QCD corrections to neutralino annihilation

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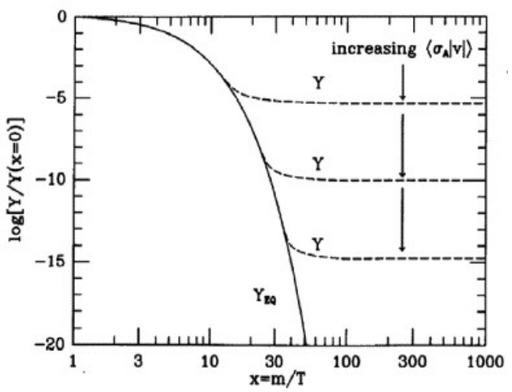
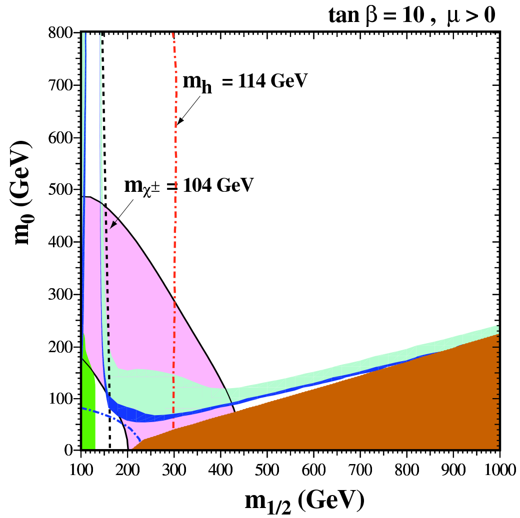
• V. Barger, W.-Y. Keung, HEL, G. Shaughnessy, A. Tregre, Phys. Lett. B633 (2006) 98-105 [hep-ph/0510257]
• V. Barger, W.-Y. Keung, HEL, G. Shaughnessy, *work in progress*

Why calculate neutralino annihilation?

- Cross section controls dark matter relic abundance



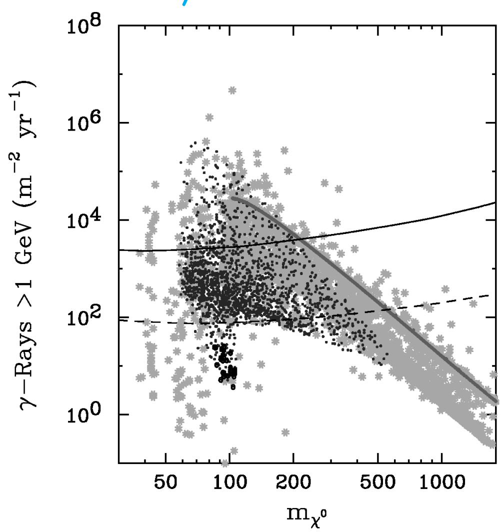
[Kolb & Turner]



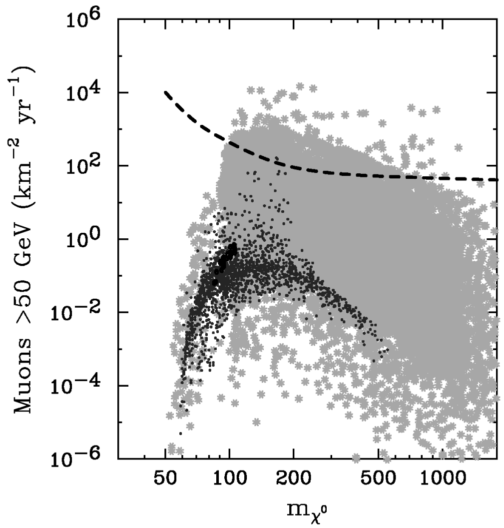
[Olive et al]

- Cross section controls indirect detection rates

EGRET/GLAST



IceCube



[Bertone, Hooper & Silk 2004]

Why calculate QCD corrections?

QCD corr's can be significant in some regions of parameter space.

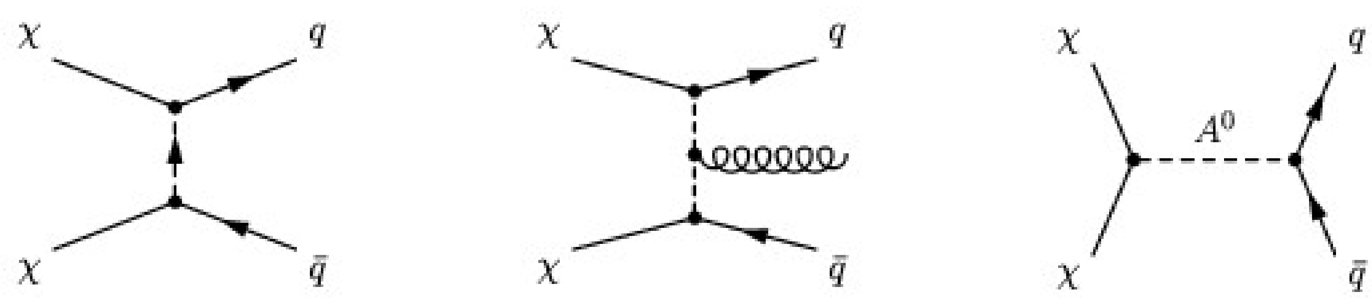
- Where $\chi\chi \to$ light fermions dominates, neutralinos are p-wave annihilators in the early universe
    - s-wave cross section is helicity-suppressed by m_f^2/m_χ^2.

- Hard QCD radiation and $\chi\chi \to gg$ through a loop lift the m_f^2 suppression – big effect on the s-wave cross section; not so much on the p-wave cross section.

$\longrightarrow$ corrections tend to be most relevant for indirect-detection rates.

Consider $\chi\chi$ annihilation through squark exchange

Some typical diagrams:



etc.

The first diagram above can be reduced to an effective vertex described by a dimension-six operator suppressed by the squark mass $\widetilde{M}$:

$$\mathcal{L} = \frac{c}{\widetilde{M}^2}\mathcal{O}_6, \qquad\qquad \mathcal{O}_6 = (\bar{\chi}\gamma_\mu\gamma_5\chi)\,(\bar{q}\gamma^\mu\gamma_5 q)$$

This is valid in the limit $m_\chi \ll \widetilde{M}$.

In the zero-velocity limit the neutralinos behave like a pseudoscalar.

$\mathcal{O}_6$ is related to the divergence of the axial vector current of the quarks:

$$\mathcal{O}_6 \to \left[\bar{\chi}\frac{i\gamma_5}{2m_\chi}\chi\right]\left[\partial_\mu\left(\bar{q}\gamma^\mu\gamma_5 q\right)\right]$$

- If $m_q = 0$, the axial vector current is conserved at tree level, $\partial_\mu\left(\bar{q}\gamma^\mu\gamma_5 q\right) = 0$.

  This is the m_f^2/m_χ^2 suppression showing up.

- There are two ways to lift this suppression:

(1) Go beyond leading order in α_s to include the anomalous triangle diagram.

(2) go to dimension-eight (or higher) by including hard gluon radiation.

Anomalous triangle diagram

The lifting of the m_f^2 suppression here is due to the well-known Partially Conserved Axial Current (PCAC):

$\partial_\mu (\bar{q}\gamma^\mu \gamma_5 q) \neq 0$ due to the anomaly, even when $m_q = 0$.

The anomaly condition reads: (including m_q and only QCD interactions)

$$\partial_\mu (\bar{q}\gamma^\mu \gamma_5 q) = 2 m_q \bar{q} i \gamma_5 q + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \widetilde{G}^{(a)\mu\nu}$$

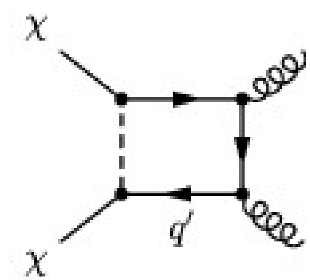
Neglecting m_q, we can write the zero-velocity dimension-six $\chi\chi$ annihilation amplitude in the form

$$\mathcal{L}_{\rm eff} = \left(\frac{c/m_\chi}{2\widetilde{M}^2} \right) (\bar{\chi} i \gamma_5 \chi) \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \widetilde{G}^{(a)\mu\nu}$$

This is $\chi\chi$ annihilation into gluons.

[Still working in $\widetilde{M}^{-2}$ approximation for squark propagator.]

Expression describes one massless quark running around the loop.



Anomalous triangle diagram

- Calculation first done for $\chi\chi \to \gamma\gamma$ [Rudaz 1989; Bergstrom 1989]

- Easy to extend to $\chi\chi \to gg$ [Flores, Olive, Rudaz 1989]

$m_{q'} = 0$ result: (sum is over 5 light quarks; top decouples)

$$v_{\rm rel}\sigma(\chi\chi \to gg) = \frac{\alpha_s^2}{32\pi^3} m_\chi^2 \left[\sum_{q'} \frac{|g_\ell|^2}{M^2_{\tilde{q}'_L}} + \frac{|g_r|^2}{M^2_{\tilde{q}'_R}} \right]^2$$

where

$$g_\ell = -\sqrt{2} N_{11} g'(T_3 - Q) + \sqrt{2} N_{12} g T_3, \qquad g_r = -\sqrt{2} N_{11} g' Q.$$

We neglect left-right squark mixing ($m_{q'} = 0$ approximation)

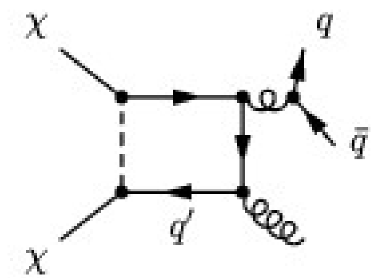
- Full $m_{q'}$, $\widetilde{M}$ dependence is also known

  [Drees, Jungman, Kamionkowski, Nojiri 1993]

What about beyond leading order?

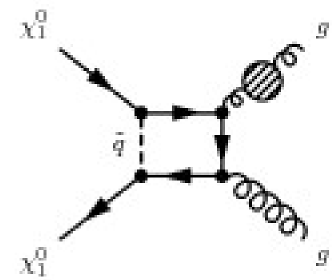
$\chi\chi \to gg$ is order α_s^2: large scale dependence at leading order.
Set scale $\mu_0 = 2m_\chi$, vary between $\mu_0/2 \ldots 2\mu_0$: $v\sigma$ varies by $\pm 16\%$.

At NLO, must include:



(1) gluon splitting into quark or gluon pairs

(2) radiation of a 3rd gluon off of the internal q' line



(3) virtual corrections: gluons crossing the box, gluons connecting the box to a gluon leg

(4) renormalization; e.g., gluon propagator bubbles containing quarks and gluons

The calculation is big and ugly.
Luckily we can use a trick to do it!

The trick:

In the zero-velocity limit, $\chi\chi \to gg$ is related to the anomaly equation:

$$\partial_\mu(\bar{q}'\gamma^\mu\gamma_5 q') \;=\; 2m_{q'}\bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

- Neglecting the $m_{q'}$ term relates the axial vector current divergence to the two-gluon operator:

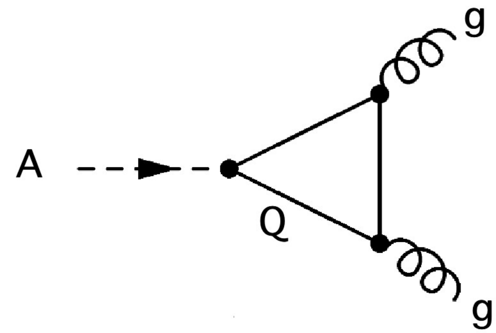
$$\partial_\mu(\bar{q}'\gamma^\mu\gamma_5 q') \;\simeq\; \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

- If we take the opposite limit, $m_{q'} \gg m_\chi$, then the anomaly equation relates a pseudoscalar coupling to the same two-gluon operator:

$$0 \;\simeq\; 2m_{q'}\bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

(the term on the left-hand side becomes negligible in the $m_{q'} \gg m_\chi$ limit)

$$0 \simeq 2m_{q'}\bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \tilde{G}^{(a)\mu\nu}$$



This describes pseudoscalar decay through a heavy quark triangle in the limit $m_Q \gg m_A$.

This helps us because of the Adler-Bardeen theorem, which tells us that the anomaly equation holds to all orders in α_s.

Should be able to relate $\chi\chi \to gg$ at NLO to $A \to gg$ at NLO.

$A \to gg$ at NLO calculated by Spira, Djouadi, Graudenz, Zerwas (1995):

$$\Gamma_{\mathrm{NLO}}(A \to gg) = \Gamma_{\mathrm{LO}}(A \to gg) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{97}{4} - \frac{7}{6}N_f + \frac{33 - 2N_f}{6} \log \frac{\mu^2}{4m_\chi^2}\right)\right]$$

Correction is multiplicative in the $m_Q \gg m_A$ approximation.

How can we use this?

Following [Chetyrkin, Kniehl, Steinhauser, Bardeen 1998]:

Start with the bare Yukawa Lagrangian for interactions of A^0 with quarks:

$$\mathcal{L} = -\frac{A}{v}\left[\sum_{i=1}^{n_l} m_{q_i}^0 \bar{q}_i^0 i\gamma_5 q_i^0 + m_t^0 \bar{t}^0 i\gamma_5 t^0\right]$$

Taking the limit $m_t \to \infty$ and setting $m_{q_i} = 0$ for the light quarks, we can write this as a combination of pseudoscalar operators:

$$\mathcal{L} = -\frac{A}{v}\left[C_1^0 O_1^0 + C_2^0 O_2^0 + \cdots\right]$$

where

$$O_1^0 = G_{\mu\nu}^{0,a} \widetilde{G}^{0,a\mu\nu}, \qquad O_2^0 = \partial_\mu J_5^{0,\mu},$$

with $$J_5^{0,\mu} = \sum_{i=1}^{n_l} \bar{q}_i^0 \gamma^\mu \gamma_5 q_i^0$$

Now the bare lagrangian must be renormalized:

- $J_5^{0,\mu}$ is the colour-singlet axial-vector current, which is renormalized multiplicatively; $\partial_\mu J_5^{0,\mu}$ likewise is renormalized multiplicatively.

- $G_{\mu\nu}^{0,a} \widetilde{G}^{0,a\mu\nu}$ mixes under renormalization: $\partial_\mu J_5^{0,\mu}$ feeds into $G_{\mu\nu}^{0,a} \widetilde{G}^{0,a\mu\nu}$ at one loop because you can close the quark loop.

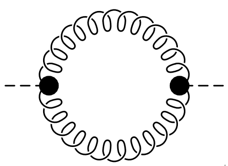
So we'll get:

$$\mathcal{L} = -\frac{A}{v} \left[C_1 O_1 + C_2 O_2 + \cdots \right],$$
$$O_1 = Z_{11} O_1^0 + Z_{12} O_2^0, \qquad O_2 = Z_{22} O_2^0.$$

The $A \to gg$ decay is the imaginary part of the $A \to A$ amplitude, which is described by correlators $\langle O_i O_j \rangle$:

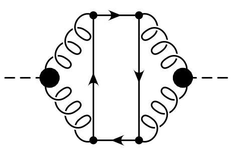
$$\Gamma(A \to gg) = \frac{\sqrt{2} G_F}{M_A} \left[C_1^2 \mathrm{Im}\langle O_1 O_1 \rangle + 2 C_1 C_2 \mathrm{Im}\langle O_1 O_2 \rangle + C_2^2 \mathrm{Im}\langle O_2 O_2 \rangle \right]$$

- $\langle O_1 O_1 \rangle$ first appears at order α_s^0.
  - Diagram $\longrightarrow$



- $\langle O_1 O_2 \rangle$ first appears at order α_s^1.
  - Need to radiate a gluon from $q\bar{q}$ in O_2 and split a gluon into quarks in O_1.
- $\langle O_2 O_2 \rangle$ first appears at order α_s^2.
  - Kinematics kills $\langle O_2 O_2 \rangle$ at leading order for $m_q = 0$. Need to make two boxes and connect the gluons.

- C_1 starts at order α_s^1, since $A G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$ is generated by the top loop.
- C_2 starts at order α_s^2, since $A \partial_\mu J_5^\mu$ is generated at two loops by attaching a quark line to the gluons that were generated by the top loop.



$C_1^2 \text{Im}\langle O_1 O_1 \rangle \sim \alpha_s^2 + \cdots$

$C_1 C_2 \text{Im}\langle O_1 O_2 \rangle \sim \alpha_s^4 + \cdots$

$C_2^2 \text{Im}\langle O_2 O_2 \rangle \sim \alpha_s^6 + \cdots$

The $A \to gg$ calculation transfers directly over to $\chi\chi \to gg$ at NLO only:

- LO: want $C_1^2 \text{Im}\langle O_1 O_1 \rangle$ at leading α_s^2 order.
    - This is just LO $A \to gg$.

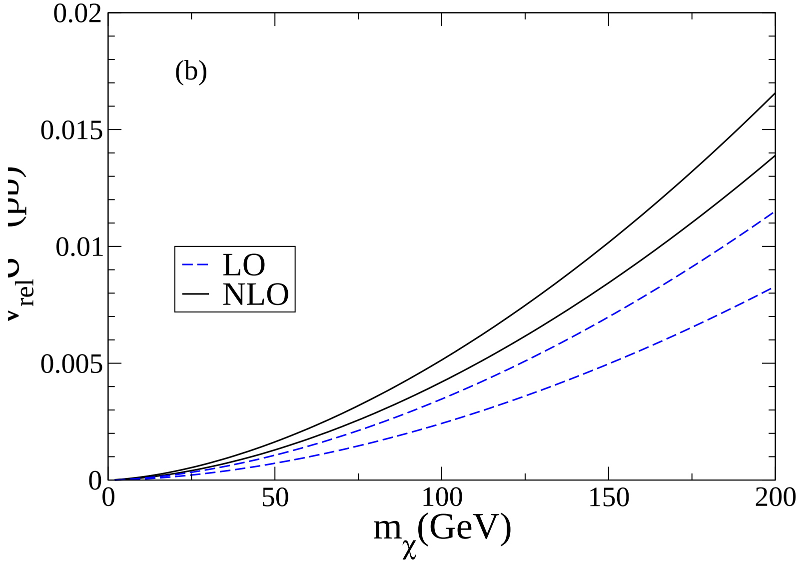
- NLO: want $C_1^2 \text{Im}\langle O_1 O_1 \rangle$ at NLO, α_s^3.
    - This is just NLO $A \to gg$.

- NNLO: want $C_1^2 \text{Im}\langle O_1 O_1 \rangle$ at NNLO, α_s^4.
    - Cannot get this simply from $A \to gg$, since $C_1 C_2 \langle O_1 O_2 \rangle$ also contributes at this order.

We get $\chi\chi \to gg$ at NLO "for free":

$$v_{\rm rel}\sigma_{\rm NLO}(\chi\chi \to gg) = v_{\rm rel}\sigma_{\rm LO}(\chi\chi \to gg) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{97}{4} - \frac{7}{6}N_f + \frac{33-2N_f}{6}\log\frac{\mu^2}{4m_\chi^2}\right)\right]$$
$$= v_{\rm rel}\sigma_{\rm LO}(\chi\chi \to gg)\,[1+0.62]$$

where the last line is for $\mu = 2m_\chi = 2\times(100\ \text{GeV})$ and $N_f = 5$.



← NLO: scale uncertainty ±9%

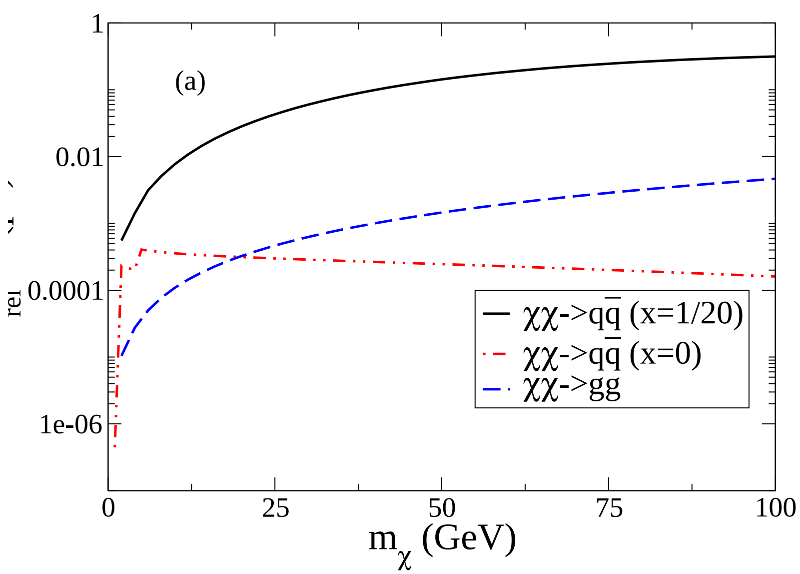
← LO: scale uncertainty ±16%

$\chi\chi \to gg$ cross section is increased by some 60% at NLO.

Where is this useful?

- Early universe: $\chi\chi \to gg$ typically only a small contribution to the total annihilation cross section. Not particularly important.

- Present day: $\chi\chi \to gg$ can be the dominant annihilation mode. Corrections are important for total annihilation cross section and branching fractions $\to$ can affect indirect DM detection rates.



$\leftarrow$ $\chi\chi \to q\bar{q}$ in early universe

$\leftarrow$ $\chi\chi \to gg$ (includes NLO)

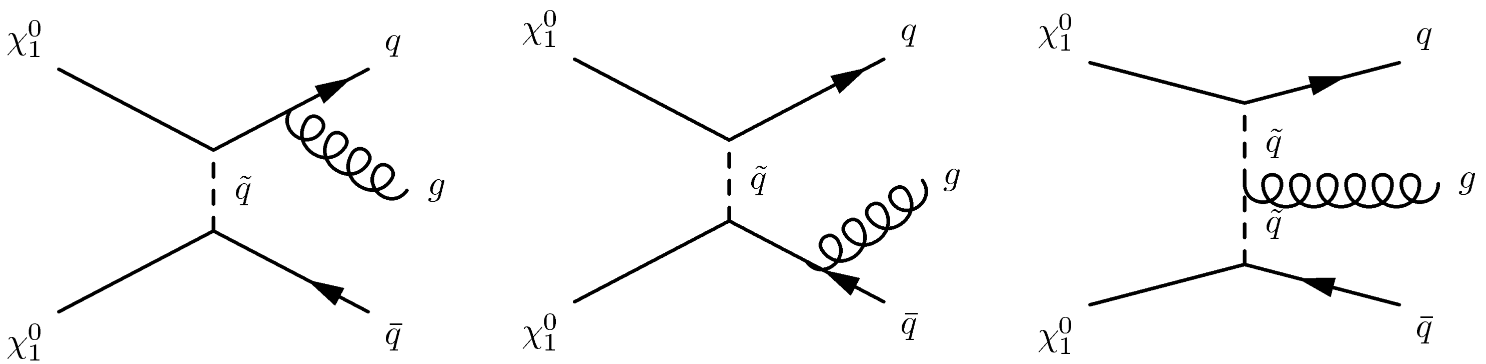
$\leftarrow$ $\chi\chi \to q\bar{q}$ today

Leading QCD corrections to p-wave $\chi\chi \to q\bar{q}$ were calculated in [Flores, Olive, Rudaz 1989] — not a huge effect

Further directions: the dimension-eight amplitude

Remember there were two ways to lift the m_f^2/m_χ^2 suppression:
(1) using the anomaly
(2) going to dimension-eight.

The dimension-eight amplitude was calculated for $\chi\chi \to f\bar{f}\gamma$ in [Flores, Olive, Rudaz 1989]



The full calculation was done in [Drees, Jungman, Kamionkowski, Nojiri 1993].

For $m_q \simeq 0$, the leading $1/M_{\tilde{q}}^8$ part is

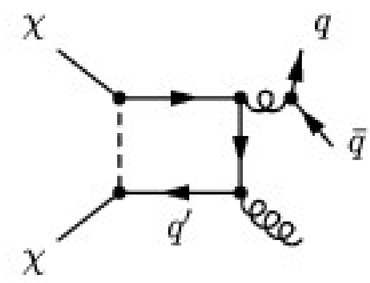
$$v_{\rm rel}\sigma(\chi\chi \to q\bar{q}g) = \frac{4\alpha_s}{15}\frac{m_\chi^6}{16\pi^2}\left[\frac{|g_\ell|^4}{M_{\tilde{q}_L}^8} + \frac{|g_r|^4}{M_{\tilde{q}_R}^8}\right].$$

Not yet computed:

interference term between

(1) dimension-eight $\chi\chi \to q\bar{q}g$, and

(2) dimension-six $\chi\chi \to q\bar{q}g$ through the box with gluon splitting to $q\bar{q}$



- Interference term is order α_s^2 – same order as LO $\chi\chi \to gg$
- Interference term is order $1/\widetilde{M}^6$ – more suppressed than $\chi\chi \to gg$ but less suppressed than pure dimension-eight cross section.

This is work in progress.

$$v_{\rm rel}\sigma \sim \frac{\alpha_s m_\chi^6}{M_{\tilde{q}}^8} \qquad (\chi\chi \to q\bar{q}g \text{ tree level})$$
$$+ \frac{\alpha_s^2 m_\chi^2}{M_{\tilde{q}}^4}\left[1 \qquad (\chi\chi \to gg \text{ LO})\right.$$
$$+\alpha_s \qquad (\chi\chi \to gg \text{ NLO})$$
$$\left.+\frac{m_\chi^2}{M_{\tilde{q}}^2} \qquad (\chi\chi \to q\bar{q}g \text{ dim6} - \text{dim8 interference})\right].$$

Conclusions

- Precision cosmology motivates calculation at higher orders.

- Neutralinos are Majorana fermions – s-wave annihilation is helicity suppressed. Processes that lift the suppression can have a big impact on present-day annihilation rates.

- We calculated NLO QCD corrections to $\chi\chi \to gg$ by using the Adler-Bardeen theorem and known NLO QCD corrections to $A^0 \to gg$: about a +60% effect.

- Calculation of interference term between $\chi\chi \to g^* g \to q\bar{q}g$ and dimension-8 $\chi\chi \to q\bar{q}g$ in progress.
  Same α_s order as LO $\chi\chi \to gg$; relative $m_\chi^2/M_{\tilde{q}}^2$ suppression.

- Implications for indirect detection still need to be worked out.