Testing the Higgs mechanism

Heather Logan (Carleton University)

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Outline:

- Introduction
 - How to test the Higgs mechanism
 - Standard Model versus New Physics
- Higgs couplings at the ILC
 - Impact of theoretical uncertainties
- Higgs couplings at the LHC
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- Invisible Higgs
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Introduction: the Higgs mechanism

Introduce a scalar "Higgs" field H

- Doublet under SU(2)_L: $H = (\phi^+, \phi^0)^T$
- Carries $U(1)_Y$ hypercharge

Write down couplings of H:

- To gauge bosons via the covariant derivative, $\mathcal{L} = |\mathcal{D}_{\mu}H|^2$.
- To itself via the Higgs potential, $-\mathcal{L} = V = m^2 H^{\dagger} H + \lambda (H^{\dagger} H)^2$.
- To fermions via Yukawa couplings, $\mathcal{L} = y_f \overline{f_R} H^{\dagger} F_L$.

e.g.,
$$F_L = (u_L, d_L)^T$$
, $f_R = d_R$.

These couplings are all gauge invariant.

Choose the signs of the terms in the Higgs potential: $V = m^2 H^{\dagger} H + \lambda (H^{\dagger} H)^2$ where m^2 is negative and λ is positive

Potential is symmetric under $SU(2)_L \times U(1)_Y$ gauge symmetry, but the minimum of the potential is away from zero field value: $SU(2) \times U(1)$ symmetry is spontaneously broken.

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At the minimum, Higgs field has a nonzero vacuum expectation value v.

Expand about the minimum:

$$H = \begin{pmatrix} G^+ \\ (h+v)/\sqrt{2} + iG^0/\sqrt{2} \end{pmatrix}$$

h is the massive excitation of the field: the physical Higgs boson.

 G^0 and G^+ are the would-be Goldstone bosons: they become the third polarization degree of freedom of the Z and W^+ gauge bosons.

With $v \neq 0$, the Higgs couplings to gauge bosons and fermions give those particles mass.

Covariant derivative gives gauge boson masses and coups to h:

 $\mathcal{L} = (\mathcal{D}_{\mu}H)^{\dagger} (\mathcal{D}^{\mu}H) + \cdots$

where

 $[Q = T_3 + Y/2]$

$$\mathcal{D}_{\mu} = \partial_{\mu} - igW_{\mu}^{a}T^{a} - ig'\frac{Y}{2}B_{\mu}$$

$$= \partial_{\mu} - i\frac{g}{\sqrt{2}} \left(W_{\mu}^{+}T^{+} + W_{\mu}^{-}T^{-}\right)$$

$$-i\frac{g}{\cos\theta_{W}}Z_{\mu} \left(T^{3} - \sin^{2}\theta_{W}Q\right) - ieQA_{\mu}$$

This gives: [extra 1/2 for the ZZ terms is a symmetry factor]

$$\mathcal{L} = (g^2 v^2 / 4) W^+ W^- + (g^2 v / 2) h W^+ W^- + (g^2 / 4) h h W^+ W^- + (g_Z^2 v^2 / 8) ZZ + (g_Z^2 v / 4) h ZZ + (g_Z^2 / 8) h h ZZ$$

where $g_Z = \sqrt{g^2 + g'^2}$.

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Yukawa couplings $y_f \overline{f_R} H^{\dagger} F_L$ give fermion masses and couplings to *h*:

 $\mathcal{L} = (y_f v / \sqrt{2}) \overline{f}_R f_L + (y_f / \sqrt{2}) h \overline{f}_R f_L + \text{h.c.}$



Because of fixed couplings, Standard Model Higgs decay modes depend only on ${\cal M}_{\cal H}$:





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This simple linear relation between masses and Higgs couplings holds in the Standard Model.

But beyond the Standard Model, Higgs couplings can vary.

An example: Minimal Supersymmetric Standard Model (MSSM)

MSSM has two Higgs doublets, H_1 and H_2 , with two different vacuum expectation values, v_1 and v_2 .

W boson mass comes from sum of two covariant derivatives: $\mathcal{L} = |\mathcal{D}_{\mu}H_1|^2 + |\mathcal{D}_{\mu}H_2|^2, \text{ which gives } M_W^2 = \frac{g^2v_1^2}{4} + \frac{g^2v_2^2}{4} = \frac{g^2v_{SM}^2}{4}.$

So v_1 and v_2 must obey $v_1^2 + v_2^2 = v_{SM}^2 = 2M_W/g$. One unknown combination is left free: $v_2/v_1 \equiv \tan \beta$.

Two complex doublets \rightarrow 8 degrees of freedom

h: lightest CP-even Higgs

H, *A*, and H^{\pm} : heavier CP-even, CP-odd, and charged Higgses

 G^0 and G^{\pm} : unphysical Goldstone bosons

Mix to form mass eigenstates:

$$H_1 \cos\beta + H_2 \sin\beta = \left(\begin{array}{c} G^+ \\ [v_{SM} + iG^0 + h\sin(\beta - \alpha) + H\cos(\beta - \alpha)]/\sqrt{2} \end{array} \right)$$

$$-H_1 \sin \beta + H_2 \cos \beta = \left(\begin{array}{c} H^+ \\ [iA^0 + h\cos(\beta - \alpha) - H\sin(\beta - \alpha)]/\sqrt{2} \end{array} \right)$$

Couplings of h get modified from their SM values:

 $\begin{array}{ll} g_{hWW} = \sin(\beta - \alpha)g_{H_{SM}WW} & \text{likewise } Z \\ g_{hb\bar{b}} = [\sin(\beta - \alpha) - \tan\beta\cos(\beta - \alpha)]g_{H_{SM}b\bar{b}} & \text{likewise } d, s, \ e, \mu, \tau \\ g_{ht\bar{t}} = [\sin(\beta - \alpha) + \cot\beta\cos(\beta - \alpha)]g_{H_{SM}t\bar{t}} & \text{likewise } u, c \end{array}$

In most MSSM parameter space, H, A, and H^{\pm} are fairly heavy.

Mixing angle:
$$\cos(\beta - \alpha) \simeq \frac{1}{2} \sin 4\beta \frac{M_Z^2}{M_A^2} \longrightarrow 0$$
 for $M_A \gg M_Z$

Couplings of h approach their SM values – the decoupling limit.

Search for coupling deviations \rightarrow test Higgs sector structure!Heather LoganTesting the Higgs mechanismMcGill 2007-03-28

Higgs couplings at the ILC

Clean environment – no large QCD backgrounds

Well-known initial state – no parton distributions; energy/momentum of initial state known





Model-independent technique: Z recoil

Use 4-momentum conservation to reconstruct Higgs events looking only at the recoiling Z.

Initial state: $e^- \rightarrow \star \leftarrow e^+$ $p(e^-) = (E_{cm}/2, 0, 0, E_{cm}/2), \quad p(e^+) = (E_{cm}/2, 0, 0, -E_{cm}/2)$ Initial 4-momentum $= p(e^-) + p(e^+) = (E_{cm}, 0, 0, 0)$

Final state: $Z \leftarrow \star \longrightarrow H$ Z decays to dileptons (e^+e^- or $\mu^+\mu^-$) and the Higgs goes off in the other direction.

Measure the 4-momenta of the Z decay leptons: $p(\ell^-)$ and $p(\ell^+)$. Require that $p(\ell^-)$ and $p(\ell^+)$ reconstruct the Z: $[p(\ell^-) + p(\ell^+)]^2 = M_Z^2$

Use energy-momentum conservation to get the Higgs 4-momentum:

$$p(Higgs) = p(e^{-}) + p(e^{+}) - p(\ell^{-}) - p(\ell^{+})$$
.



H.J. Schreiber et al., DESY-ECFA Conceptual LC Design Report (1997)

"Recoil mass" is $[p(Higgs)]^2 = M_H^2$.

See a Higgs mass peak in the Z recoil spectrum.

Count events in the recoil Higgs mass peak: get the ZH cross section.

Count Higgs decay products in the recoil Higgs mass peak: get the Higgs branching ratios.

Model-independent!!

ZH cross section measurement does not depend on Higgs decay mode.

BR measurements do not depend on production cross-section assumptions.

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From $WW \rightarrow H$ cross section, get WWH coupling

- \rightarrow predict $H \rightarrow WW$ partial width
- \rightarrow Combine with BR($H \rightarrow WW$) to extract total width

 \rightarrow Extract all the other Higgs couplings from respective BRs Totally model independent!

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Measure Higgs branching ratios to high precision:

Table 1: Summary of expected precisions on Higgs boson branching ratios from existing studies within the ECFA/DESY workshops. (a) for 500 fb⁻¹ at 350 GeV; (b) for 500 fb⁻¹ at 500 GeV; (c) for 1 ab⁻¹ at 500 GeV; (d) for 1 ab⁻¹ at 800 GeV; (e) as for (a), but method described in [35] (see text).

Mass(GeV)	120	140	160	180	200	220	240	280	320
Decay				Relative P	recision (%)			
bb	2.4 (a) / 1.9 (e)	2.6 (a)	6.5 (a)	12.0 (d)	17.0 (d)	28.0 (d)			
$c\overline{c}$	8.3 (a) / 8.1 (e)	19.0 (a)							
au au	5.0 (a) / 7.1 (e)	8.0 (a)							
$\mu\mu$	30. (d)								
gg	5.5 (a) /4.8 (e)	14.0 (a)							
WW	5.1 (a) / 3.6 (e)	2.5 (a)	2.1 (a)		3.5 (b)		5.0 (b)	7.7 (b)	8.6 (b)
ZZ			16.9 (a)		9.9 (b)		10.8 (b)	16.2 (b)	17.3 (b)
$\gamma\gamma$	23.0 (b) / 35.0 (e)								
$\mathrm{Z}\gamma$		27.0 (c)							

review talk by K. Desch, hep-ph/0311092

With a 1 TeV ILC one does even better (larger cross sections, more statistics):

	Higgs Mass (GeV)					
	115	120	140	160	200	
$\Delta(\sigma \cdot B_{bb})/(\sigma \cdot B_{bb})$	± 0.003	± 0.004	± 0.005	± 0.018	± 0.090	
$\Delta(\sigma \cdot B_{WW})/(\sigma \cdot B_{WW})$	± 0.021	± 0.013	± 0.005	± 0.004	± 0.005	
$\Delta(\sigma \cdot B_{gg})/(\sigma \cdot B_{gg})$	± 0.014	± 0.015	± 0.025	± 0.145		
$\Delta(\sigma \cdot B_{\gamma\gamma})/(\sigma \cdot B_{\gamma\gamma})$	± 0.053	± 0.051	± 0.059	± 0.237		
$\Delta(\sigma \cdot B_{ZZ}) / (\sigma \cdot B_{ZZ})$					± 0.013	

from Barklow, hep-ph/0312268

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ILC at 1000 GeV, 1000 fb<sup>-1</sup>
-80% e^- polarization, +50% e^+ polarization
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With experimental uncertainties at the percent level, must consider theory uncertainties too.

 $H \rightarrow q\bar{q}$: QCD corrections to 3 loops, EW corrections to 1 loop. Dominant corrections absorbed by using $\overline{m_q}(M_H)$ in partial width. Uncertainty ~ 1% remaining.

 $H \rightarrow \ell \ell$: EW corrections to 1 loop. Uncertainty negligible for our purposes.

 $H \rightarrow W^{(*)}W^{(*)}/Z^{(*)}Z^{(*)} \rightarrow 4f$: NLO EW + QCD corrections including off-shell gauge boson effects now available – PROPHECY4F. Uncertainty ~ 0.5% remaining.

 $H \rightarrow gg$: N³LO QCD corrections known, plus leading EW. Remaining scale dependence ~ 3%.

 $H \rightarrow \gamma \gamma$: NLO EW + NNLO QCD corrections known. Uncertainty negligible for our purposes. WBF \rightarrow *H* production cross section: 1-loop EW known. Uncertainty ~ 0.5% remaining.

Summary:

	Theory uncertainty		
Higgs partial width	in literature	in HDECAY	
$\Gamma_{b\overline{b}}, \ \Gamma_{c\overline{c}}$	1%	1%	
$\Gamma_{ au au}$, $\Gamma_{\mu\mu}$	0.01%	0.01%	
Γ_{WW} , Γ_{ZZ}	0.5%	5%	
Γ_{gg}	3%	16%	
$\Gamma_{\gamma\gamma}$	0.1%	4%	
${\sf \Gamma}_{Z\gamma}$	4%	4%	
Higgs production cross section			
$\sigma_{e^+e^- \rightarrow \nu \bar{\nu} H}$	0.5%	_	

[Droll & H.L., hep-ph/0612317]

There are al	so uncertainties in th	ne "inputs": mostly m	$a_b, m_c, \alpha_s.$
Parameter	Value	Percent uncertainty	Source
$\overline{\alpha_s(m_Z)}$	0.1185 ± 0.0020	1.7%	PDG
$\overline{m_b}(M_b)$	$4.20\pm0.04~{ m GeV}$	0.95%	B decays
$\overline{m_c}(M_c)$	$1.224\pm0.057~{ m GeV}$	4.7%	B decays

 $\overline{m_b}(M_b)$ and $\overline{m_c}(M_c)$ extracted from fits to semileptonic *B* meson decay spectra using HQET.

Can also get the masses from $e^+e^- \rightarrow$ hadrons or unquenched lattice QCD. Methods developing; close to being competitive to *B* decays.

Input uncertainties propagate into uncertainties in the SM Higgs partial widths:

Normalized derivatives of Higgs partial widths									
	$\alpha_s(m_Z)$			$\overline{m_b}(M_b)$			$\overline{m_c}(M_c)$		
m_H	120 GeV	140 GeV	160 GeV	120 GeV	140 GeV	160 GeV	120 GeV	140 GeV	160 GeV
$\Gamma_{b\overline{b}}$	-1.177	-1.217	-1.249	2.565	2.567	2.568	0.000	0.000	0.000
$\Gamma_{c\overline{c}}$	-4.361	-4.400	-4.432	-0.083	-0.084	-0.084	3.191	3.192	3.192
${\sf \Gamma}_{gg}$	2.277	2.221	2.175	-0.114	-0.112	-0.104	-0.039	-0.032	-0.027
$\Gamma_{\gamma\gamma}$	0.002	0.002	0.001	0.010	0.008	0.005	0.012	0.009	0.005

[Droll & H.L., hep-ph/0612317]

Concentrate on lower Higgs mass region.

Precisions from before:

SM Higgs	BR uncertainties fro	om 500 fb $^{-1}$ at 350 GeV (no beam pol'n)
	$m_H = 120 { m GeV}$	140 GeV
$BR(b\overline{b})$	2.4%	2.6%
$BR(c\overline{c})$	8.3%	19.0%
BR(au au)	5.0%	8.0%
BR(WW)	5.1%	2.5%
BR(gg)	5.5%	14.0%

[Desch, hep-ph/0311092]

SM Higgs $\sigma \times BR$ statistical uncertainties from 1000 fb ⁻¹ at 1000 GeV							
	$m_H = 115 { m GeV}$	120 GeV	140 GeV				
$\sigma imes BR(b\overline{b})$	0.3%	0.4%	0.5%				
$\sigma imes BR(WW)$	2.1%	1.3%	0.5%				
$\sigma imes BR(gg)$	1.4%	1.5%	2.5%				
$\sigma imes BR(\gamma\gamma)$	5.3%	5.1%	5.9%				

Beam pol'ns of -80% for electrons and +50% for positrons assumed.

[Barklow, hep-ph/0312268]

To evaluate impact of theory uncertainties, need a "benchmark": choose differentiation of SM from MSSM Higgs.

Choose a particular MSSM scenario: m_h^{max} benchmark scenario.

Compute a $\Delta \chi^2$ both without and with theory and parametric uncertainties; see how this affects the "distinguishing power" of ILC.

Consider "ILC early phase": 500 fb⁻¹ at 350 GeV C.o.M. energy, and "ILC late phase": 1000 fb⁻¹ at 1000 GeV C.o.M. energy. chosen to match experimental studies.



Theory/param uncerts not important in early-phase running. Reduce "reach" in M_A by about 15% in late-phase running.

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No single source gives majority of the effect.

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Breakdown of sources of parametric uncertainty:

No single source gives majority of the effect.

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Results:

- Theory and parametric uncertainties not an issue for initial phase of ILC: we are in good shape.

- After TeV-phase ILC running, though, thy/param uncerts reduce the "reach" in M_A by about 15%.

Starting to become relevant.

- No single source dominates the theory uncertainties: Need to take multiple calculations to the next level to improve this situation.

Higgs couplings at the LHC

Higgs will be accessible via multiple production mechanisms:



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Higgs production cross sections are reasonably large: 1 pb \times 1 fb⁻¹ = 1000 events



M. Spira, Fortsch. Phys. 46, 203 (1998)

If the Higgs is Standard Model-like, LHC will discover it!



S. Asai et al., Eur. Phys. J. C 32S2, 19 (2004)

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Testing the Higgs mechanism

Higgs will be accessible in many production and decay channels: (GF = gluon fusion, WBF = weak boson fusion)

 $\mathsf{GF} \ gg \to H \to ZZ$ Inclusive $H \rightarrow \gamma \gamma$ WBF $qqH \rightarrow qqZZ$ WBF $qqH \rightarrow qq\gamma\gamma$ $t\bar{t}H$, $H \rightarrow \gamma\gamma$ $\mathsf{GF} qq \to H \to WW$ WH, $H \rightarrow \gamma \gamma$ WBF $qqH \rightarrow qqWW$ $ZH, H \rightarrow \gamma \gamma$ $t\bar{t}H$, $H \rightarrow WW$ $WH, H \rightarrow WW$ WBF $qqH \rightarrow qq\tau\tau$

$$t\bar{t}H$$
, $H \rightarrow b\bar{b}$

Testing the Higgs mechanism

Higgs couplings determine production cross sections and decay branching ratios \longrightarrow determine the rates in each channel.



LHC, 200 fb⁻¹ (except 300 fb⁻¹ for $ttH, H \rightarrow bb, WH, H \rightarrow bb$). Zeppenfeld, hep-ph/0203123

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If there's a discrepancy, want to know where it comes from.

Take ratios of rates with same production and different decays: production cross section and Higgs total width cancel out.



LHC, 200 fb⁻¹ (except 300 fb⁻¹ for $ttH, H \rightarrow bb, WH, H \rightarrow bb$). Zeppenfeld, hep-ph/0203123

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Testing the Higgs mechanism

Ratios of couplings are nice.

But can we measure each coupling independently?

Difficulties:

- No measurement of total production rate.

- Some decays cannot be directly observed at LHC due to backgrounds: $H \rightarrow gg$, $H \rightarrow$ light quarks, etc.

Incomplete data: can't extract individual couplings in a totally model-independent way.

Multi-dimensional "error ellipsoid" is unbounded in some directions.

Observation of Higgs production

 \longrightarrow lower bound on production couplings

 \rightarrow lower bound on Higgs total width.

But: no model-independent upper bound on Higgs total width.

To make progress, have to make some theoretical assumptions.

Testing the Higgs mechanism

Consider Higgs models containing only SU(2) doublets/singlets.

- hWW, hZZ couplings related by custodial SU(2).
- hWW, hZZ couplings bounded from above by SM values.

This is a mild assumption!

- True in most good models: MSSM, NMSSM, 2HDM, etc.
- Larger Higgs multiplets stringently constrained by ρ parameter.

Theoretical constraint $\Gamma_V \leq \Gamma_V^{SM}$ \oplus measurement of Γ_V^2/Γ_{tot} from WBF $\rightarrow H \rightarrow VV$ \rightarrow upper bound on Higgs total width.

...slicing the error ellipsoid...

Combine with lower bound on Higgs total width from production couplings.

- Interplay constrains remaining Higgs couplings.
- Make no assumptions on unexpected/unobserved Higgs decay modes.

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Must include the appropriate systematic uncertainties:

5% overall Luminosity normalization

Theory uncertainties on Higgs production: 20% Gluon Fusion 15% ttH assoc. prod. 7% WH, ZH assoc. prod. 4% Weak Boson Fusion

Reconstruction/identification efficiencies:

2% leptons 2% photons 3% b quarks 3% τ jets 5% forward tagging jets and veto jets (for WBF)

Background extrapolation from side-bands (shape): from 0.1% for $H \rightarrow \gamma \gamma$

to 5% for $H \to W W$ and $H \to \tau \tau$

to 10% for $H \rightarrow b\overline{b}$

Result: fit of Higgs couplings-squared



Dührssen, Heinemeyer, H.L., Rainwater, Weiglein & Zeppenfeld, hep-ph/0406323

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Another approach: fit observed rates to a particular model.

Fits within a model are more constrained than a general fit of independent Higgs couplings.

Model constraints \rightarrow fewer parameters: taking a slice through the "error ellipsoid."

Get tighter constraints as a result of the model assumption. We saw this already when taking $g_{HWW,HZZ} \leq g_{HWW,HZZ}^{SM}$.

Lose generality, but gain constraining power: This is fine as long as you know what your assumptions are!

Can use this approach to test consistency with individual models.

Example: chi-squared fits in MSSM, m_h^{max} scenario



from Dührssen, Heinemeyer, H.L., Rainwater, Weiglein & Zeppenfeld, hep-ph/0406323

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The SM Higgs is very narrow for $M_H \lesssim 160$ GeV.

If Higgs couples with electroweak strength to a neutral (quasi)stable particle (e.g., dark matter) with mass $< M_h/2$, then $h \rightarrow$ invisible can be the dominant decay mode.





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The Higgs *could* decay invisibly

- $h
 ightarrow { ilde \chi}_1^0 { ilde \chi}_1^0$ in MSSM, NMSSM
- $\bullet \ h \to SS$ in simple models of scalar dark matter
- $h \rightarrow KK$ neutrinos in extra dimensions
- $h \rightarrow$ Majorons
- . . .

Shouldn't just assume Higgs will be SM-like.

Even small additions (e.g., singlet scalar dark matter) can make $BR(h \rightarrow invis.)$ large.

Let's cover all our bases!

"Invisible" Higgs is not that hard to "see": missing transverse momentum (p_T) . $h \rightarrow jj$ is much harder. Limits on invisible decay modes:



95% CL exclusion limits with 30 fb⁻¹ at LHC [ATL-PHYS-PUB-2006-009]

 ξ^2 is a scaling factor: $\sigma \times BR(H \rightarrow invis) \equiv \xi^2 \sigma_{SM}$

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Extracting the mass of an invisible Higgs: Mass of h_{inv} accessible only through production process.



Measure signal rate.

Assuming SM production cross section and 100% invisible decay:

- $Z + h_{inv}$: $\Delta m_h = 30-40$ (12-14) GeV with 10 (100) fb⁻¹.
- WBF: $\Delta m_h \simeq 40$ (30) GeV with 10 (100) fb⁻¹.

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What if production rate is not SM-like? What if decay is not 100% invisible?

For a more model-independent M_h extraction, take the ratio of $Z + h_{inv}$ and WBF rates. Davoudiasl, Han & H.L. (2004)

 $Z + h_{inv} \sim hZZ$ coupling; WBF $\sim hWW, hZZ$ couplings – related by SU(2) in models with only Higgs doublets/singlets.



Ratio method: $\Delta m_h \simeq 35-50 \ (15-20) \ \text{GeV}$ with 10 (100) fb⁻¹.

Not great, but rather model-independent.

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500 fb⁻¹ at 350 GeV. Dashes = invisible rate; dots = Higgsstrahlung cross section

M. Schumacher, LC-PHSM-2003-096

Get the Higgs mass from recoil method.

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Conclusions

If the Higgs mechanism is realized in nature, LHC and ILC data will let us test it.

In high precision measurements, theory uncertainties begin to play a role.

- Production cross section, decay partial widths, SM input parameters at late-phase ILC

- Higgs production cross sections at LHC

Model-independent measurements are always best, but model assumptions are sometimes necessary.

- Nothing wrong with testing individual models

- Keep assumptions as mild as possible for maximum generality

- Appropriate theory assumptions can reveal interesting relations in the data