Neutrinos and extended Higgs sectors

Heather Logan

(Carleton University)

Workshop on TeV-scale Physics and Neutrino Masses

TRIUMF, October 2007

Outline

Triplet model for neutrino mass

Triplet from Littlest Higgs model
- Neutrino mass from triplet versus dimension-5 operator
- Phenomenology

Constraints on triplet models – the ρ parameter

Custodial SU(2) and the Georgi-Machacek model

Renormalization of triplet models

Summary

Triplet-Higgs model for neutrino mass

Consider a complex SU(2)$_L$-triplet scalar with hypercharge 2:

$Q = T^3 + Y/2$

$$\phi = \left(\begin{array}{cc} \phi^{++} & \phi^{+}/\sqrt{2} \\ \phi^{+}/\sqrt{2} & \phi^{0} \end{array} \right)$$

There is only one gauge-invariant dimension-four coupling of ϕ to fermions:

$$\begin{aligned} \mathcal{L} &= Y_{ij} L_i^T \phi\, C^{-1} L_j + \text{h.c.} \\ &= Y_{ij} \left(\ell_L^T, \nu_L^T \right)_i \left(\begin{array}{cc} \phi^{++} & \phi^{+}/\sqrt{2} \\ \phi^{+}/\sqrt{2} & \phi^{0} \end{array} \right) \left(\begin{array}{c} C^{-1}\ell_L \\ C^{-1}\nu_L \end{array} \right)_j + \text{h.c.} \end{aligned}$$

This coupling violates lepton number!

Giving ϕ a vev, $\langle \phi^0 \rangle = v'$, generates Majorana neutrino masses:

$$\mathcal{M}_{ij} = Y_{ij} v'$$

Neutrino masses are experimentally $\sim \mathcal{O}(0.1\text{ eV})$:
$Y_{ij} v'$ must be very small, $\sim 10^{-10}$ GeV.

Feynman rules for $\Delta L = 2$ couplings (all particles outgoing):

$\phi^{--}\ell_i^+\ell_j^+ \quad (i \leq j)$	$2iY_{ij}^* P_R C$
$\phi^-\ell_i^+\bar{\nu}_j$	$i\sqrt{2}Y_{ij}^* P_R C$
$\phi^s\nu_i\nu_j \quad (i \leq j)$	$i\sqrt{2}Y_{ij}C^{-1}P_L$
$\phi^s\bar{\nu}_i\bar{\nu}_j \quad (i \leq j)$	$i\sqrt{2}Y_{ij}^* P_R C$
$\phi^p\nu_i\nu_j \quad (i \leq j)$	$-\sqrt{2}Y_{ij}C^{-1}P_L$
$\phi^p\bar{\nu}_i\bar{\nu}_j \quad (i \leq j)$	$\sqrt{2}Y_{ij}^* P_R C$

[Han, H.L., Mukhopadhyaya & Srikanth, hep-ph/0505260]

- ϕ^s, ϕ^p are the real scalar and pseudoscalar components of ϕ^0.
- C is the charge-conjugation operator.
- If we ignore CP-violating phases then Y_{ij} is symmetric: we've combined the symmetric vertices involving ϕ^{--}, ϕ^s and ϕ^p and written them only for $i \leq j$.
- Flavour structure of leptonic decays is related to Majorana neutrino mass matrix.

This is completely generic.

Framework: Little Higgs models

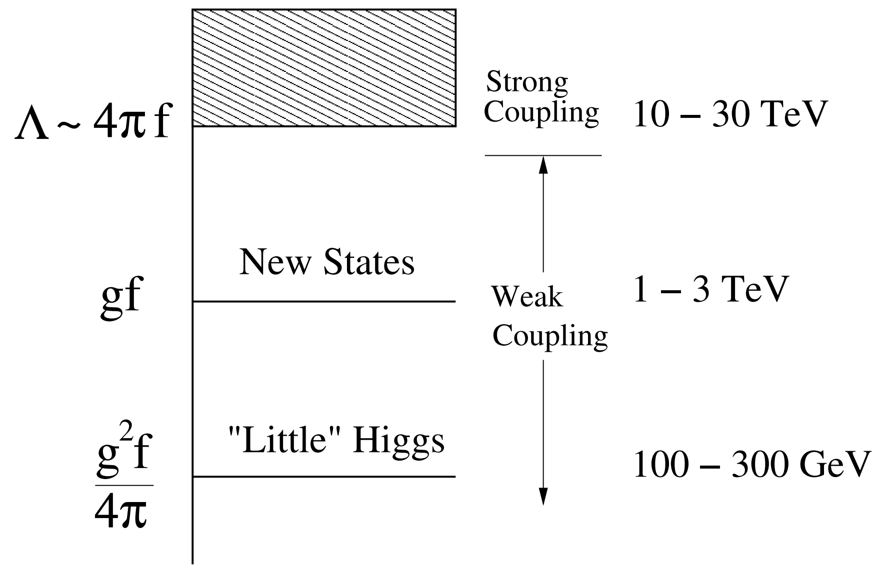
Little Higgs models stabilize the weak scale against one-loop radiative corrections, thereby pushing the cutoff to $\sim$ 10 TeV while maintaining a naturally light Higgs boson.

New particles at $\sim$ 1 TeV cancel off the 1-loop SM quadratic divergence of the Higgs mass.

Higgs is a pseudo-Goldstone boson from global symmetry breaking at scale $\Lambda \sim 4\pi f \sim 10-30$ TeV;

Quadratic divergences cancelled at one-loop level by new states $M \sim gf \sim 1-3$ TeV;

Higgs acquires a mass radiatively at the EW scale $v \sim g^2 f/4\pi \sim 100-300$ GeV.



EW precision constraints $\leftrightarrow$ more complicated model-building. Here we're only interested in the triplet Higgs.

Little Higgs models with triplets:

- Littlest Higgs: 1 complex triplet

- Littlest Higgs with custodial symmetry [Chang]:
  1 complex triplet + 1 real triplet

- Minimal Moose: 1 light complex triplet

- Minimal Moose with custodial symmetry [Chang & Wacker]:
  1 real triplet

- Moose with T-parity [Cheng & Low]: 3 real triplets

Here I'll focus on Littlest Higgs.
Some comments later on models with custodial symmetry.

Littlest Higgs model

[Arkani-Hamed, Cohen, Katz, Nelson, JHEP 0207, 034 (2002)]

The Littlest Higgs model is a nonlinear sigma model broken by a condensate $f \sim$ TeV.

Global symmetry: SU(5) $\longrightarrow$ SO(5)
Nonlinear sigma model field Σ (5$\times$5) contains H and a triplet ϕ.
H and ϕ are Nambu-Goldstone bosons of the global symmetry breaking.

Gauge symmetry: $[\mathrm{SU}(2)]^2 \times [\mathrm{U}(1)]^2 \longrightarrow \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
Embedded in the SU(5) global symmetry $\longrightarrow$ Explicitly breaks global symmetry; makes H and ϕ pseudo-Nambu-Goldstone bosons.

Yukawa interactions: Extra SU(2)-singlet vector-like pair of quarks T, $\bar{T}$ added to top sector.
$\longrightarrow$ Explicitly breaks global symmetry; makes H a pseudo-Nambu-Goldstone boson.

Pseudo-Goldstone bosons and gauge structure

Nonlinear sigma model:

$$\Sigma = e^{2i\Pi/f}\Sigma_0 = \Sigma_0 + \frac{2i}{f}\begin{pmatrix} i\phi^\dagger & h^\dagger/\sqrt{2} & \\ h^*/\sqrt{2} & & h/\sqrt{2} \\ & h^T/\sqrt{2} & -i\phi \end{pmatrix} + \cdots$$

Gauged $[SU(2)\times U(1)]^2$ subgroup:

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & & \\ & & \\ & & \end{pmatrix} \qquad Q_2^a = \begin{pmatrix} & & \\ & & \\ & & -\sigma^a/2 \end{pmatrix}$$

$Y_1 = \mathrm{diag}(-3,-3,2,2,2)/10 \qquad Y_2 = \mathrm{diag}(-2,-2,-2,3,3)/10$

Gauge generators each preserve part of the global symmetry:

$$SU(3)_1 \rightarrow \left(\begin{array}{c|c} 0_{2\times 2} & \\ \hline & V_3 \end{array}\right) \qquad SU(3)_2 \rightarrow \left(\begin{array}{c|c} V_3 & \\ \hline & 0_{2\times 2} \end{array}\right)$$

These symmetries keep H light, but ϕ gets a mass $\sim f \sim$ TeV.

New particle content at the TeV scale:

Z_H, $W_H^\pm$ – SU(2) triplet of gauge bosons from the breaking $[SU(2)]^2 \to SU(2)_L$. Cancels the Higgs mass divergence from $W^\pm$, W^3.

T – vectorlike charge-2/3 quark. Cancels the Higgs mass divergence from the top quark.

$\Phi^{0,+,++}$ – SU(2) triplet of scalars. Cancels the Higgs mass divergence from the Higgs self-interaction.

A_H – U(1) gauge boson from the breaking $[U(1)]^2 \to U(1)_Y$. Cancels the Higgs mass divergence from B^Y. [EW precision favors only one U(1) → no A_H particle]

Model parameters:

f – new physics scale $\sim$ TeV

g_1/g_2 – $SU(2)_{1,2}$ gauge boson coupling ratio [Z_H, $W_H^\pm$]

λ_1/λ_2 – top sector parameter [T]

v' – triplet Φ vev

g_1'/g_2' – $U(1)_{1,2}$ gauge boson coupling ratio [EW precision favours variant with only one U(1)]

Scalar potential

The Littlest Higgs model contains a doublet h and a triplet ϕ:

$$h = (h^+, h^0), \qquad \phi = \begin{pmatrix} \phi^{++} & \phi^+/\sqrt{2} \\ \phi^+/\sqrt{2} & \phi^0 \end{pmatrix}$$

Most general gauge-invariant renormalizable Higgs potential is:

$$\begin{aligned} V &= \lambda_{\phi^2} f^2 \mathrm{Tr}(\phi^\dagger \phi) - \lambda_{h\phi h} f (h \phi^\dagger h^T + h^* \phi h^\dagger) - \mu^2 h h^\dagger + \lambda_{h^4} (h h^\dagger)^2 \\ & \quad + \lambda_{h\phi\phi h} h \phi^\dagger \phi h^\dagger + \lambda_{h^2\phi^2} h h^\dagger \mathrm{Tr}(\phi^\dagger \phi) + \lambda_{\phi^2\phi^2} [\mathrm{Tr}(\phi^\dagger \phi)]^2 \\ & \quad + \lambda_{\phi^4} \mathrm{Tr}(\phi^\dagger \phi \phi^\dagger \phi). \end{aligned}$$

Little Higgs framework: take $\mu^2 \sim f^2/16\pi^2$.
Minimizing the potential gives vevs, $\langle h^0 \rangle = v/\sqrt{2}$ and $\langle \phi^0 \rangle = v'$:

$$v^2 = \frac{\mu^2}{\lambda_{h^4} - \lambda_{h\phi h}^2/\lambda_{\phi^2}}, \qquad v' = \frac{\lambda_{h\phi h} v^2}{2\lambda_{\phi^2} f}.$$

- Neglected subleading contributions from $h^2\phi^2$, ϕ^4 terms.
- Notice $v' \sim v^2/f$ and is induced by the $h\phi h$ term.

Scalar mass eigenstates

Scalar mass eigenstates are mixtures of h and ϕ.
Mass hierarchy leads to small mixing, $\sim \mathcal{O}(v/f)$.

To leading order in v/f, (ϕ^{++} does not mix)

$$\begin{aligned} \Phi^p &= c_p\sqrt{2}\,\mathrm{Im}\phi^0 - s_p\sqrt{2}\,\mathrm{Im}h^0, \\ \Phi^+ &= c_+\phi^+ - s_+h^+, \\ \Phi^s &= c_0\sqrt{2}\,\mathrm{Re}\phi^0 - s_0\sqrt{2}\,\mathrm{Re}h^0 \end{aligned}$$

where $s_p = 2\sqrt{2}v'/v$, $s_+ = 2v'/v$ (fixed by Goldstone boson) and $s_0 \simeq 2\sqrt{2}v'/v$ (leading contrib from potential).
Small mixing with doublet yields couplings to ordinary fermions.

Masses to leading order in v/f are

$$\begin{aligned} M_\Phi^2 &\simeq \lambda_{\phi^2} f^2 \qquad \text{(degenerate)} \\ m_H^2 &\simeq 2(\lambda_{h^4} - \lambda_{h\phi h}^2/\lambda_\phi^2)v^2 = 2\mu^2 \end{aligned}$$

Littlest Higgs model: potential for h and ϕ is generated radiatively (Coleman-Weinberg potential).

Model structure leads to $\lambda_{h^4} = \lambda_{\phi^2}/4$.

Gives a relation between parameters:

$$M_\Phi^2 = \frac{2m_H^2 f^2}{v^2} \frac{1}{[1 - (4v' f/v^2)^2]}$$

Must have $M_\Phi^2 > 0$:

$$\frac{v'^2}{v^2} < \frac{v^2}{16 f^2}.$$

$M_\Phi \gtrsim \sqrt{2} f m_H / v$

Neutrino masses from triplet in Littlest Higgs model

$$\mathcal{M}_{ij} = Y_{ij} v'$$

Neutrino masses are experimentally $\sim \mathcal{O}(0.1 \text{ eV})$:
$Y_{ij} v'$ must be very small, $\sim 10^{-10}$ GeV.

Two possibilities:

(1) $v' \sim v^2/f \sim 1$ GeV and $Y_{ij} \sim 10^{-10}$
Physics behind small neutrino masses is in Y_{ij}

(2) $Y_{ij} \sim 1$ and $v' \sim 10^{-10}$ GeV
Physics behind small neutrino masses is in Coleman-Weinberg potential.
Technically natural: $\lambda_{h\phi h} = 0$ preserves lepton number.

or (3) somewhere in between.

Other sources of neutrino mass?

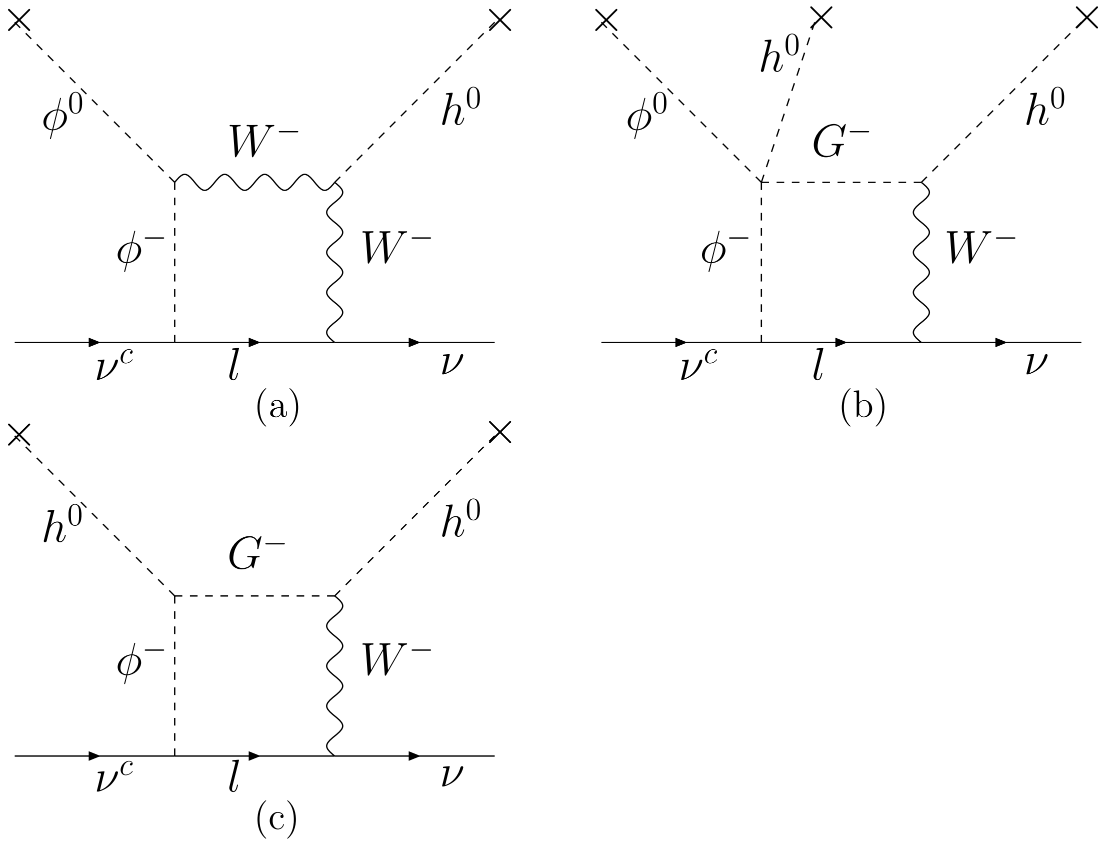
Low cutoff $\Lambda \sim 10$ TeV in Little Higgs models:
dimension-5 operator might be significant.

$$\mathcal{L}_5 = Y_5 \frac{(hL)^2}{\Lambda}$$

Unless Y_5 is tiny, neutrino masses will be way too large.

To avoid this, have to postulate that there is no additional lepton-number violating physics at scale Λ, aside from our $L\phi L$ coupling.

Have to also avoid ops induced by Coleman-Weinberg potential:



(a) and (b) suppressed by [loop factor]$\times v^2/f^2$ relative to $Y_{ij}v'$

(c) suppressed by [loop factor] relative to $Y_{ij}v'$

Phenomenology: triplet decays to leptons

Proceeds through LNV coupling.

$$\Gamma(\phi^{++} \to \ell_i^+ \ell_j^+) = \begin{cases} \frac{|Y_{ii}|^2 m_\phi}{8\pi}, & (i = j) \\ \frac{|Y_{ij}|^2 m_\phi}{4\pi} & (i < j) \end{cases}$$

$$\Gamma(\phi^{+} \to \ell_i^+ \bar{\nu}_j) = \frac{|Y_{ij}|^2 m_\phi}{8\pi},$$

$$\Gamma(\phi^{s} \to \nu_i \nu_j + \bar{\nu}_i \bar{\nu}_j) = \begin{cases} \frac{|Y_{ii}|^2 m_\phi}{8\pi}, & (i = j) \\ \frac{|Y_{ij}|^2 m_\phi}{4\pi}, & (i < j) \end{cases}$$

$$\Gamma(\phi^{p} \to \nu_i \nu_j + \bar{\nu}_i \bar{\nu}_j) = \begin{cases} \frac{|Y_{ii}|^2 m_\phi}{8\pi}, & (i = j) \\ \frac{|Y_{ij}|^2 m_\phi}{4\pi}, & (i < j) \end{cases}$$

All of order $(m_\nu / v')^2 m_\phi$.

Phenomenology: triplet decays to gauge and Higgs bosons

Decays to Higgs, longitudinal gauge bosons (Goldstones):

$$\begin{aligned}
\Gamma(\phi^{++} \to W_L^+ W_L^+) &\approx \frac{v'^2 m_\phi^3}{2\pi v^4}, \\
\Gamma(\phi^{+} \to W_L^+ Z_L) &\approx \Gamma(\phi^+ \to W_L^+ h) \approx \frac{v'^2 m_\phi^3}{4\pi v^4}, \\
\Gamma(\phi^{s} \to Z_L Z_L) &\approx \Gamma(\phi^s \to hh) \approx \frac{v'^2 m_\phi^3}{4\pi v^4} \\
\Gamma(\phi^{p} \to Z_L h) &\approx \frac{v'^2 m_\phi^3}{2\pi v^4}
\end{aligned}$$

Coupling is via $h\phi h$ term:

$$\Gamma(V_L V_L, hh, V_L h) = \frac{v'^2 m_\phi^3}{2\pi v^4} = (\lambda_{h\phi h} f)^2 \frac{m_\phi}{8\pi}.$$

Phenomenology: triplet decays to transverse gauge bosons

Decays to transverse gauge bosons are through $g^2 v'$ vertex:

$$\Gamma(\phi^{++} \to W_T^+ W_T^+) \approx \frac{g^4 v'^2}{4\pi m_\phi},$$

$$\Gamma(\phi^{+} \to W_T^+ Z_T) \approx \frac{g^4 v'^2}{8\pi m_\phi c_W^2},$$

$$\Gamma(\phi^{s} \to Z_T Z_T) \approx \frac{g^4 v'^2}{8\pi m_\phi c_W^4}$$

- Smaller by $g^4 v^4 / m_\phi^4$ than transverse modes.

- Same dependence on v'^2.

Phenomenology: triplet decays to heavy quarks

ϕ couples to 3rd-generation quarks through mixing with h:
$\sim v'/v$, tiny when $v' \ll 1$ GeV.

However: structure of nonlinear sigma model leads to direct higher-dim coupling of ϕ to fermions!

$$\mathcal{L}_{\text{Yuk}} = \frac{\lambda_1 f}{2} \epsilon_{ijk} \epsilon_{xy} \chi_i \Sigma_{jx} \Sigma_{ky} t^c + \lambda_2 f \bar{T} \bar{t}'^c \quad \text{with } \chi = (b, t, T),$$

$$\Sigma = e^{2i\Pi/f} \begin{pmatrix} & & \mathbf{1}_{2\times 2} \\ & 1 & \\ \mathbf{1}_{2\times 2} & & \end{pmatrix} \quad \text{and} \quad \Pi = \begin{pmatrix} \mathbf{0}_{2\times 2} & h^\dagger/\sqrt{2} & i\phi^\dagger \\ h^*/\sqrt{2} & 0 & h/\sqrt{2} \\ -i\phi & h^T/\sqrt{2} & \mathbf{0}_{2\times 2} \end{pmatrix}$$

Expansion generates higher-dim terms like $h^T + \phi h^\dagger / f + \cdots$

Inserting h vev generates $\phi \bar{f} f$ couplings $\sim y_f v/f$.
Not suppressed by v'!

Couplings generated through h–ϕ mixing would be $\sim y_f v'/v$.

$$\Gamma(\phi^+ \to t\bar{b}) \approx \frac{N_c m_t^2 m_\phi}{32\pi f^2},$$

$$\Gamma(\phi^{s,p} \to t\bar{t}) \approx \frac{N_c m_t^2 m_\phi}{16\pi f^2}$$

$$\Gamma(\phi^{s,p} \to b\bar{b}) \approx \frac{N_c m_b^2 m_\phi}{16\pi f^2}$$

Partial widths $\sim (m_t^2/v^2)(v^2/f^2)m_\phi$. Not suppressed by v'!

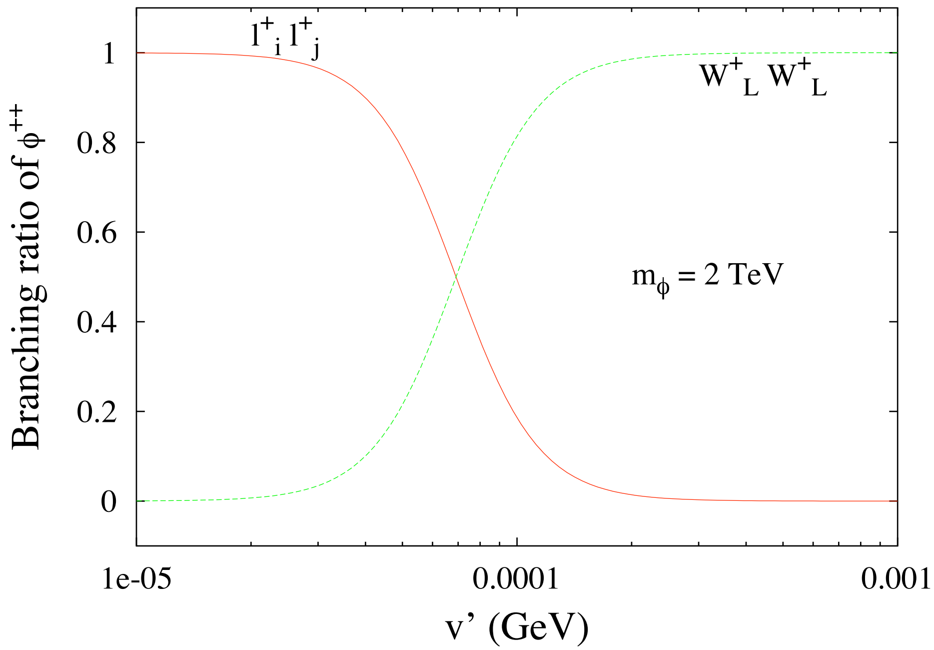
Unique feature of little Higgs models: not present in general triplet neutrino mass model.

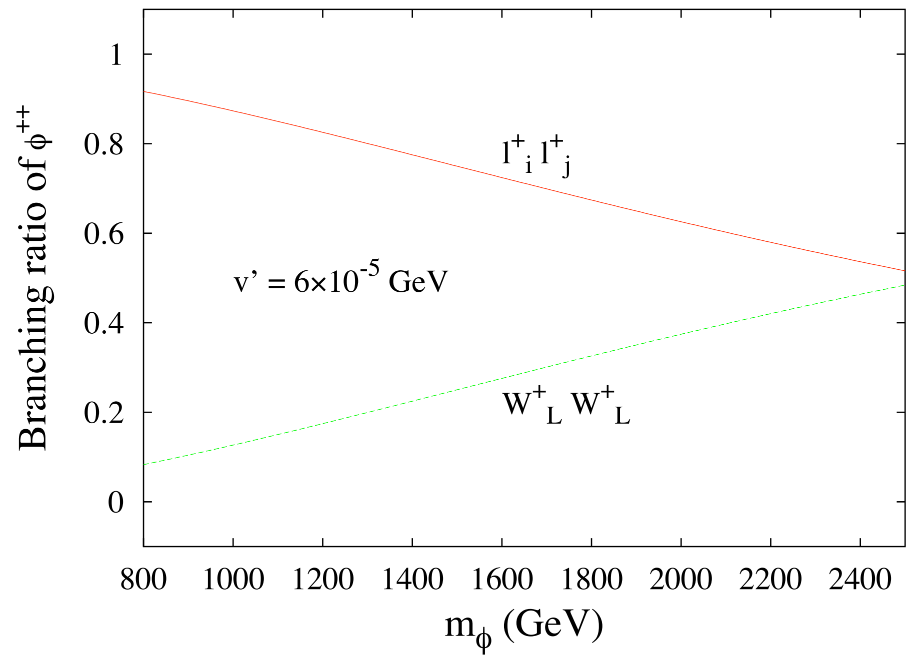
Littlest Higgs model: ϕ can also decay to heavy top-partner T:

$$\Gamma(\phi^+ \to T\bar{b}) \approx \frac{N_c m_t^2 m_\phi}{32\pi f^2} \left(\frac{\lambda_1}{\lambda_2}\right)^2 \left[1 - \left(\frac{m_T}{m_\phi}\right)^2\right]^2$$

$$\Gamma(\phi^{s,p} \to T\bar{t} + t\bar{T}) \approx \frac{N_c m_t^2 m_\phi}{16\pi f^2} \left(\frac{\lambda_1}{\lambda_2}\right)^2 \left[1 - \left(\frac{m_T}{m_\phi}\right)^2\right]^2$$

Phenomenology: triplet branching fractions: ϕ^{++}





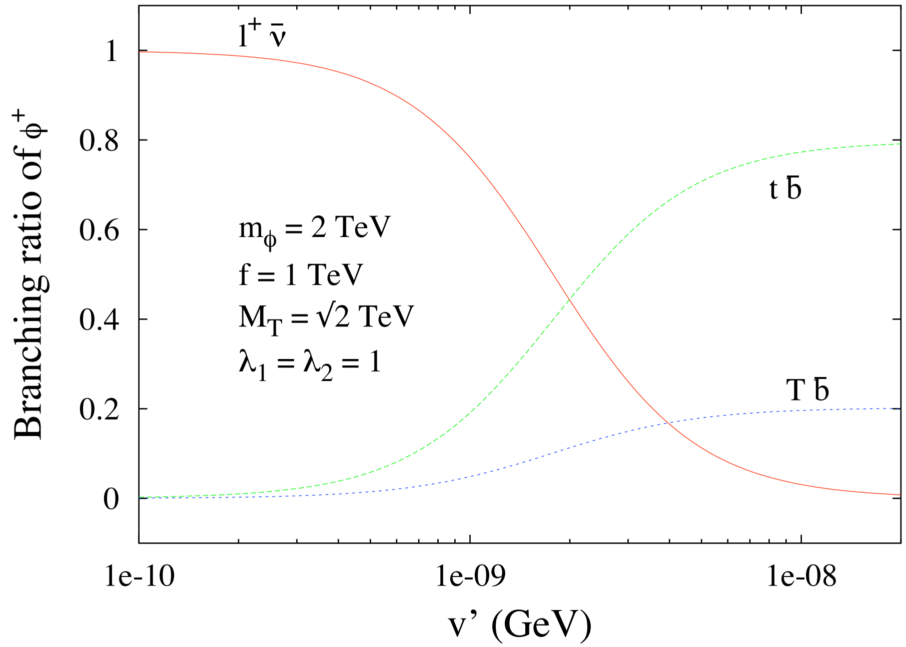
[Han, H.L., Mukhopadhyaya & Srikanth, hep-ph/0505260]

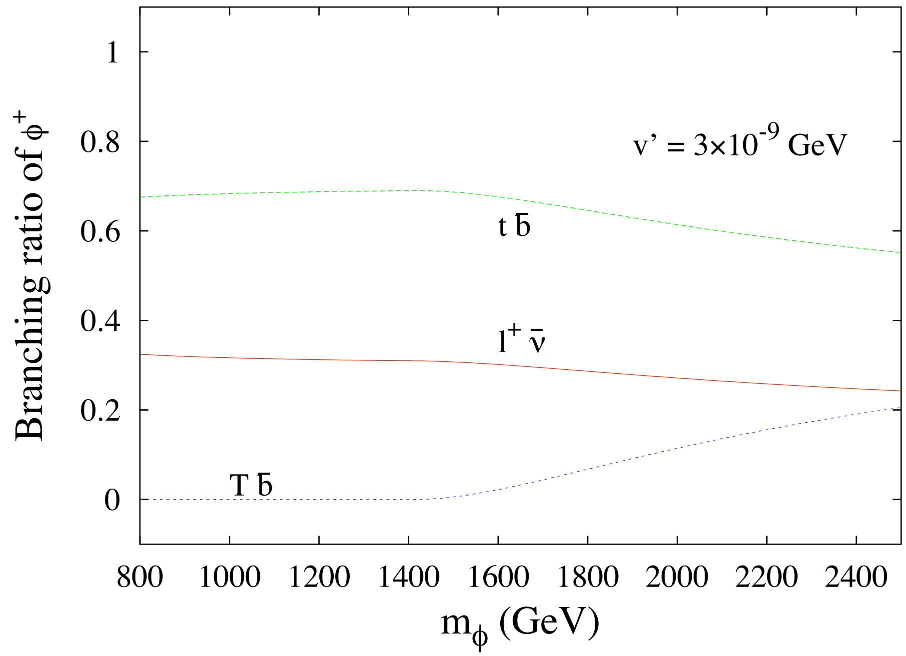
$$\Gamma(\ell_i^+\ell_j^+) = \begin{cases} \frac{|Y_{ii}|^2 m_\phi}{8\pi}, & (i=j) \\ \frac{|Y_{ii}|^2 m_\phi}{4\pi} & (i<j) \end{cases}, \qquad \Gamma(W_L^+W_L^+) \approx \frac{v'^2 m_\phi^3}{2\pi v^4}, \qquad Y_{ij}v' \sim 10^{-10}\ \text{GeV}$$

Crossover for $v' \sim$ few 10^{-5} GeV ($Y \sim$ few 10^{-6}).

Note relative m_ϕ^2 growth of $W_L^+W_L^+$ vs $\ell^+\ell^+$: longitudinal pol'n vector $\sim (E_W/m_W)^2$ enhancement.

Phenomenology: triplet branching fractions: ϕ^+





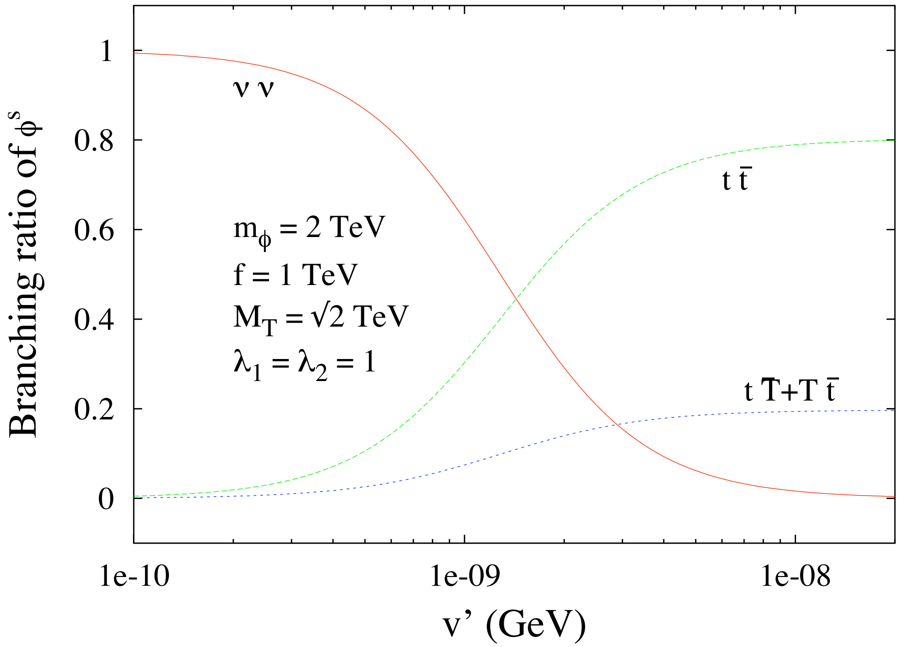
[Han, H.L., Mukhopadhyaya & Srikanth, hep-ph/0505260]

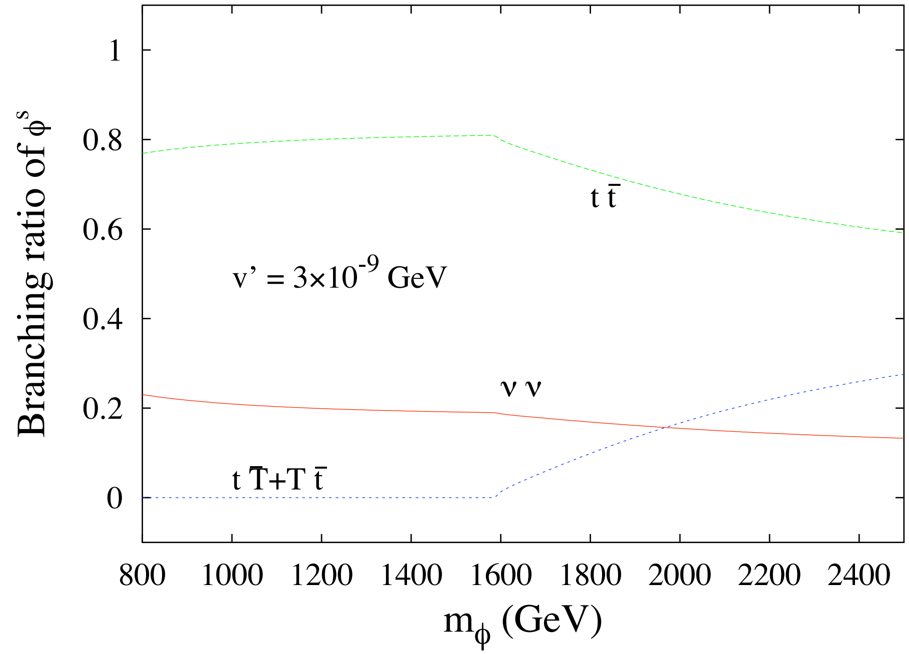
Crossover to $t\bar{b}$ decays for $v' \sim 10^{-9}$ GeV ($Y \sim 0.1$).

Due entirely to nonlinear sigma model expansion!

WZ mode remains insignificant.

Phenomenology: triplet branching fractions: ϕ^s





[Han, H.L., Mukhopadhyaya & Srikanth, hep-ph/0505260]

Again crossover to $t\bar{t}$ decays for $v' \sim 10^{-9}$ GeV ($Y \sim 0.1$).
Due entirely to nonlinear sigma model expansion!

ZZ, hh modes remain insignificant.

ϕ^p decays are the same.

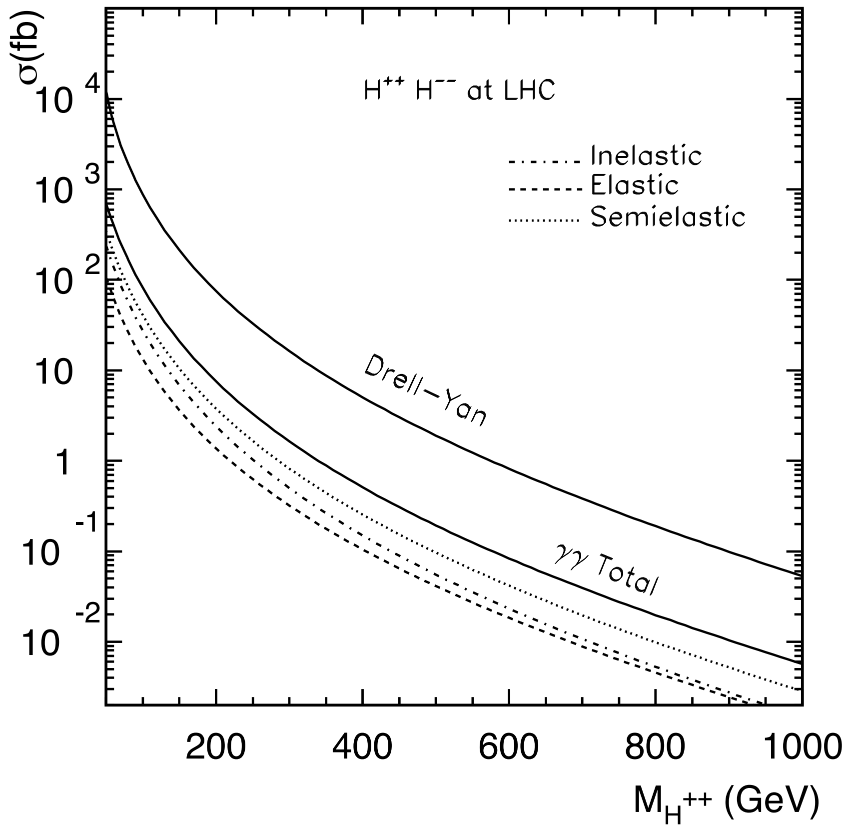
Phenomenology: production of triplet states

Single-production from couplings to gauge bosons:
- $\mathcal{L} = |\mathcal{D}_\mu \phi|^2$ gives ϕVV couplings $\sim g v'$.
- On the edge of observability at LHC when $v' \sim 1$ GeV.
- When LNV decays are relevant, $v' \ll$ GeV and ϕVV is extremely tiny.

Pair production from couplings to gauge bosons:
- $\mathcal{L} = |\mathcal{D}_\mu \Phi|^2$ gives $\phi\phi VV$ couplings $\sim g^2$.
- Unsuppressed couplings!
- Pair production kinematically suppressed for heavy ϕ.

Single-production from LNV coupling to leptons:
- $e^- e^- \to \phi^{--}$, s-channel production in $e^- e^-$ mode of ILC.
- Coupling $\sim Y_{ee}$.

Focus on pair production of $\phi^{++}\phi^{--}$ at LHC.



Production mainly electromagnetic.

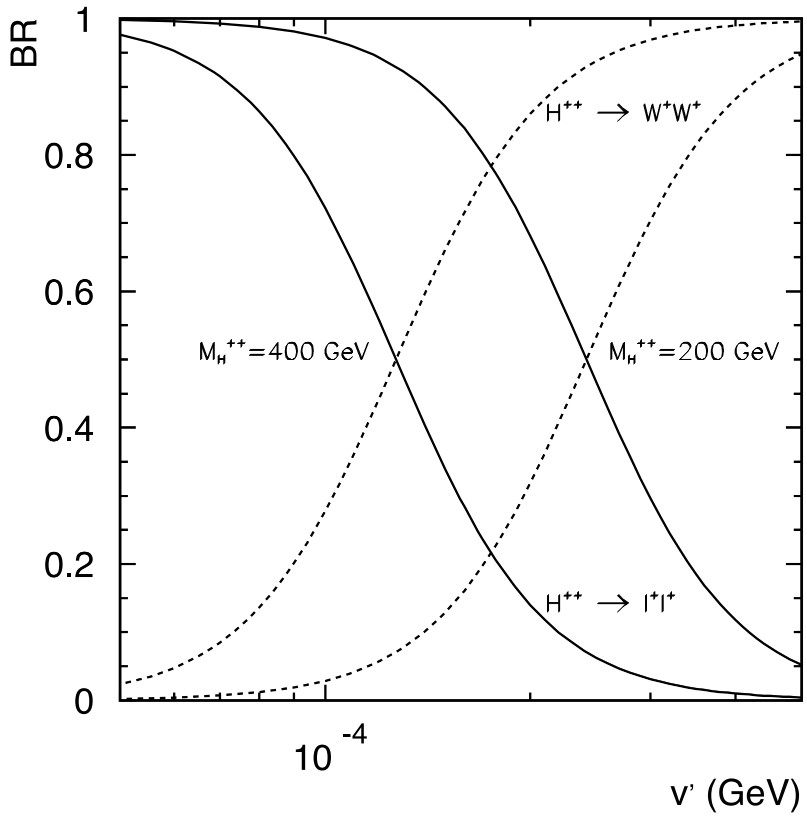
[Han, Mukhopadhyaya, Si, & Wang, arXiv:0706.0441]

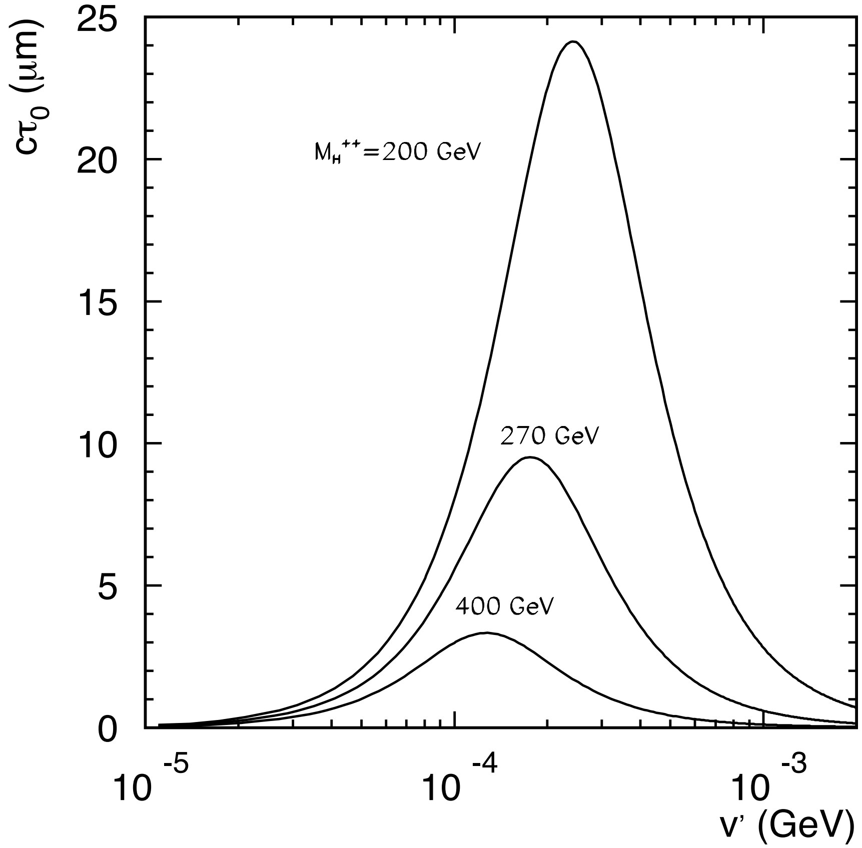
Decay modes:

- $\ell_i^+\ell_j^+$ and W^+W^+: interplay of Y_{ij} and v'.
- ϕ^+W^+, $\phi^+\phi^+$: depend on ϕ mass splittings.

Study ignored these – mass splittings due to EWSB typically $\sim m_W^2/m_\phi$.

Decay length maximized at BR crossover:

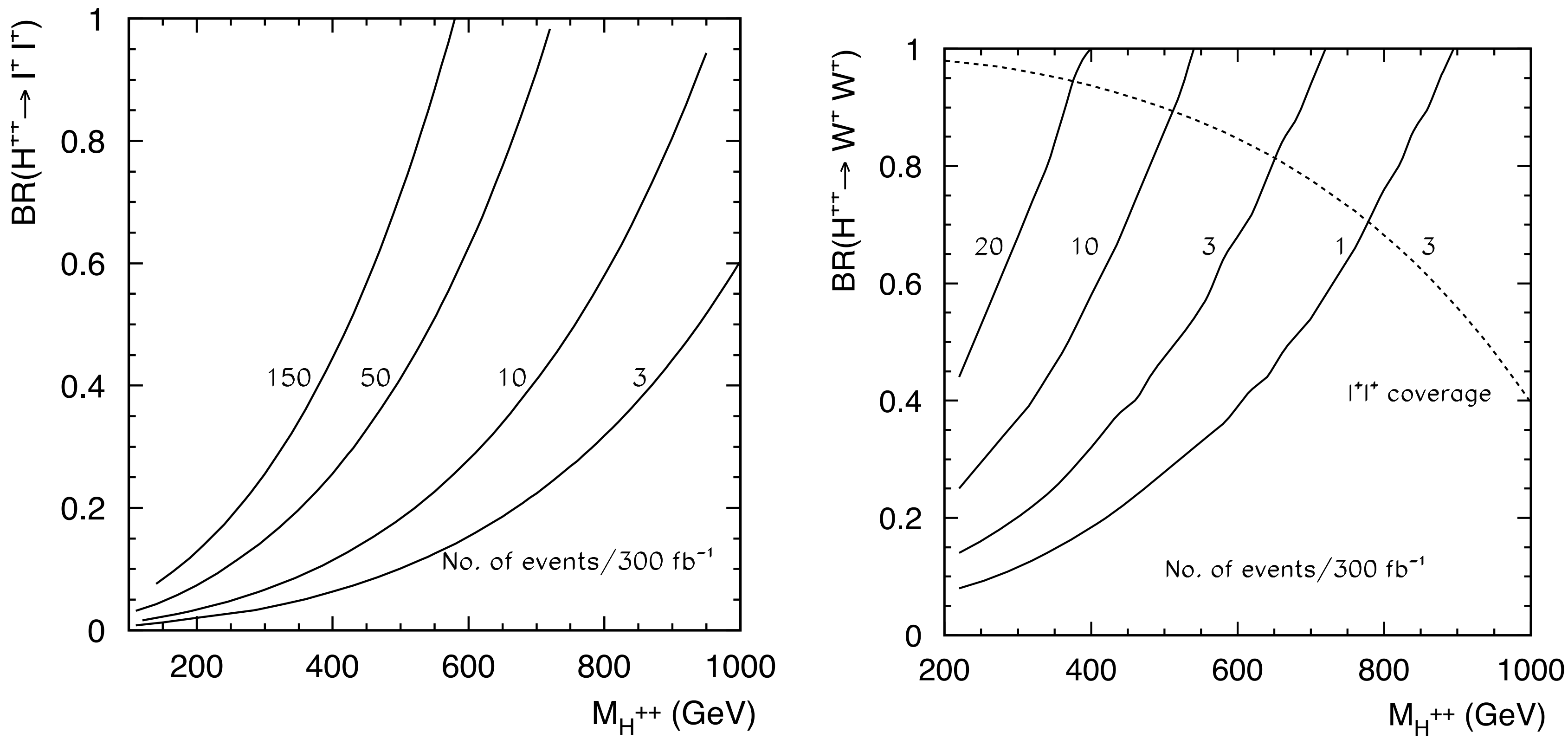




[Han, Mukhopadhyaya, Si, & Wang, arXiv:0706.0441]

... maybe detectable with enough boost, but ϕ is not long-lived.

Event numbers after cuts: (LHC, 300 fb^{-1})



[Han, Mukhopadhyaya, Si, & Wang, arXiv:0706.0441]

- Nearly background-free: mainly a question of statistics.
- Reach in $\ell^+\ell^+$ channel up to 1000 GeV for BR > 0.6.
- Reach in one channel or the other up to ~ 600 GeV.

Flavour structure of decays reflects $\mathcal{M}_{ij}$!

Electroweak precision constraints on triplet models

SM has an accidental "custodial" SU(2) symmetry that keeps $\rho \equiv \frac{m_W^2}{m_Z^2 c_W^2} = 1$ at tree level.

SM Higgs : $$h = \begin{pmatrix} h^{0*} & h^+ \\ -h^{+*} & h^0 \end{pmatrix} \quad \text{with} \quad \langle h \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}$$

Higgs potential and kinetic terms are invariant under SU(2) rotations on either left or right side: SU(2)$_L \times$SU(2)$_R$.

Higgs vev breaks SU(2)$_L \times$SU(2)$_R$ down to the diagonal SU(2) subgroup: this is the custodial SU(2).

SU(2)$_L$ is gauged; SU(2)$_R$ is a global symmetry.

Triplet vev violates custodial SU(2): $\rho \neq 1$ at tree level!
Tight constraints on triplet models from ρ parameter.

[One-loop: coming in a few slides.]

Easy to work out general case:
- Need at least one doublet to give mass to quarks.
- Add a $Y = 2$ complex triplet or $Y = 0$ real triplet:

$$Y = 2: \quad \phi = \begin{pmatrix} \phi^{++} \\ \phi^{+} \\ \phi^{0} \end{pmatrix} \qquad\qquad Y = 0: \quad \xi = \begin{pmatrix} \xi^{+} \\ \xi^{0} \\ \xi^{-} \end{pmatrix}$$

Vacuum expectation values: $\langle \phi^0 \rangle = v'$, $\langle \xi^0 \rangle = v_0$.

Complex triplet with vev v' plus doublet with vev $v_d \sim 246$ GeV:

$$m_W^2 = \frac{g^2}{4}(v_d^2 + 4v'^2), \qquad m_Z^2 = \frac{g^2}{4c_W^2}(v_d^2 + 8v'^2)$$

$$\text{so} \qquad \rho = \frac{v_d^2 + 4v'^2}{v_d^2 + 8v'^2} \simeq 1 - \frac{4v'^2}{v_d^2}$$

Real triplet with vev v_0 plus doublet with vev $v_d \sim 246$ GeV:

$$m_W^2 = \frac{g^2}{4}(v_d^2 + 4v_0^2), \qquad m_Z^2 = \frac{g^2}{4c_W^2}v_d^2$$

$$\text{so} \qquad \rho = \frac{v_d^2 + 4v_0^2}{v_d^2} = 1 + \frac{4v_0^2}{v_d^2}$$

Rho parameter constraint forces $v', v_0 \lesssim$ few GeV.

Not a concern for our tiny ϕ vev!
Single production via $VV\phi$ can never be very strong.

There is a way around this ρ parameter constraint.

Consider a model with one doublet, one $Y = 2$ complex triplet, and one $Y = 0$ real triplet.

$$\rho = \frac{v_d^2 + 4v'^2 + 4v_0^2}{v_d^2 + 8v'^2} = 1 \quad \text{if} \quad v' = v_0.$$

Cancellation between ρ parameter shifts from complex and real triplet. Now triplet vevs can be large!

This is the Georgi-Machacek model.
Higgs sector combines one complex triplet and one real triplet into a multiplet of SU(2)$_L$×SU(2)$_R$:

$$\chi = \begin{pmatrix} \phi^{0*} & \xi^{+} & \phi^{++} \\ -\phi^{+*} & \xi^{0} & \phi^{+} \\ \phi^{++*} & \xi^{-} & \phi^{0} \end{pmatrix}, \qquad \langle\chi\rangle = \begin{pmatrix} v' & 0 & 0 \\ 0 & v' & 0 \\ 0 & 0 & v' \end{pmatrix}$$

so that $v_0 = v'$.

SU(2)$_L$ is gauged; SU(2)$_R$ is a global symmetry.
Higgs potential constructed to be symmetric under SU(2)$_L$×SU(2)$_R$.

[Chanowitz & Golden]

v' breaks $SU(2)_L \times SU(2)_R$ down to the diagonal SU(2) subgroup: this is the (global) custodial SU(2).

Model preserves $\rho = 1$ at tree level and v' is not constrained!

Physical states:

- $\mathrm{SU}(2)_c$ 5-plet: consists entirely of triplets ϕ and ξ. No couplings to fermions. Couples to gauge boson pairs $\sim gv'$.
- $\mathrm{SU}(2)_c$ 3-plet: mixture of doublet and complex triplet. (SM Goldstones form such a triplet.) Couples to fermions via doublet component. No ϕVV coupling.
- Two $\mathrm{SU}(2)_c$ singlets (which mix in general): one is the SM (doublet) Higgs and the other is a mixture of the neutral triplet states. SM Higgs part couples to fermions; both couple to gauge boson pairs.

Georgi-Machacek structure is used in Littlest Higgs with Custodial Symmetry model [Chang].

Breaking of custodial SU(2)

For a doublet the custodial SU(2) is unbroken in the gauge sector: ρ remains a prediction of the model even after radiative corrections are included.

But for our combined triplets, the SU$(2)_R$ is broken by the gauging of hypercharge as its T^3 generator.

Custodial SU(2)-breaking feeds in to the Higgs potential at one-loop and ρ gets a counterterm beyond tree level.

Triplet-Higgs models beyond tree level

To use electroweak precision constraints, want to work at 1-loop.

First consider the SM.

Precision measurements are all in the EW gauge sector: determined by
- 3 parameters g, g' and v
- fermion quantum numbers (these don't get renormalized)

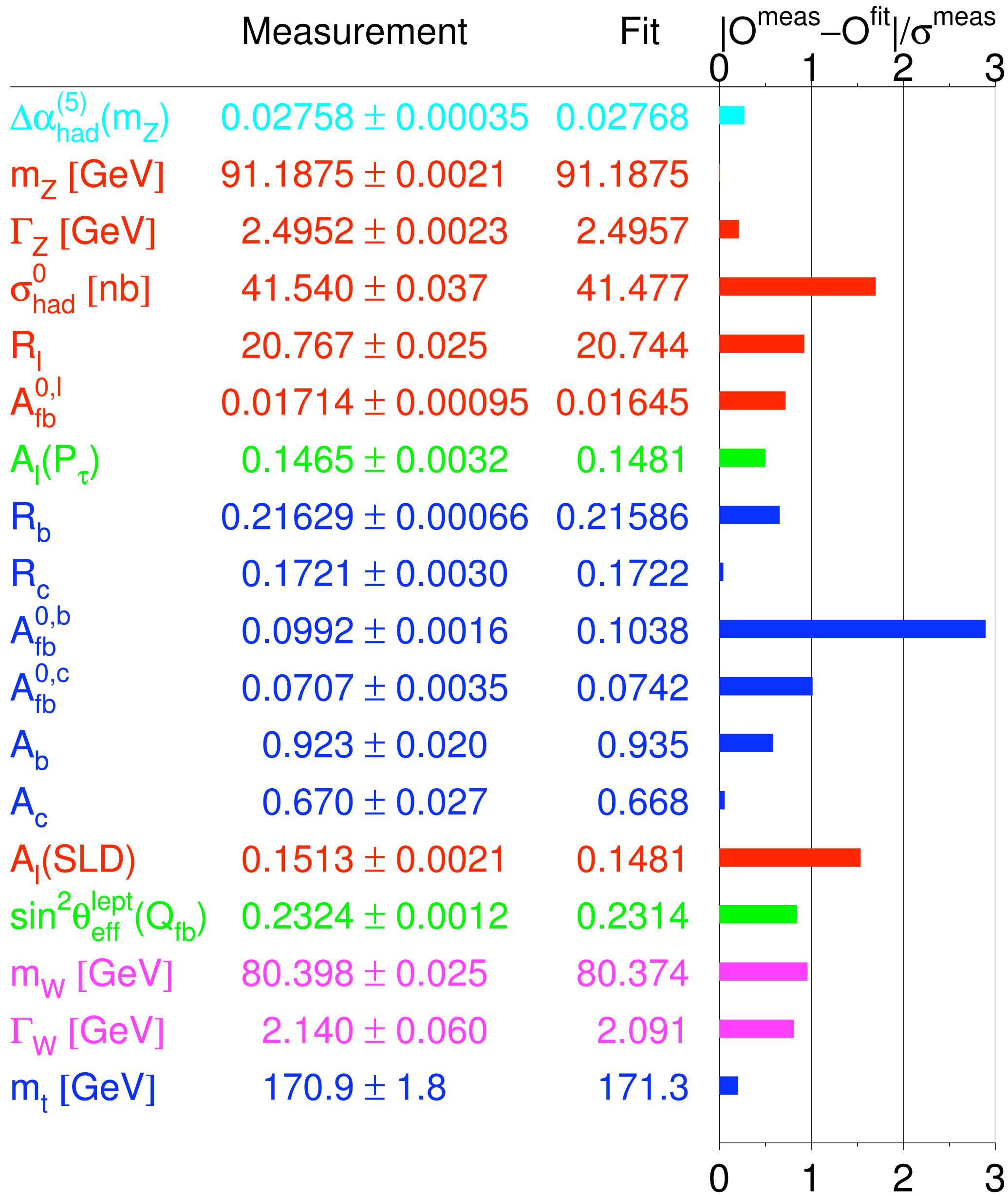
These 3 renormalizable parameters of the SM get counterterms beyond tree level.

Fix them with 3 "inputs"; then the rest of the EW precision observables are predicted. Usual choice is:

$$G_\mu = 1.16639(1) \times 10^{-5}\ \text{GeV}$$
$$M_Z = 91.1876(21)\ \text{GeV}$$
$$\alpha(M_Z)^{-1} = 127.934 \pm 0.027$$

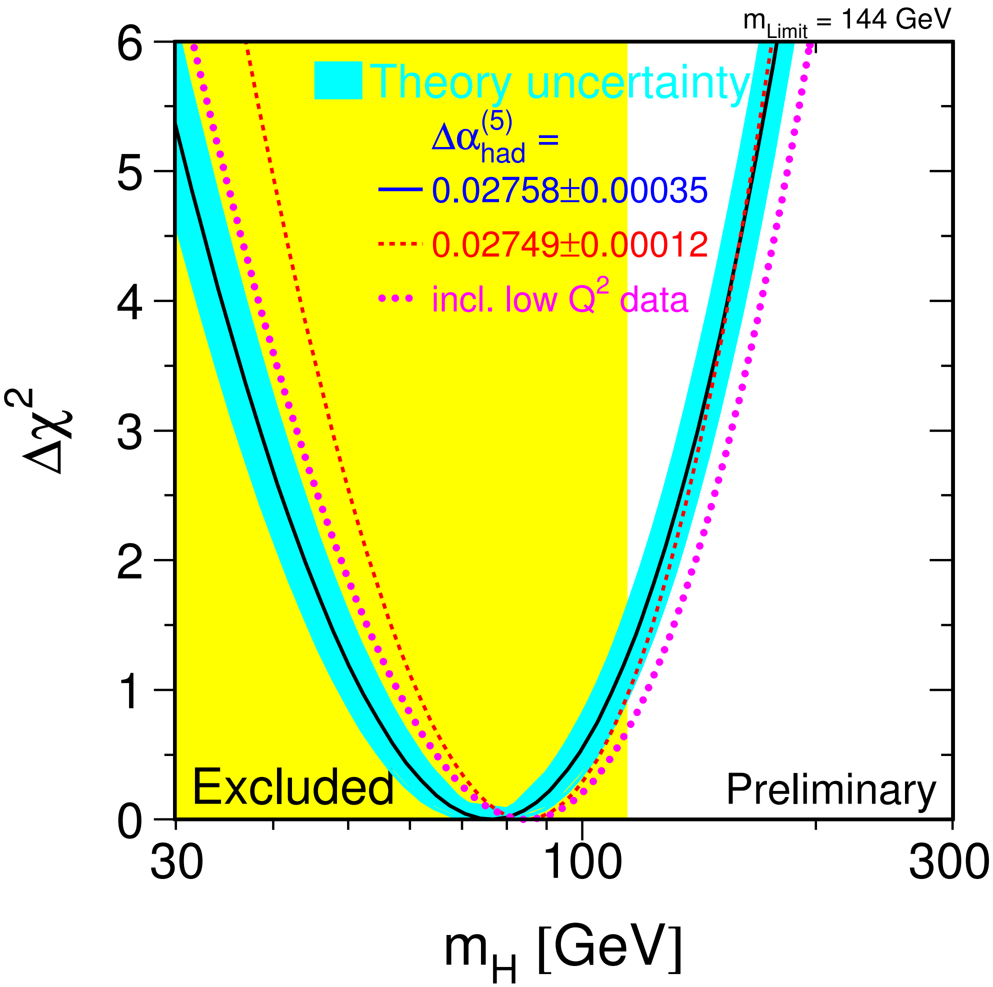
In practice, long list of EW precision observables are fitted to the 1-loop SM with three free parameters (counterterms).

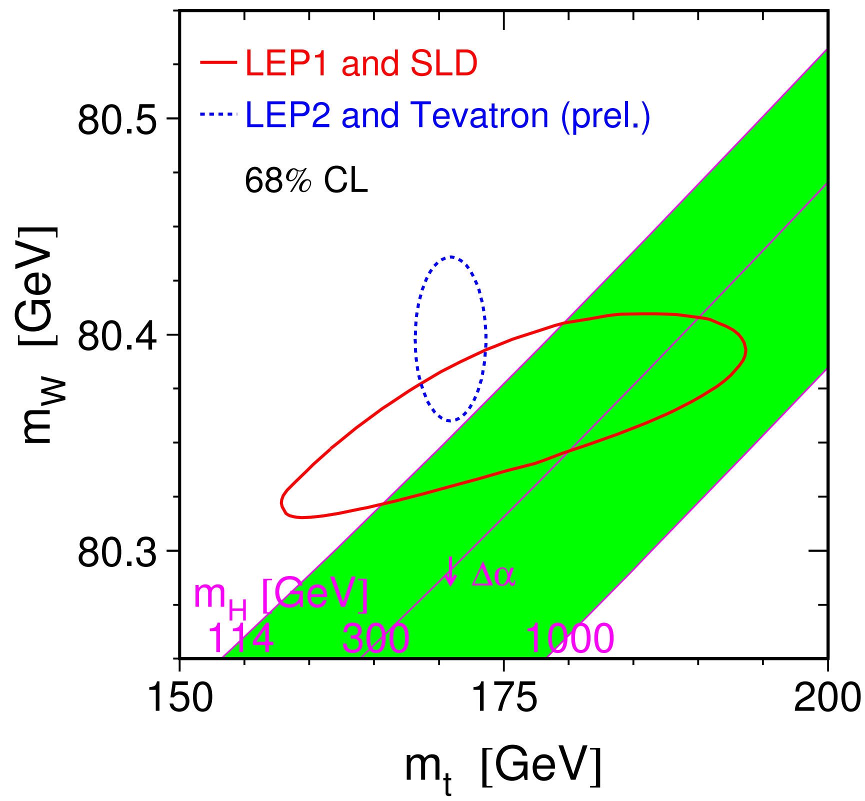
Top mass is not used to fix a counterterm; it shows up only at 1-loop in the corrections.



Higgs mass is also not used to fix a counterterm; it shows up at 1-loop in the corrections.

EW precision fit can then be used to constrain the SM Higgs mass.





Much of the power of the Higgs mass fit comes from M_W.

SM prediction at 1-loop: M_W is defined through muon decay:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2 \bar{s}_\theta^2}[1 + \Delta r_{\rm SM}] \qquad \text{with} \quad \bar{s}_\theta^2 = 1 - \frac{M_W^2}{M_Z^2} \quad \text{(on shell)}$$

$$\text{and} \quad \Delta r_{\rm SM} = -\frac{\delta G_\mu}{G_\mu} + \frac{\delta\alpha}{\alpha} - \frac{\delta \bar{s}_\theta^2}{\bar{s}_\theta^2} - \frac{\delta M_W^2}{M_W^2}.$$

All these are fixed in terms of:

- self-energies as enter through the counterterm-fixing conditions
- vertex and box diagrams contributing to muon decay

$$\frac{\delta G_\mu}{G_\mu} = -\frac{\Pi_{WW}(0)}{M_W^2} + \delta_{\text{box,vertex}}$$

$$\frac{\delta\alpha}{\alpha} = \Pi'_{\gamma\gamma}(0) + 2\frac{\bar{s}_\theta^2}{\bar{c}_\theta^2}\frac{\Pi_{\gamma Z}(0)}{M_Z^2}$$

$$\frac{\delta M_W^2}{M_W^2} = \frac{\Pi_{WW}(M_W^2)}{M_W^2}$$

$$\frac{\delta \bar{s}_\theta^2}{\bar{s}_\theta^2} = \frac{\bar{c}_\theta^2}{\bar{s}_\theta^2}\left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right] = \frac{\bar{c}_\theta^2}{\bar{s}_\theta^2}\left[\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2}\right]$$

Top mass dependence enters quadratically through $\delta\bar{s}_\theta^2/\bar{s}_\theta^2$.

Key difference with triplet models:
4 gauge sector parameters instead of 3. g, g', v, and v'.

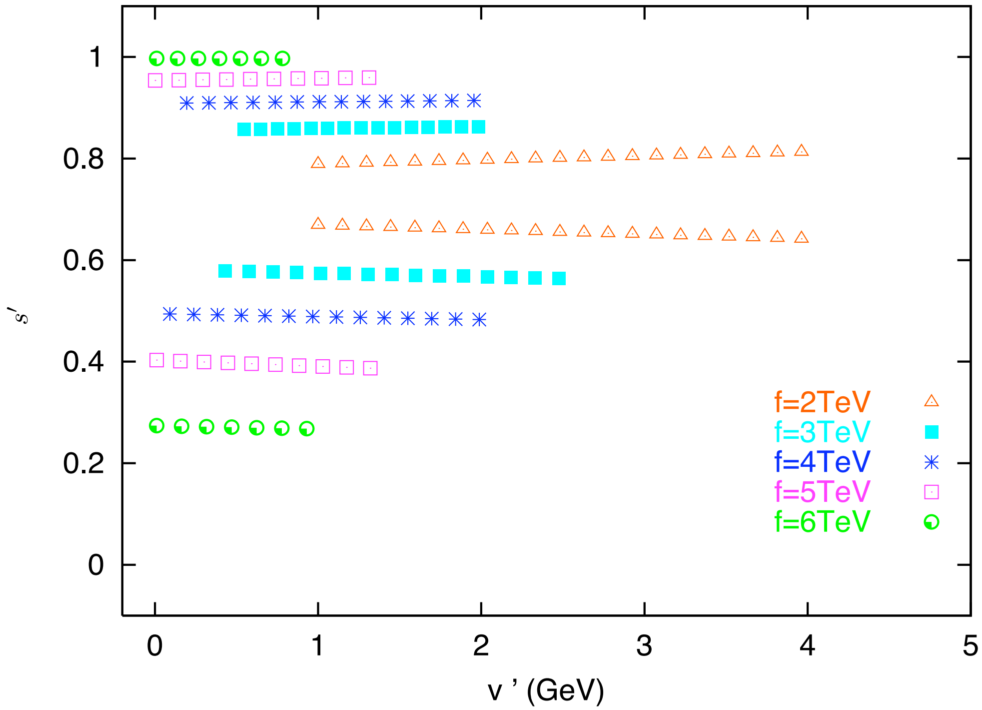
No more on-shell definition for s_θ^2:
$\rho = M_W^2 / M_Z^2 \bar{c}_\theta^2 \neq 1$ at tree level!

Need 4 inputs to fix 4 free counterterms:
$G_\mu = 1.16639(1) \times 10^{-5}$ GeV
$M_Z = 91.1876(21)$ GeV
$\alpha(M_Z)^{-1} = 127.934 \pm 0.027$
$s_\theta^2 = 0.23150 \pm 0.00016$ (effective leptonic mixing angle)

$$\text{where} \quad \frac{\text{Re}(g_V^e)}{\text{Re}(g_A^e)} = 4 s_\theta^2 - 1$$

- Use these 4 inputs to fix the 4 counterterms at 1-loop level.
- Predict M_W in terms of other free model parameters that enter through 1-loop diagrams.
- Use the measured M_W value to constrain the model parameters.

Littlest Higgs model: allowed parameter space for v' at one loop



[Chen & Dawson, hep-ph/0311032]

Other masses used:

$m_t = 175$ GeV

$m_b = 3$ GeV ($\overline{\text{MS}}$)

$m_H = 120$ GeV

Plotted against $s' = g_2'/\sqrt{g_1'^2 + g_2'^2}$.

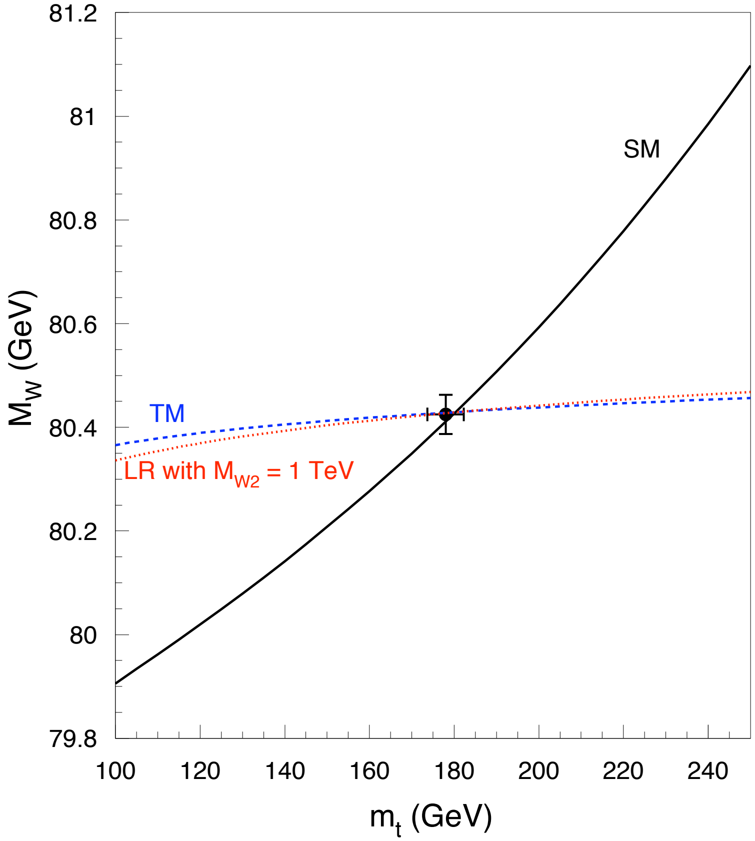
Scanned over $s = g_2/\sqrt{g_1^2 + g_2^2}$ and $x_L = \lambda_1^2/(\lambda_1^2 + \lambda_2^2)$.

Note $v' \gtrsim 1$ GeV required for low $f \sim 2$ TeV.

Chen & Dawson used $\Delta M_W \simeq 60$ MeV at 1σ; plot presumably a 2σ constraint.

Looks straightforward, but there is a lot of subtle physics there.

$\delta s_\theta^2/s_\theta^2$ no longer fixed in terms of W and Z self-energies. Determined instead by RC's to $\sin^2\theta_{\rm eff}^{\rm lept}$: γ–Z mixing, vertex & box diagrams.



1-loop M_W correction no longer quadradically sensitive to top mass!

Data point: expt values, 1σ error bars.
SM: includes complete top/bottom quark, SM gauge, and SM Higgs contributions with $m_H = 120$ GeV.
TM = 1 doublet + 1 real triplet: plot includes only top quark contributions; other free params chosen to intersect data point.

[Chen, Dawson & Krupovnickas, hep-ph/0504286]

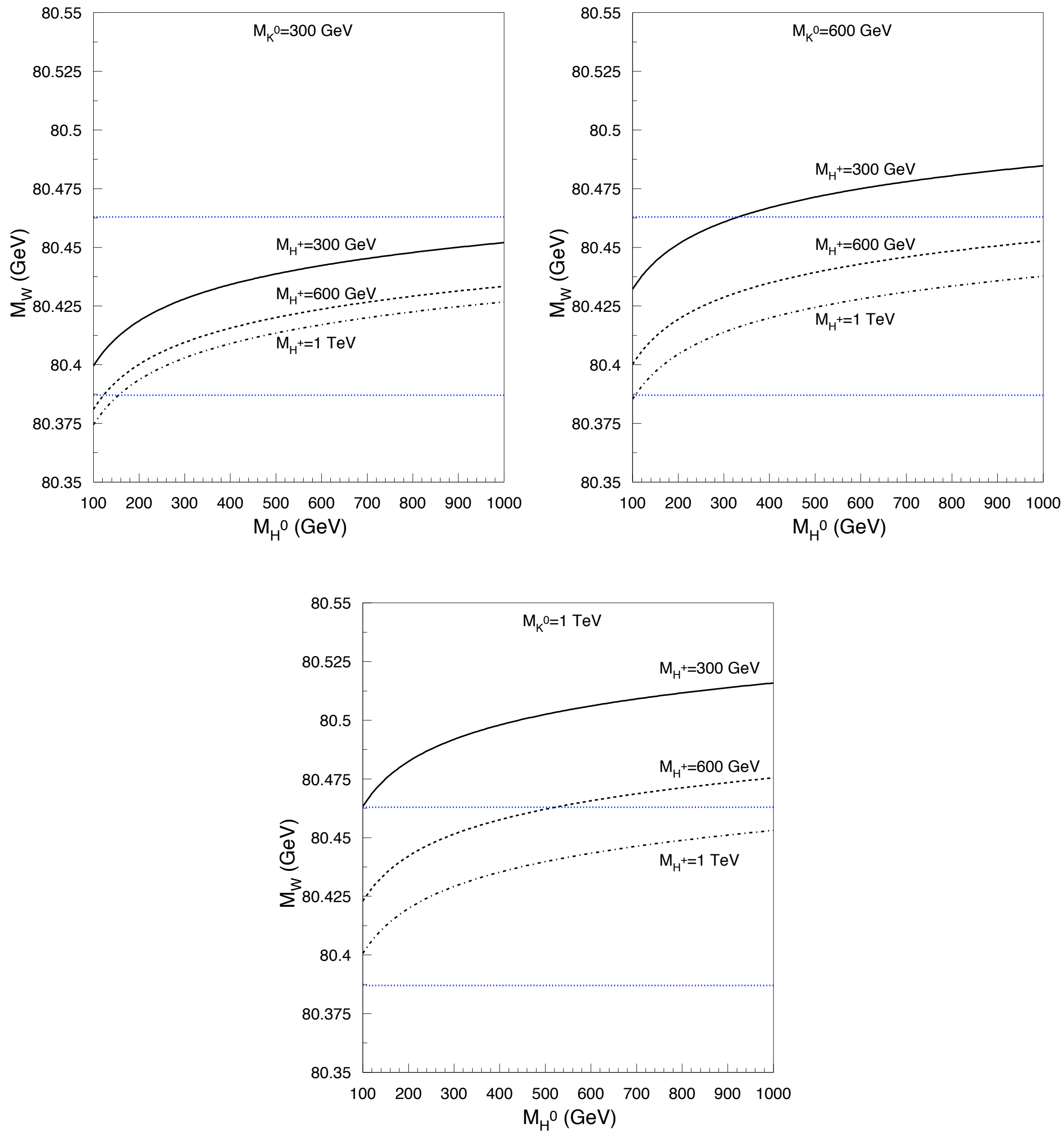
New fit in triplet model replaces SM "blue band" Higgs mass fit.

SM-like Higgs H^0 does not have to be light!

W mass constraint can be satisfied even for $m_H \sim$ TeV.

Triplet contribution to ρ essentially canceling heavy SM Higgs contribution.

[Chen, Dawson & Krupovnickas, hep-ph/0504286]



Summary

Triplet-Higgs models interesting from neutrino mass perspective

Dimension-4 operator for neutrino mass: renormalizable Lagrangian!

Rich LHC phenomenology
- Opportunity to directly probe Majorana neutrino Yukawa matrix
Y_{ij} in LNV triplet decays

$\rho \neq 1$ at tree level: EW precision constraints on v'
Neutrino mass: nice to have $v' \ll$ GeV: no worries about ρ.
Model-building with real and complex triplets – Georgi-Machacek.

Renormalization of triplet models: very different from SM.
4 fundamental gauge-sector parameters.
An excellent lesson in field theory renormalization.