., fermion self-energy diagram with a gauge boson loop gives

$$\Delta m_f \sim \frac{g^2}{16\pi^2} m_f \log \left(\frac{\Lambda^2}{m_f^2} \right)$$

Notice that $\Delta m_f \propto m_f$.

This is a manifestation of **chiral symmetry**:

In the limit $m_f = 0$ the system has an extra symmetry: the left- and right-handed components of the fermion are separate objects.

In this limit, radiative corrections **cannot** give $m_f \neq 0$: fermion mass is protected by chiral symmetry.

Scalars have no such symmetry protection (in a non-SUSY theory).

But supersymmetry relates a scalar to a partner fermion:

it links the scalar mass to the fermion mass!

(In unbroken SUSY they are degenerate.)

So the scalar mass is also protected by chiral symmetry – the Λ^2 divergences all cancel and only $\log(\Lambda^2/m^2)$ divergences are left.

References:

S.P. Martin, “A Supersymmetry Primer,” [hep-ph/9709356](#)

- Nice accessible introduction to supersymmetry algebra and the MSSM

J. Wess and J. Bagger, “Supersymmetry and Supergravity”

- Quite formal little book on supersymmetry algebra, building supersymmetric Lagrangians, and supersymmetry breaking

H. Baer and X. Tata, “Weak Scale Supersymmetry: From Superfields to Scattering Events”

I. Aitchison, “Supersymmetry in Particle Physics”

M. Drees, R.M. Godbole and P. Roy, “Theory and Phenomenology of Sparticles”

- Recent textbooks on supersymmetry and MSSM phenomenology

Recap:

We've seen two motivations for SUSY:

- Mathematical beauty (the only possible extension of the space-time symmetry)
- A solution of the hierarchy problem

Next we need to look at how SUSY is implemented.

Implementing SUSY

We need at least all the observed SM particles.

The **Minimal Supersymmetric Standard Model** is defined by adding the minimal set of new particles for a working supersymmetric theory that contains the SM.

Each fermion gets a boson (scalar) partner:

$$e_L, e_R \leftrightarrow \tilde{e}_L, \tilde{e}_R \quad \text{“selectrons”}$$

$$t_L, t_R \leftrightarrow \tilde{t}_L, \tilde{t}_R \quad \text{“top squarks” (or “stops”)}$$

and similarly for the rest of the quarks and leptons

The number of degrees of freedom match:

chiral fermion has 2 d.o.f \leftrightarrow complex (charged) scalar has 2 d.o.f.

Each gauge boson gets a fermionic partner:

$$W^\pm \leftrightarrow \tilde{W}^\pm \quad \text{“winos”}$$

$$Z, \gamma \leftrightarrow \tilde{Z}, \tilde{\gamma} \quad \text{“zino”, “photino”}$$

$$\text{(or } W^0, B \leftrightarrow \tilde{W}^0, \tilde{B} \quad \text{“neutral wino”, “bino”)}$$

Again the number of degrees of freedom match:

Transverse gauge boson has 2 d.o.f. (polarizations) \leftrightarrow chiral fermion

Building a supersymmetric Lagrangian

In a supersymmetric theory, the Lagrangian must be invariant under supersymmetry transformations.

This can be constructed (tediously):

- free chiral supermultiplet
- interactions of chiral supermultiplets (Yukawa couplings, etc)
- gauge interactions

Invariance under supersymmetry transformations turns out to be a really strict requirement.

The upshot is that the interactions and masses of all particles in a renormalizable, supersymmetric theory are determined just by their **gauge transformation properties** and by the so-called **superpotential W** .

The superpotential: (not actually a potential in the usual sense)

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k$$

- * M^{ij} is a mass matrix
- * y^{ijk} will turn out to be Yukawa coupling matrices
- * W is gauge invariant and analytic in the ϕ 's
- * $\begin{pmatrix} \phi \\ \psi \end{pmatrix}$ is a chiral supermultiplet (change ∂^μ to \mathcal{D}^μ for gauge ints)

$$\mathcal{L}_{\text{free}} = -\partial^\mu\phi^{*i}\partial_\mu\phi_i - i\psi^\dagger{}^i\bar{\sigma}^\mu\partial_\mu\psi_i + F^{*i}F_i$$

where $F_i = -W_i^*$ ("F-terms"), $W^i = \frac{\delta W}{\delta\phi_i} = M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k$

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}W^{ij}\psi_i\psi_j + W^iF_i + \text{c.c.}$$

where $W^{ij} = \frac{\delta^2 W}{\delta\phi_i\delta\phi_j} = M^{ij} + y^{ijk}\phi_k$ (will give Yukawa couplings)

Adding in the gauge interactions gives more Lagrangian pieces:

* $\begin{pmatrix} \lambda^a \\ A_\mu^a \end{pmatrix}$ is a gauge supermultiplet, λ^a is a gaugino

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - i\lambda^{a\dagger}\bar{\sigma}^\mu\mathcal{D}_\mu\lambda^a + \frac{1}{2}D^a D^a$$

where $D^a = -g(\phi^*T^a\phi)$ (“D-terms”)

$$\mathcal{L}_{\text{gauge int}} = -\sqrt{2}g [(\phi^*T^a\psi)\lambda^a + \text{h.c.}] + g(\phi^*T^a\phi)D^a$$

summed over all the gauge groups and chiral multiplets, with g being the relevant gauge coupling for each group.

Notice the parts that involve only scalars: (we will use later: Higgs)

$$\begin{aligned} \mathcal{L} &\supset F^{*i}F_i + W^iF_i + W_i^*F^{*i} + \frac{1}{2}D^a D^a + g(\phi^*T^a\phi)D^a \\ &= -W_i^*W_i - \frac{1}{2}\sum_a g_a^2(\phi^*T^a\phi)^2 \end{aligned}$$

$$\text{recall } W^i = M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k$$

Summary of allowed Lagrangian terms:

Gauge interactions

- Higgs, squark, slepton self-interactions through $-\frac{1}{2} \sum_a g_a^2 (\phi^* T^a \phi)^2$
- gaugino interactions through $-\sqrt{2}g [(\phi^* T^a \psi) \lambda^a + \text{h.c.}]$

Fermion Yukawa couplings through $-\frac{1}{2} W^{ij} \psi_i \psi_j$ ($W^{ij} = M^{ij} + y^{ijk} \phi_k$)

SM Yukawas: $y_u \bar{u}_R \tilde{\Phi}^\dagger Q + y_d \bar{d}_R \Phi^\dagger L$. SUSY: no conjugate fields allowed: need a second Higgs doublet with opposite hypercharge.

- also show up in squark and slepton interactions through $W_i^* W_i$

A Higgsino mass term called the μ parameter from $-\frac{1}{2} W^{ij} \psi_i \psi_j$

And some **problematic** fermion-fermion-sfermion Yukawa couplings also from $-\frac{1}{2} W^{ij} \psi_i \psi_j$.

“Problematic” ?

$$\mathcal{L} \supset -\frac{1}{2}y^{ijk}\phi_k\psi_i\psi_j$$

Taking $\phi_k\psi_i\psi_j = \tilde{d}^c QL$ violates lepton number.

Taking $\phi_k\psi_i\psi_j = \tilde{d}^c u^c d^c$ violates baryon number.

These two couplings together allow **very fast proton decay**:
 $uu \rightarrow e^+ \bar{d}$ via t-channel down-type squark $\Rightarrow p \rightarrow e^+ \pi^0$

Very very bad! Need to forbid at least one of these two couplings.

R-parity gets rid of them both: $R = (-1)^{2S+3B+L}$

$S = \text{spin}$, $B = \text{baryon number}$, $L = \text{lepton number}$.

Upshot: familiar SM particles are R-parity even; SUSY partners are R-parity odd.

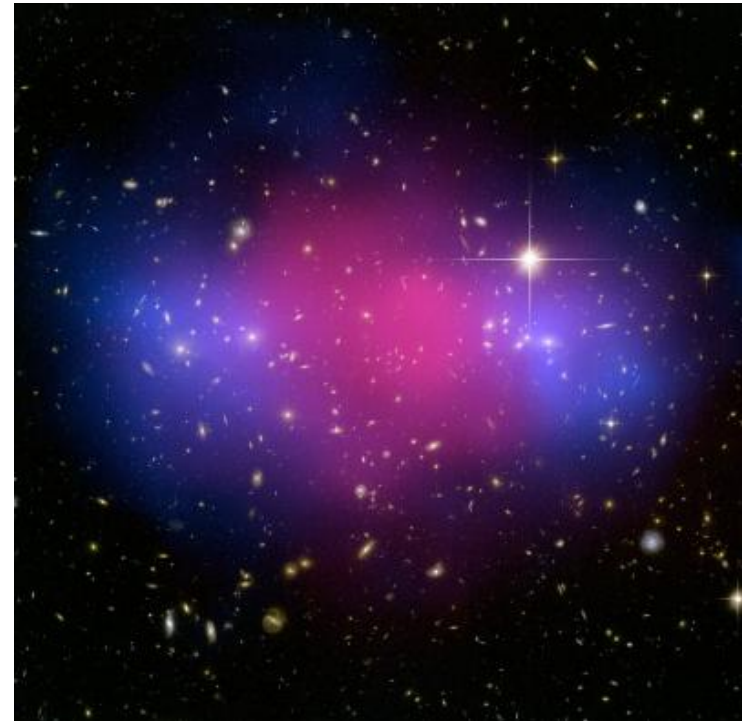
Conserved R-parity \rightarrow lightest R-odd particle (LSP) is stable
 \rightarrow **dark matter candidate!**

Why is this good?

We need a particle explanation for dark matter!



Bullet cluster



MACS J0025

Pink – hot gas via x-ray emission (Chandra)

Blue – mass density as reconstructed from gravitational lensing (Hubble)

Particle dark matter: what do we know?

- Needs to be neutral (“dark”).
- Needs to be stable (around since early universe).
- Limits on interaction cross section from direct detection searches.
- Thermal production \leftrightarrow EW-strength coupling, 0.1–1 TeV mass.

Note: without thermal production, all bets are off.

- Axions: super-light particles, produced coherently in a “cold” state, search via resonant conversion to photons in a microwave cavity.
- WimpZillas: way too heavy to produce in colliders, number density too low to detect.
- SuperWimps: coupling extremely weak; produced in decay of some other relic particle. Collider: search for parent particle?

Dark matter: direct experimental evidence that we need something new. Not guaranteed to be a new weak-scale particle. Many BSM models provide a dark matter candidate.

(Weakly-Interacting Massive Particle = WIMP)

WIMP needs to be stable \rightarrow some conserved quantum number.

- Lightest particle carrying the conserved quantum number is forced to be stable.

- SUSY: **R-parity**, a Z_2 parity wanted for proton stability.

- Universal extra dimensions: **KK-parity**, also an imposed Z_2

- Little Higgs with **T-parity**: an imposed Z_2 parity motivated to improve EWP consistency.

- Twin Higgs, inert doublet model, singlet scalar dark matter, etc... all have a conserved parity and a dark matter candidate.

Z_2 parities:

Particle has quantum number $+1$ or -1 under the parity:

$$\phi \rightarrow +\phi \text{ (even)} \qquad \psi \rightarrow -\psi \text{ (odd)}$$

A Lagrangian invariant under the Z_2 can only contain terms with **even powers** of odd-charged fields.

This means that interaction vertices must involve only **even numbers** of odd-charged fields.

– Starting from a Z_2 -even initial state, Z_2 -odd particles can be produced only in **pairs**. [SUSY particles must be pair produced.]

– A Z_2 -odd particle must decay to an **odd number of Z_2 -odd particles** plus any number of Z_2 even particles.

[SUSY particles decay via a decay chain to the lightest SUSY particle (LSP), which is stable.]

– Two Z_2 -odd particles can **annihilate** into a final state involving only Z_2 -even particles.

[Two LSPs in the galactic halo can annihilate to SM particles.]

The particle content of the MSSM

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0 \ H_d^0 \ H_u^+ \ H_d^-$	$h^0 \ H^0 \ A^0 \ H^\pm$
squarks	0	-1	$\tilde{u}_L \ \tilde{u}_R \ \tilde{d}_L \ \tilde{d}_R$ $\tilde{s}_L \ \tilde{s}_R \ \tilde{c}_L \ \tilde{c}_R$ $\tilde{t}_L \ \tilde{t}_R \ \tilde{b}_L \ \tilde{b}_R$	(same) (same) $\tilde{t}_1 \ \tilde{t}_2 \ \tilde{b}_1 \ \tilde{b}_2$
sleptons	0	-1	$\tilde{e}_L \ \tilde{e}_R \ \tilde{\nu}_e$ $\tilde{\mu}_L \ \tilde{\mu}_R \ \tilde{\nu}_\mu$ $\tilde{\tau}_L \ \tilde{\tau}_R \ \tilde{\nu}_\tau$	(same) (same) $\tilde{\tau}_1 \ \tilde{\tau}_2 \ \tilde{\nu}_\tau$
neutralinos	1/2	-1	$\tilde{B}^0 \ \tilde{W}^0 \ \tilde{H}_u^0 \ \tilde{H}_d^0$	$\tilde{N}_1 \ \tilde{N}_2 \ \tilde{N}_3 \ \tilde{N}_4$
charginos	1/2	-1	$\tilde{W}^\pm \ \tilde{H}_u^+ \ \tilde{H}_d^-$	$\tilde{C}_1^\pm \ \tilde{C}_2^\pm$
gluino	1/2	-1	\tilde{g}	(same)
gravitino/ goldstino	3/2	-1	\tilde{G}	(same)

... plus the usual SM quarks, leptons, and gauge bosons.

MSSM particle content: a few details

In the MSSM we are forced to expand to **two Higgs doublets**

- Structure of MSSM couplings require a second Higgs to give masses to both up and down type fermions
- Since Higgses now have fermionic partners, anomaly cancellation requires two Higgs doublets with opposite hypercharges

Instead of 1 Higgs boson, get 5 d.o.f.: h^0, H^0, A^0, H^\pm

Each Higgs boson gets a fermionic partner:

$$\begin{aligned} H_u &= (H_u^+, H_u^0) \leftrightarrow (\widetilde{H}_u^+, \widetilde{H}_u^0) \\ H_d &= (H_d^0, H_d^-) \leftrightarrow (\widetilde{H}_d^0, \widetilde{H}_d^-) \end{aligned} \quad \text{“Higgsinos”}$$

Again the number of degrees of freedom match:

Complex scalar has 2 d.o.f. \leftrightarrow chiral fermion.

Dealing with the chiral fermions:

- Have 4 neutral chiral fermions: $\widetilde{B}, \widetilde{W}^0, \widetilde{H}_u^0, \widetilde{H}_d^0$. These mix and give four Majorana **neutralinos** \widetilde{N}_i or $\widetilde{\chi}_i^0$.
- Have 4 charged chiral fermions: $\widetilde{W}^\pm, \widetilde{H}_u^\pm, \widetilde{H}_d^\pm$. These pair up (and mix) and give two Dirac **charginos** \widetilde{C}_i or $\widetilde{\chi}_i^\pm$.

If supersymmetry were an exact symmetry, the SUSY particles would have the same masses as their SM partners.

Clearly they are not: SUSY must be **broken**.

- Not easy to break SUSY without extra model content.
- Almost always SUSY-breaking happens in a “hidden sector” with no SM interactions.
- SUSY-breaking must be communicated to the “visible sector” by some “mediation” mechanism.

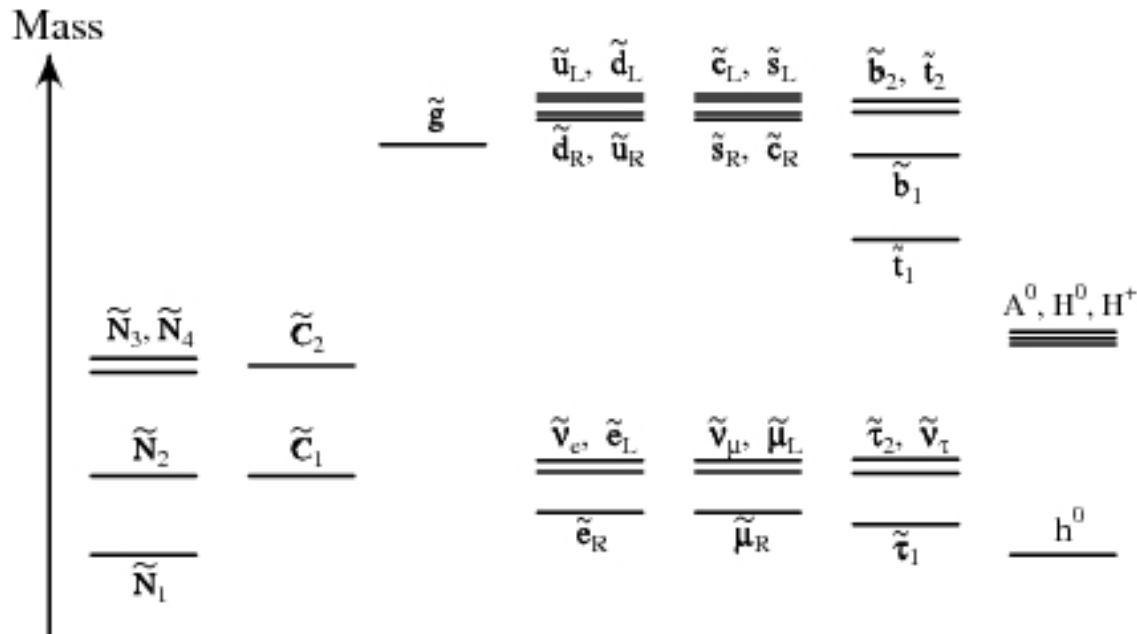
Most general set of SUSY-breaking Lagrangian terms introduces more than 100 new parameters (so much for the beauty of SUSY ...)

Specific SUSY-breaking-mediation models introduce $\mathcal{O}(5-10)$ new parameters.

Most of the SUSY phenomenology is controlled by the (unknown) SUSY-breaking parameters.

A schematic sample SUSY spectrum:

(This may or may not have anything to do with reality)



Some features:

- \tilde{N}_1 is the LSP
- \tilde{t}_1 and \tilde{b}_1 are the lightest squarks
- $\tilde{\tau}_1$ is the lightest charged slepton
- Colored particles are heavier than uncolored particles

from Martin, hep-ph/9709356

(More on this coming soon.)

Summary: So far we looked at:

- Motivations to consider supersymmetry
- How to build a supersymmetric Lagrangian
- R-parity and dark matter
- Why SUSY must be broken

Still to come:

- SUSY Higgs sectors and their phenomenology
- Superpartner spectra in various schemes of SUSY breaking, and how to observe them
- Techniques for measuring couplings and spins and testing the supersymmetry coupling relations
- Techniques for measuring masses!