TRIUMF Summer Institute 2006
Collider and Energy Frontier Physics

Beyond the Standard Model

Lecture 2

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Plan

Lecture 1 Monday July 17

- Why BSM?
- Supersymmetry

Lecture 2 Monday July 17

- Supersymmetry continued: phenomenology

Lecture 3 Wednesday July 19

- Large extra dimensions: ADD
- Universal extra dimensions; particle spins and UED vs. SUSY

Lecture 4 Thursday July 20

- Deconstruction and the Little Higgs
- T-parity

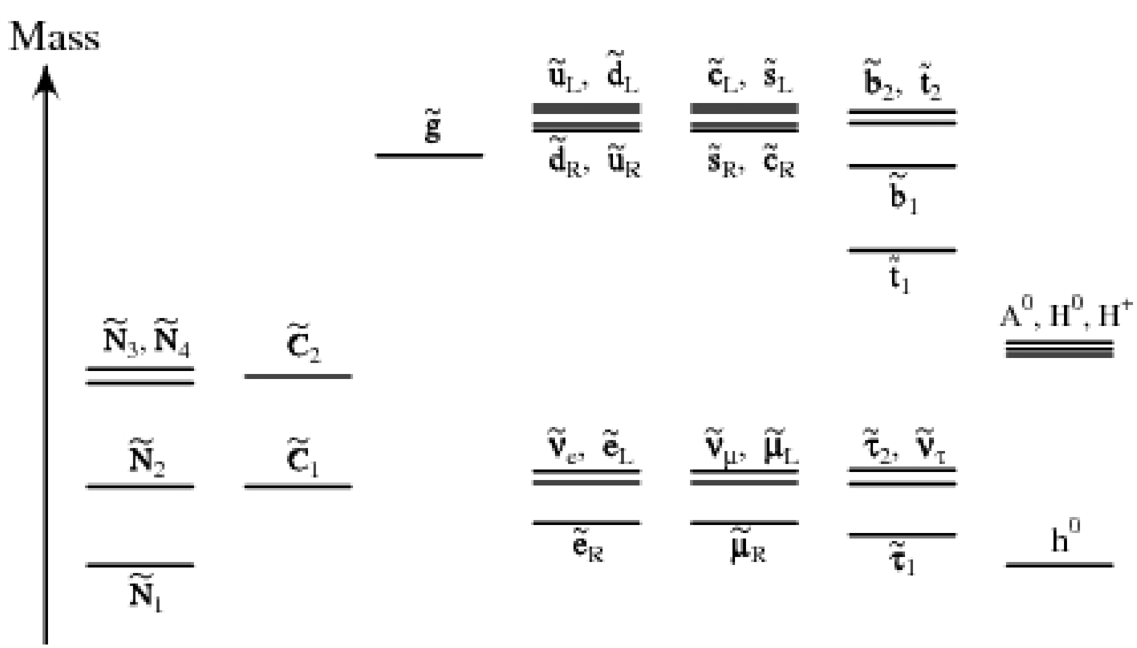
Lecture 5 Friday July 21

- Warped extra dimensions: RS
- RS and Technicolour

In the last lecture we went over the basic ideas behind SUSY and its motivation as a solution to the hierarchy problem.

In this lecture I want to give you a flavour of SUSY phenomenology at colliders.

A schematic sample SUSY spectrum:
(This may or may not have anything to do with reality)



Some features:

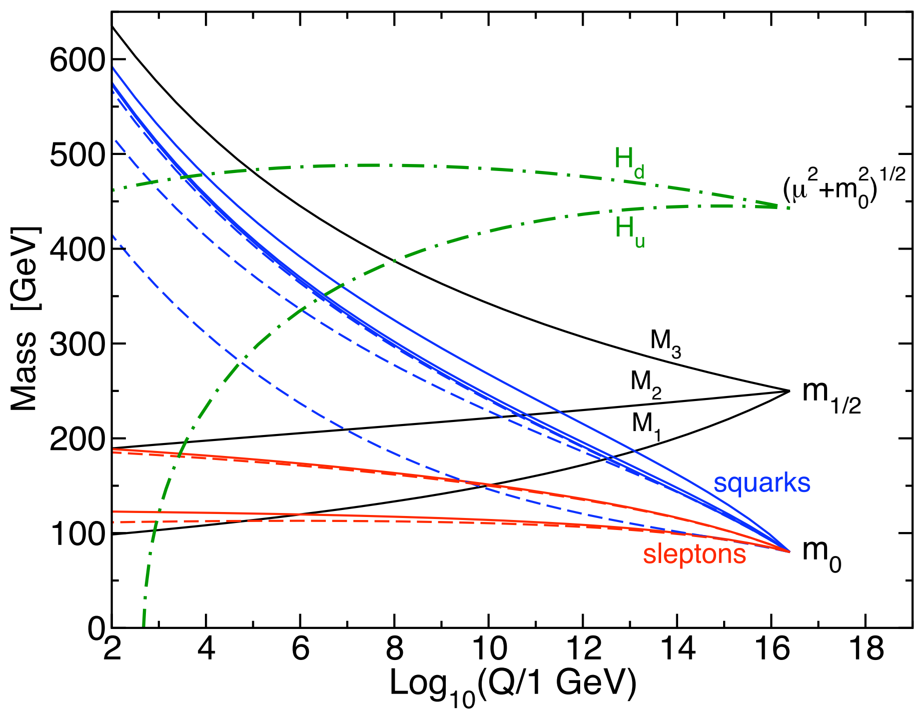
- $\widetilde{N}_1$ is LSP
- $\tilde{t}_1$ and $\tilde{b}_1$ are the lightest squarks
- $\tilde{\tau}_1$ is the lightest charged slepton
- Coloured particles are heavier than uncoloured particles

Where do these features come from?

SUSY particle masses are set at a high scale by SUSY-breaking mechanism.

Masses "run" down by Renormalization Group equations.

E.g., "Constrained MSSM" (CMSSM) model (a.k.a. mSUGRA):



from Martin, hep-ph/9709356

The SUSY breaking terms get set at a high scale $\gg$ TeV.
E.g.: Gravity mediated: set at M_P

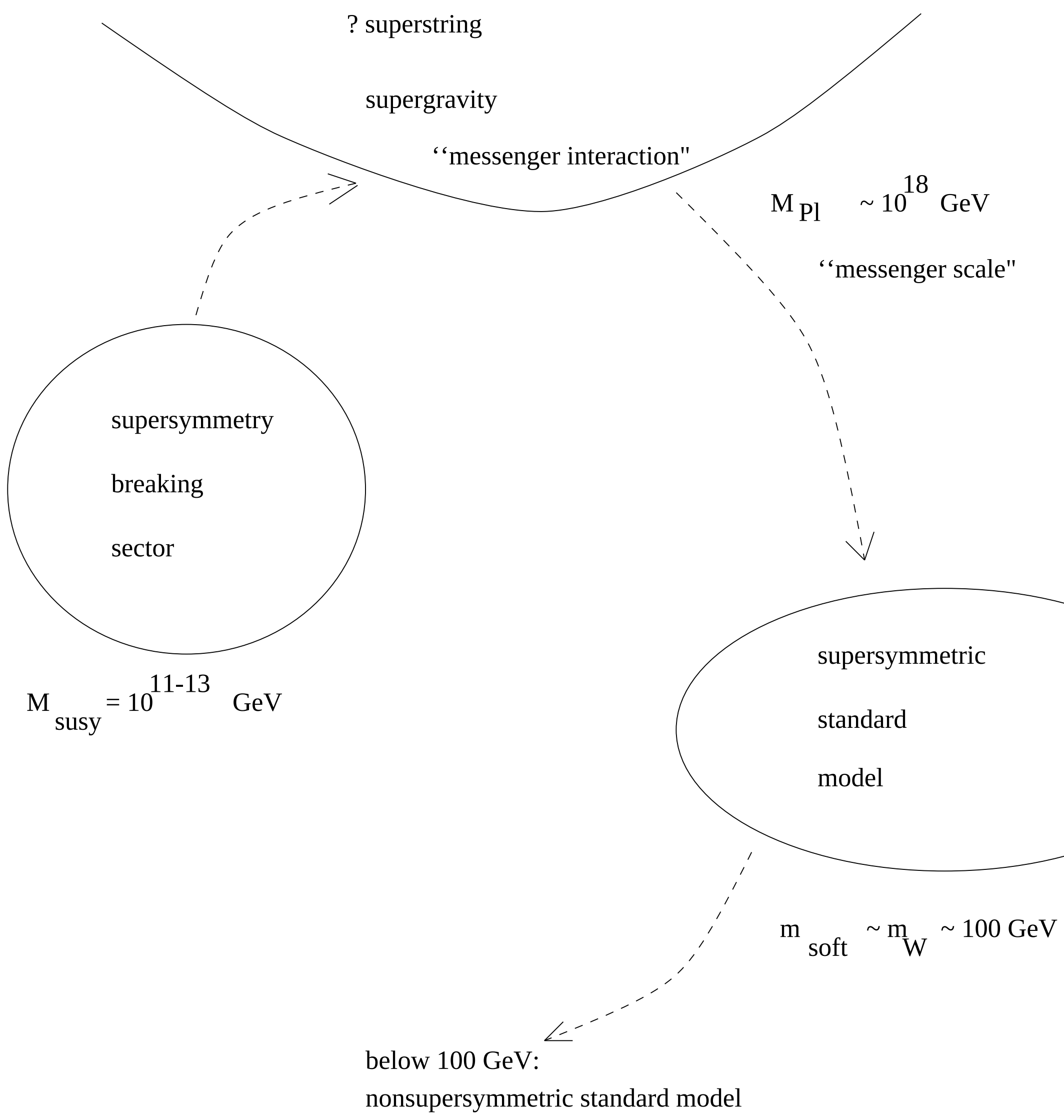


figure from Poppitz, hep-ph/9710274

The SUSY breaking terms get set at a high scale $\gg$ TeV.

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Gauge mediated: set at $M_{mess} \gg$ TeV

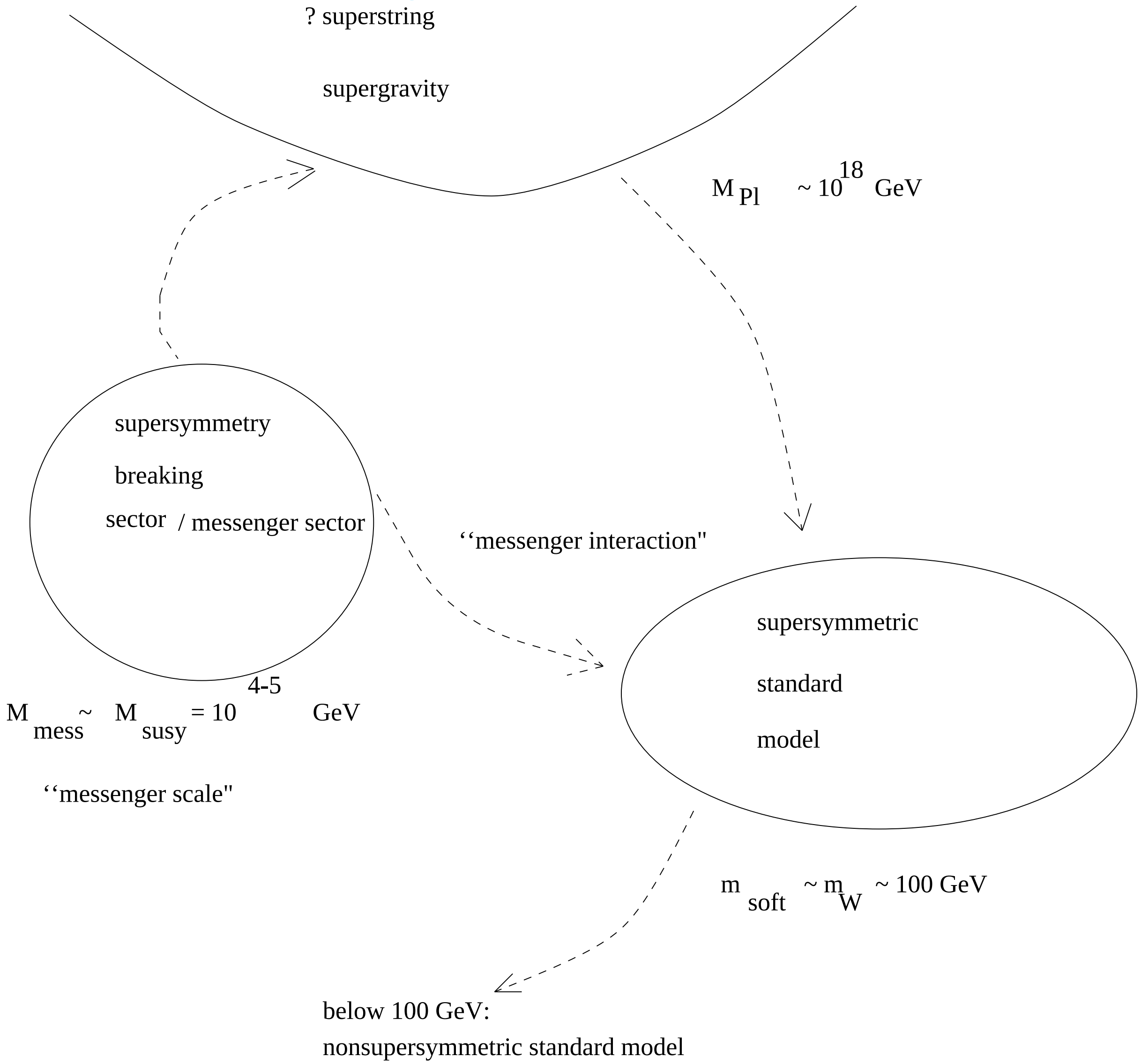


figure from Poppitz, hep-ph/9710274

The SUSY breaking terms get set at a high scale $\gg$ TeV.

E.g.: Gravity mediated: set at M_P

Gauge mediated: set at $M_{mess} \gg$ TeV

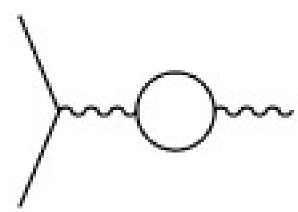
Use renormalization group equations (RGEs) to determine the parameters of the Lagrangian at the EW scale.

- Must "run down" the parameters to the low scale.
- SUSY breaking terms are the boundary conditions at high scale.

Predict mass spectrum, mixing angles, new particle interactions.

Gauge couplings: Running is given by the beta functions b_a.

$$\frac{d}{dt}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad (a = 1, 2, 3)$$



where

$$t = \ln(Q/Q_0) \qquad b_a^{SM} = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) \qquad b_a^{MSSM} = \left(\frac{33}{5}, 1, -3\right)$$

(Q is the "current" scale; Q_0 is the starting scale)

Running of the gauge couplings

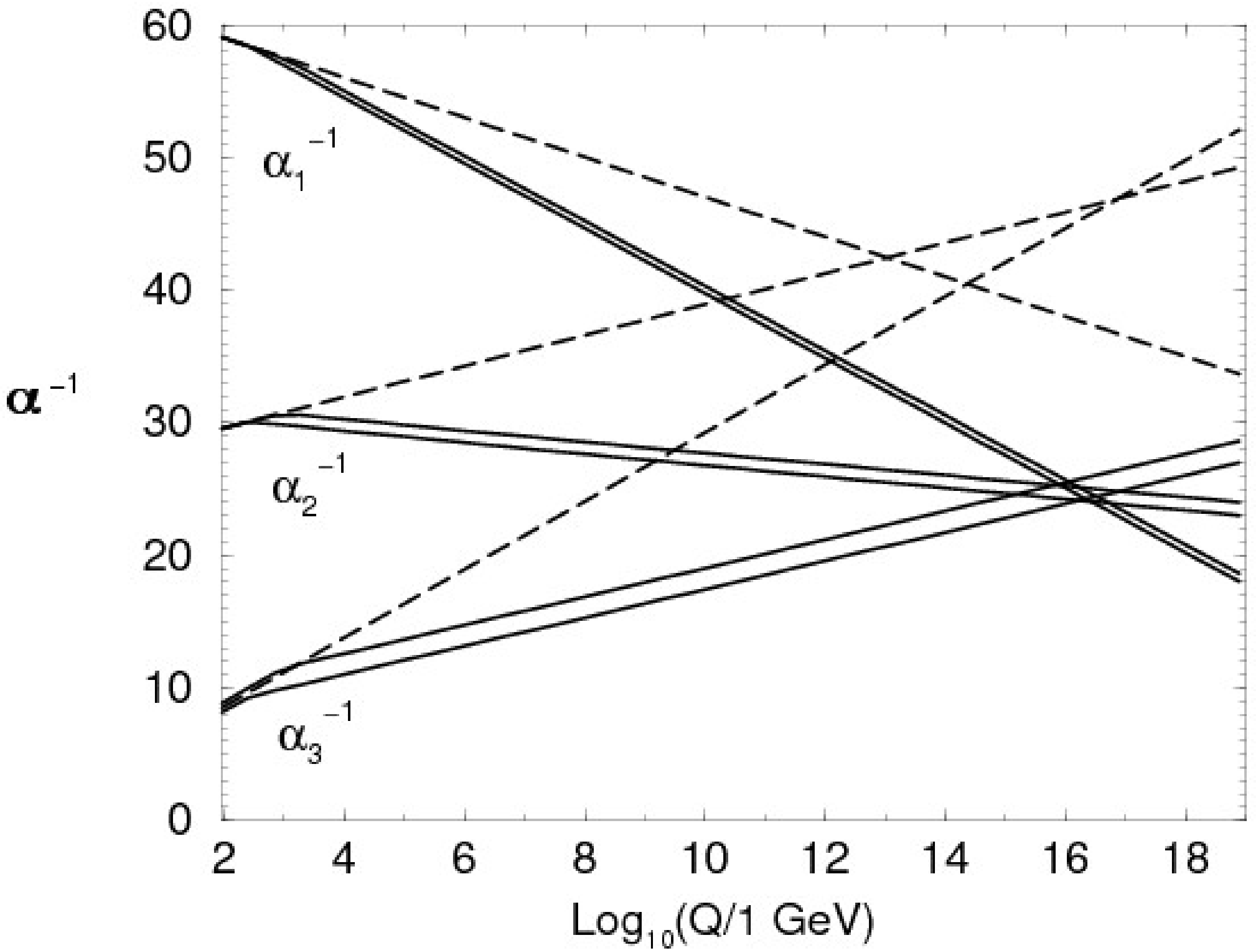


figure from Martin, hep-ph/9709356

Dashed lines: SM   Solid lines: MSSM

(Bands are the uncertainties in the low-energy values.)

Here's another glory of SUSY: gauge coupling unification!

Gaugino mass parameters:

Running determined by same b_a as gauge couplings:

$$\frac{d}{dt} M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a \qquad\qquad b_a^{MSSM} = \left(\frac{33}{5}, 1, -3 \right)$$

Ratios M_a / g_a^2 are scale independent up to small 2-loop effects.

In mSUGRA (Constrained MSSM), the gaugino masses unify:
$M_1(M_{\rm Pl}) = M_2(M_{\rm Pl}) = M_3(M_{\rm Pl}) \equiv m_{1/2}$
Gauge couplings also unify nearby, at $M_{\rm GUT} \simeq 0.01 M_{\rm Pl}$, so
$g_1^2(M_{\rm Pl}) \approx g_2^2(M_{\rm Pl}) \approx g_3^2(M_{\rm Pl}) \approx g_{\rm GUT}^2$ [$g_1 = \sqrt{5/3} g'$: GUT norm'n]
Therefore in the CMSSM (and any model with gaugino mass unification near $M_{\rm Pl}$),

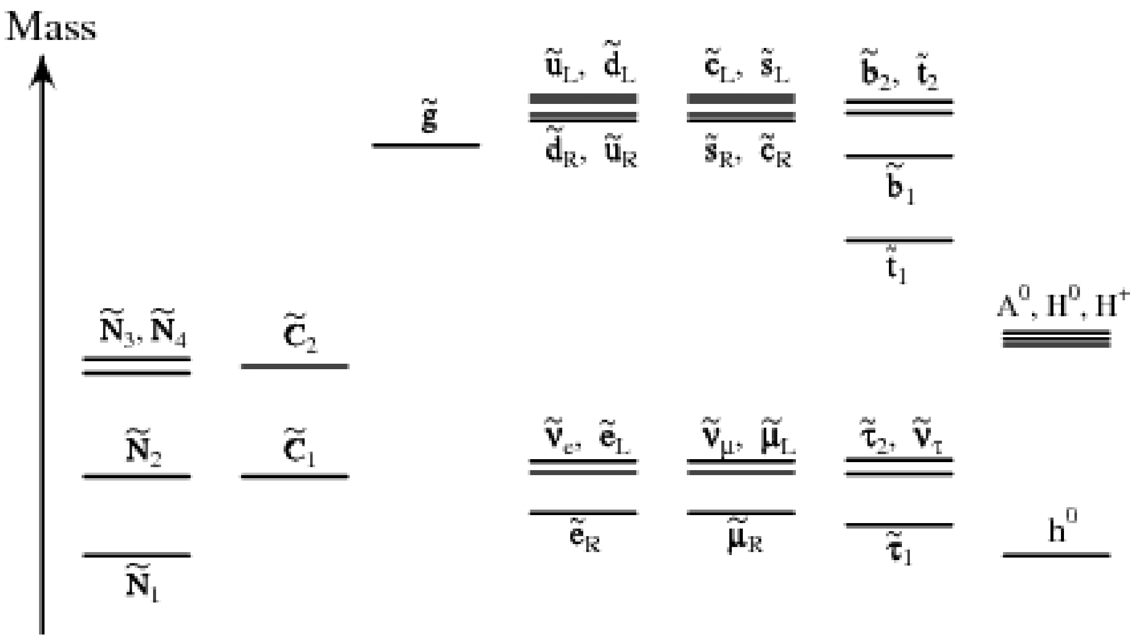
$$\frac{M_1}{g_1^2} \simeq \frac{M_2}{g_2^2} \simeq \frac{M_3}{g_3^2} \simeq \frac{m_{1/2}}{g_{\rm GUT}^2}$$

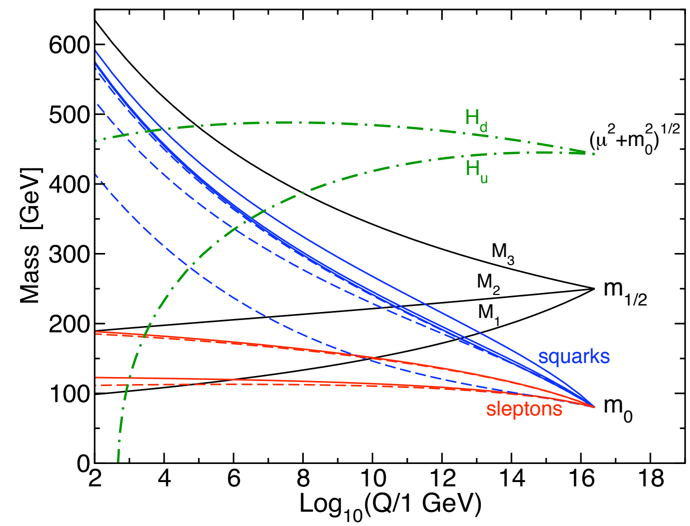
The low-scale gaugino mass params satisfy unification relations:

$$M_1 = \frac{g_1^2}{g_2^2} M_2 \simeq 0.5 M_2 \qquad\qquad M_3 = \frac{g_3^2}{g_2^2} M_2 \simeq 3.5 M_2$$

These relations can be avoided in models in which the gaugino masses do not unify at the GUT scale; e.g. gauge mediated models.

Sample spectrum





from Martin, hep-ph/9709356

Gaugino mass unification:

$$M_1 = \frac{g_1^2}{g_2^2} M_2 \simeq 0.5 M_2 \qquad\qquad M_3 = \frac{g_3^2}{g_2^2} M_2 \simeq 3.5 M_2$$

$$M_{\tilde{N}_1} \simeq 0.5\ M_{\tilde{N}_2,\tilde{C}_1} \qquad\qquad M_{\tilde{g}} \simeq 3.5\ M_{\tilde{N}_2,\tilde{C}_1}$$

These relations can be avoided in models in which the gaugino masses do not unify at the GUT scale; e.g. gauge mediated models.

Higgs sector mass parameters:

$$\mathcal{L}_{soft}^{MSSM} = -m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d$$

The RGEs are:

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3X_t - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2$$
$$16\pi^2 \frac{d}{dt} m_{H_d}^2 = 3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2$$

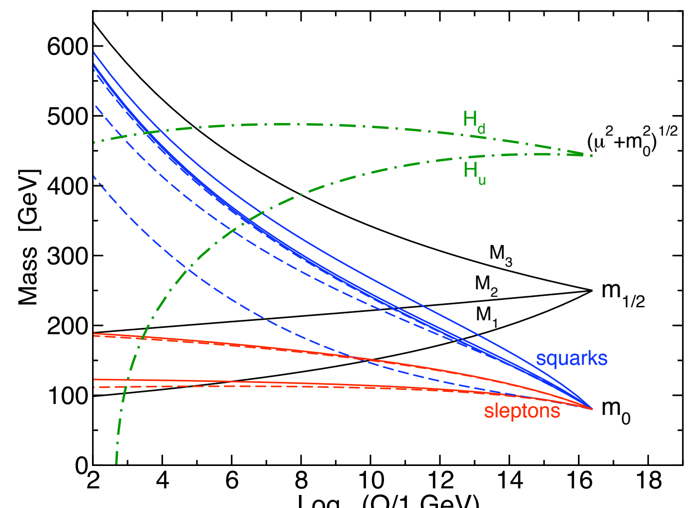
X_t, X_b, X_τ are some convenient positive-definite parameter combinations,

$$X_t = 2|y_t|^2(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2$$
$$X_b = 2|y_b|^2(m_{H_d}^2 + m_{Q_3}^2 + m_{\bar{d}_3}^2) + 2|a_b|^2$$
$$X_\tau = 2|y_\tau|^2(m_{H_d}^2 + m_{L_3}^2 + m_{\bar{e}_3}^2) + 2|a_\tau|^2$$

$X_{t,b,\tau}$ decrease the Higgs masses as you evolve down from the GUT scale.

Can start with positive m_{H_u} and m_{H_d} at the GUT scale and have them run negative by the EW scale.

This is radiative electroweak symmetry breaking – usually caused by X_t because y_t is large.



from Martin, hep-ph/9709356

Squark and slepton mass parameters:

The RGEs for the 3rd generation are:

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m^2_{Q_3} &= X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 \\
16\pi^2 \frac{d}{dt} m^2_{\bar{u}_3} &= 2X_t - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2 \\
16\pi^2 \frac{d}{dt} m^2_{\bar{d}_3} &= 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 \\
16\pi^2 \frac{d}{dt} m^2_{L_3} &= X_\tau - 6 g_2^2 |M_2|^2 - \frac{3}{5} g_1^2 |M_1|^2 \\
16\pi^2 \frac{d}{dt} m^2_{\bar{e}_3} &= 2X_\tau - \frac{24}{5} g_1^2 |M_1|^2
\end{aligned}
$$

RGEs for 1st and 2nd generations are the same but without the $X_{t,b,\tau}$ Yukawa contributions.

Large g_3^2 contribution runs squarks heavier than sleptons.

$X_{t,b,\tau}$ contributions run 3rd gen lighter than 1st & 2nd. [dashed lines]

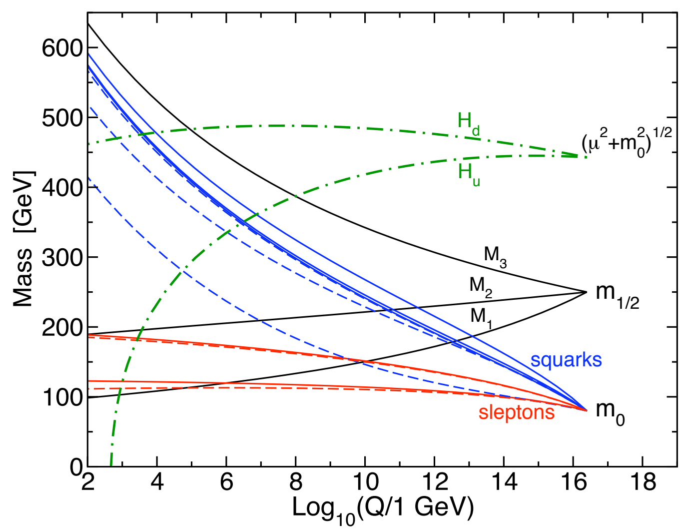


figure from Martin, hep-ph/9709356

What have we learned from the RGEs?

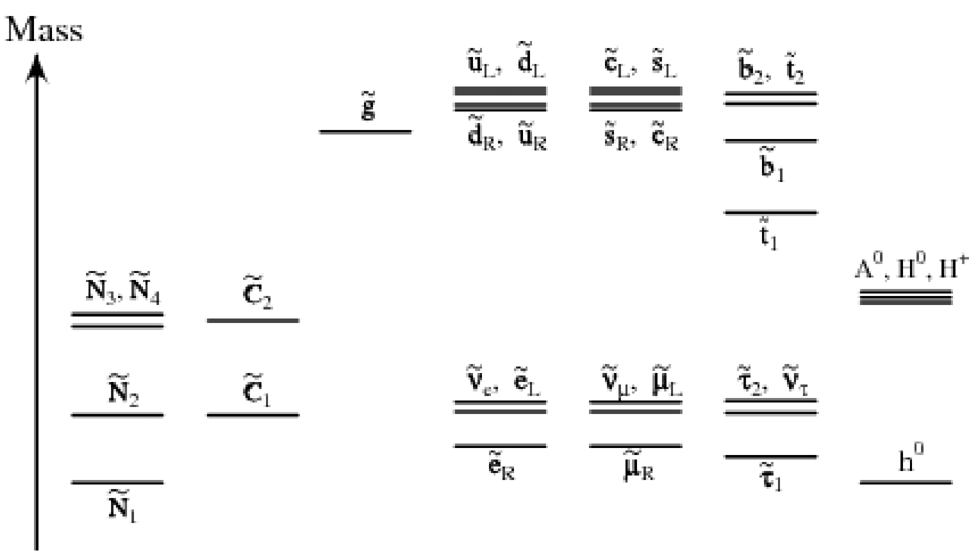
- Squarks run heavier than sleptons due to g_3^2 contribution.
- Gluino runs heavier than weak gauginos due to strong g_3.

Expect coloured sparticles to be heavier than uncoloured sparticles. [if their high-scale masses are not too different]

- Third generation runs lighter due to Yukawa contributions.

Combined with $\tilde{f}_L$–$\tilde{f}_R$ mixing in 3rd gen, expect lightest squark/slepton to be 3rd-gen.

Collider complementarity!



LHC: Produce heavy coloured particles via QCD; lighter uncoloured particles harder to see (lower rates).

ILC: Produce lighter uncoloured particles via EW interactions; heavy coloured particles beyond kinematic reach.

Sparticle decays and collider phenomenology

The general features of SUSY particle decays are controlled by:

R-parity conservation [introduced to avoid fast proton decay]

Lightest R-odd particle (LSP) is stable

Decay chains of R-odd (SUSY) particles must end in LSP

LSP as dark matter: require LSP to be neutral and uncoloured

$\longrightarrow$ escapes from detector $\longrightarrow$ missing energy

Mass spectrum [controlled by SUSY breaking and RGEs]

Heavier particles decay through a cascade of lighter particles

$\longrightarrow$ High multiplicity of objects in SUSY events – multijets, multileptons

NLSP affects event content:

- light stau $\longrightarrow$ events with taus
- light sbottom $\longrightarrow$ events with b-jets

Let's take a closer look at the likely decay modes...

Neutralino and chargino decays

Each neutralino and chargino contains at least a small amount of electroweak gaugino ($\widetilde{B}$, $\widetilde{W}^0$, or $\widetilde{W}^{\pm}$):

$\widetilde{N}_i$, $\widetilde{C}_i$ inherit weak-interaction couplings to fermion+sfermion

$$\widetilde{N}_i \to \ell\widetilde{\ell},\ \nu\widetilde{\nu};\ q\widetilde{q} \text{ [if kinematically allowed]}$$
$$\widetilde{C}_i \to \ell\widetilde{\nu},\ \nu\widetilde{\ell};\ q\widetilde{q}' \text{ [if kinematically allowed]}$$

Each neutralino and chargino contains at least a small amount of Higgsino ($\widetilde{H}_u^+$, $\widetilde{H}_u^0$, $\widetilde{H}_d^0$, or $\widetilde{H}_d^-$):

$\widetilde{N}_i$, $\widetilde{C}_i$ inherit gaugino-Higgsino-Higgs and gaugino-Higgsino-vector boson couplings

$$\widetilde{N}_i \to Z\widetilde{N}_j,\ W\widetilde{C}_j,\ h^0\widetilde{N}_j;\ A^0\widetilde{N}_j,\ H^0\widetilde{N}_j,\ H^{\pm}\widetilde{C}_j \text{ [if kin. allowed]}$$
$$\widetilde{C}_i \to W\widetilde{N}_j,\ Z\widetilde{C}_1,\ h^0\widetilde{C}_1;\ A^0\widetilde{C}_1,\ H^0\widetilde{C}_1,\ H^{\pm}\widetilde{N}_j \text{ [if kin. allowed]}$$

Typical hadron-collider signatures:

$$p+p(\bar{p}) \to \widetilde{C}_1\widetilde{N}_2 \to W\widetilde{N}_1 Z\widetilde{N}_1 \to \ell^+\ell^-\ell' + \text{MET} \qquad \text{(trileptons)}$$
$$p+p(\bar{p}) \to \widetilde{C}_1\widetilde{N}_2 \to W\widetilde{N}_1 \tau^+\widetilde{\tau}_1^- \to \ell\tau^+\tau^- + \text{MET} \qquad \text{(tau-rich)}$$
$$p+p(\bar{p}) \to \widetilde{C}_1\widetilde{N}_2 \to W\widetilde{N}_1 h^0\widetilde{N}_1 \to \ell b\bar{b} + \text{MET} \qquad \text{(b-rich)}$$
$$p+p(\bar{p}) \to \widetilde{N}_2\widetilde{N}_2 \to \ell^+\widetilde{\ell}^-\ell^+\widetilde{\ell}^- \to \ell^+\ell^+W^-W^- + \text{MET} \qquad \text{(like-sign dileptons)}$$

Heavier charginos/neutralinos can have more complicated cascade decays.

Slepton decays

Sleptons decay to lepton+chargino or lepton+neutralino:

$\tilde{\ell} \to \ell \widetilde{N}_i,\ \nu \tilde{C}_i;\qquad \tilde{\nu} \to \nu \widetilde{N}_i,\ \ell \tilde{C}_i$

If $\widetilde{N}_1$ is the LSP, then $\tilde{\ell} \to \ell \widetilde{N}_1$ and $\tilde{\nu} \to \nu \widetilde{N}_1$ are always allowed.

[except for $\tilde{\tau}$ when $m_{\tilde{\tau}_1} - m_{\widetilde{N}_1} < m_\tau$; $\to$ 3-body decay]

For sufficiently heavy sleptons, decays to charginos and heavier neutralinos are important: $\tilde{\ell} \to \nu \tilde{C}_1,\ \ell \widetilde{N}_2;\qquad \tilde{\nu} \to \ell \tilde{C}_1$

These are followed by decays of $\tilde{C}_1$, $\widetilde{N}_2$: cascade!

Left-handed sleptons may prefer decays to (heavier) winos $\tilde{C}_1$, $\widetilde{N}_2$ over decays to (lighter) bino $\widetilde{N}_1$ because SU(2) gauge charge is larger.

Right-handed sleptons are not charged under SU(2):

$\tilde{\ell}_R \to \ell \widetilde{N}_1$ is preferred for bino-like $\widetilde{N}_1$

Squark decays

Squark decay to quark+gluino will always dominate if kinematically allowed – QCD-strength coupling

If $M_{\tilde{q}} < M_{\tilde{g}}$, then squark decays to quark+neutralino or quark+chargino
Direct decay $\tilde{q} \to q\widetilde{N}_1$ is kinematically favored
Dominates for right-handed squarks because $\widetilde{N}_1$ is mostly bino
Left-handed squarks may strongly prefer decay into (heavier) winos, because SU(2) gauge coupling is larger
Heavier neutralino/chargino subsequently decays $\to$ cascade!
Squark decays to Higgsinos are less important, except for $\tilde{t}$, $\tilde{b}$ with large Yukawa couplings
Higgsino subsequently decays $\to$ cascade!

Cascade decays: can have large numbers of jets/leptons/etc in the final state.
Always get at least one jet plus missing p_T from each squark.

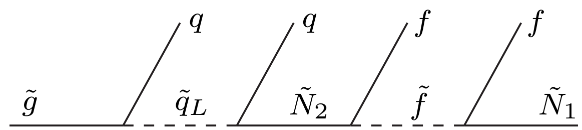
Gluino decays

The gluino can only decay to quark+squark

If $M_{\widetilde{g}} > M_{\widetilde{q}}$, then $\widetilde{g} \to \widetilde{q}q$ dominates.
Detailed mass spectrum matters:
- If only $\widetilde{g} \to \widetilde{t}_1 t$ is open, final state will contain tops.
- If only $\widetilde{g} \to \widetilde{b}_1 b$ is open, final state will contain bottoms.
- If $\widetilde{g} \to \widetilde{q}q$ is open, final state contains more generic looking jets.

All these are followed by decay chain of the squark.

For example:



If $M_{\widetilde{g}} < M_{\widetilde{q}}$, then gluino will decay via an off-shell squark.
$\widetilde{g} \to q\widetilde{q}^* \to q\bar{q}\widetilde{N}_i$ or $q\bar{q}'\widetilde{C}_i$ – three-body decays!

A (perhaps crazy) possibility: Split Supersymmetry
Gluino, gauginos, Higgsinos, and h^0 are at the EW/TeV scale.
All other scalars (squarks, sleptons, heavier Higgses) are VERY heavy, like 10^{11} GeV.
Gluino decays via $\widetilde{g} \to q\widetilde{q}^* \to q\bar{q}\widetilde{N}_i$ or $q\bar{q}'\widetilde{C}_i$, but $\widetilde{q}$ is VERY heavy:
$\to$ Long-lived gluino!

Displaced vertices; gluino air-showers; early-universe constraints

Decays to the gravitino/goldstino

In Gauge-Mediated SUSY Breaking (GMSB) models the LSP is the gravitino $\tilde{G}$ (superpartner of the graviton)

Gravitino itself couples with gravity-strength couplings: basically irrelevant.

However, once local SUSY is broken, gravitino "eats" goldstino to get a mass.

Goldstino has non-gravitational coupling to all sparticle-particle pairs: can be relevant for collider phenomenology.

Decay $\widetilde{X} \to X\tilde{G}$:

Typically too slow to compete with other decays of $\widetilde{X}$, unless $\widetilde{X}$ is the NLSP (LSP is $\tilde{G}$).

NLSP will always decay to its superpartner and $\tilde{G}$.

Phenomenology depends on what is the NLSP.

Lightest neutralino: Contains a photino component: $\widetilde{N}_1 \to \gamma\tilde{G}$

Events have $\gamma\gamma$+MET; may have "non-pointing photons".

Charged slepton: Leptons, esp. τ's in final state.

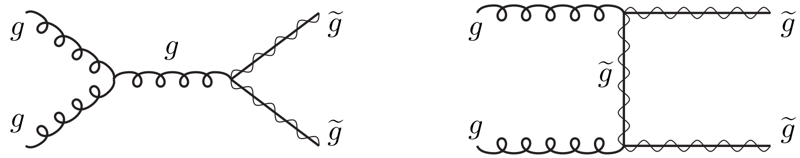
Finite decay length $\to$ heavy/slow charged tracks, decay kinks.

Superparticle production at hadron colliders

SUSY particles are always produced in pairs (because of R-parity).

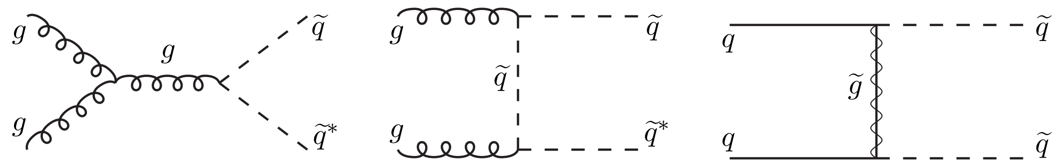
Production via QCD:

Gluino pairs



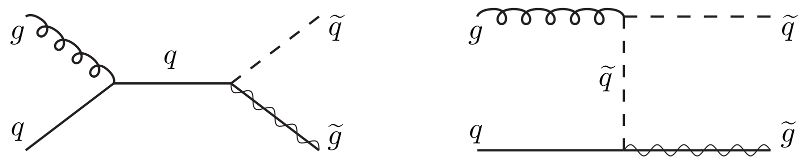
etc.

Squark pairs



etc.

Squark + gluino



etc.

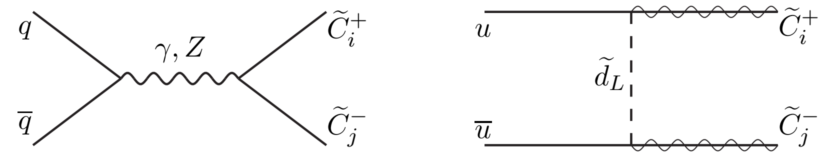
LHC reach for gluinos, squarks typically out to about 1 to 2 TeV. Although coloured particles are typically heavier than colour-neutral particles (due to RG running), large QCD production cross sections make them typically easier to see at LHC.

Superparticle production at hadron colliders

SUSY particles are always produced in pairs (because of R-parity).

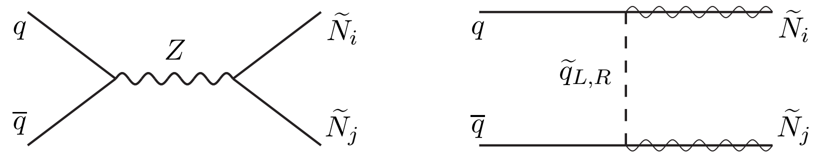
Production via EW int's:

Chargino pairs

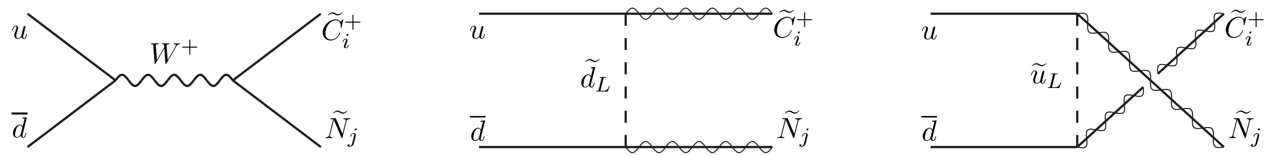


etc.

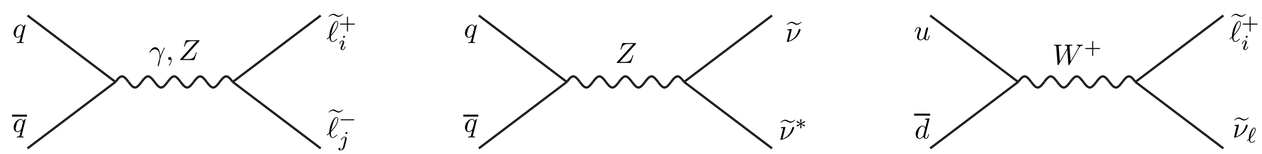
Neutralino pairs



Chargino + neutralino



Slepton pairs



Rates are smaller than for coloured particles because production cross sections involve EW couplings.

Can also have associated $\widetilde{N}_i \widetilde{q}$, $\widetilde{C}_i^{\pm} \widetilde{q}$ production – EW strength.

Some generic signatures of SUSY at hadron colliders:

Missing transverse energy
From two escaping LSPs

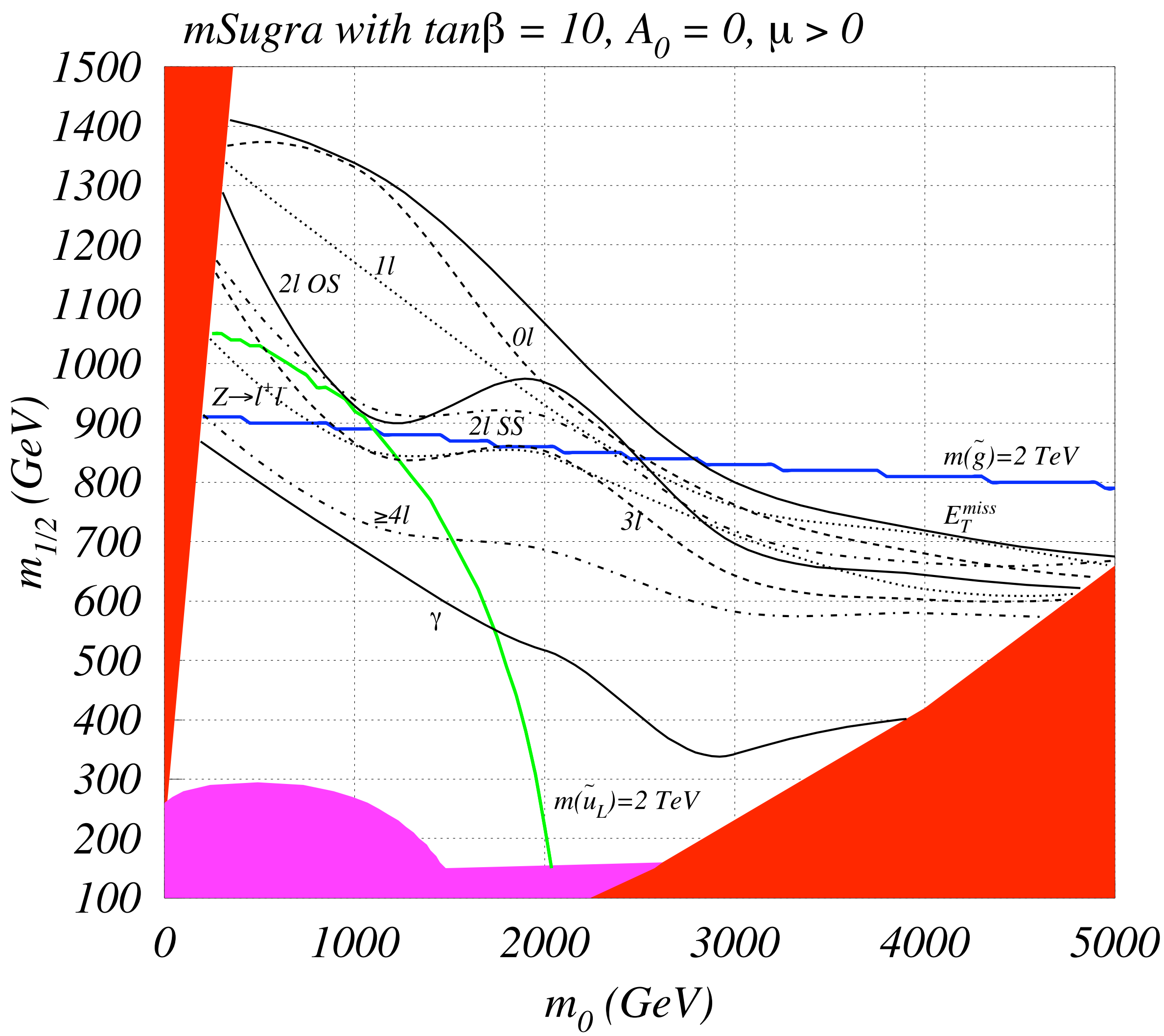
Large jet multiplicity
Produce heavier SUSY particles via QCD; long decay chains

Large $\sum E_T$ in event
Decay of heavy particles produces energetic jets, leptons
Relatively spherical distribution in detector

Like-sign leptons or b-jets
Gluino is Majorana – decays equally likely to $\widetilde{q}$ or $\widetilde{q}^*$
Decay chain gives leptons – like-sign if $\widetilde{q}\widetilde{q}$ or $\widetilde{q}^*\widetilde{q}^*$

Many more specific signatures have been studied in detail.
Signatures depend strongly on mass spectrum.

LHC "reach" for discovering SUSY: [an example in CMSSM]



from Baer, Balázs, Belyaev, Krupovnickas, & Tata, hep-ph/0304303

After discovery, next priority is measuring SUSY masses and couplings.

A new challenge:

Each SUSY event contains *two* invisible massive particles.

Can't reconstruct SUSY masses directly

Can't even measure transverse mass like for W

Need to use more sophisticated techniques:

take advantage of decay chains.

- Kinematic endpoints
- Kinematic shapes
- Four-momentum conservation relations

Kinematic endpoints at LHC

LHC is a proton-proton collider

- $\sqrt{s}$ not known; varies event-by-event
- Boost of CM along beam direction not known

LHC can produce heavy SUSY particles $\to$ long decay chains.

More kinematic variables to play with.

Don't know the boost of individual events:

$\to$ use kinematic invariants, like invariant masses.

Consider the decay chain $\widetilde{N}_2 \to \tilde{\ell}^{\pm}_R \ell^{\mp} \to \widetilde{N}_1 \ell^+ \ell^-$

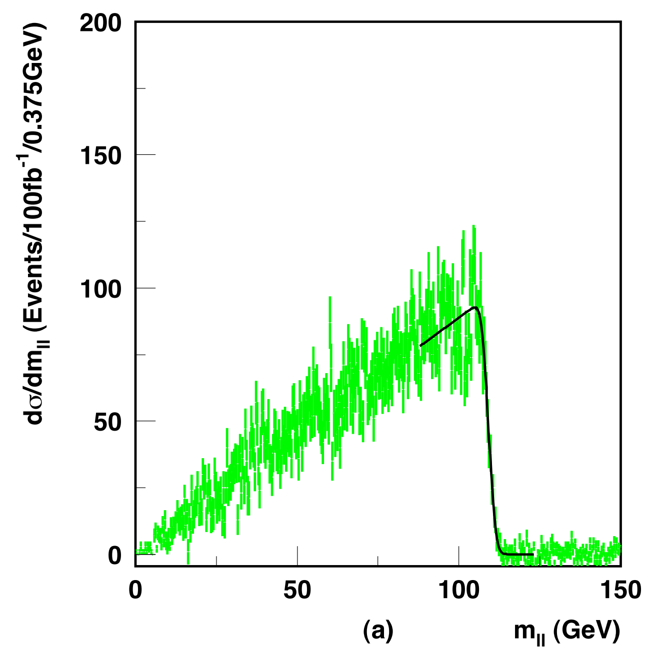
Need to select events that contain $\widetilde{N}_2$ and identify the $\ell^+\ell^-$ from $\widetilde{N}_2$ decay.

Observable: invariant mass $M_{\ell\ell}$ of $\ell^+\ell^-$

4-momentum cons. + $m_\ell \simeq 0$ $\to$ derive $M^2_{\ell\ell}$ distribution in terms of SUSY masses.

$$M_{\ell\ell}|^{\max} = \left[\frac{\left(m^2_{\widetilde{N}_2} - m^2_{\tilde{\ell}}\right)\left(m^2_{\tilde{\ell}} - m^2_{\widetilde{N}_1}\right)}{m^2_{\tilde{\ell}}} \right]^{1/2}$$

Endpoint of $M_{\ell\ell}$ constrains combination of 3 SUSY masses.



from Paige, hep-ph/0211017

LHC can do more if we look at longer decay chains:
$\longrightarrow$ more kinematic invariants to play with.

Add a squark to the top of our decay chain:

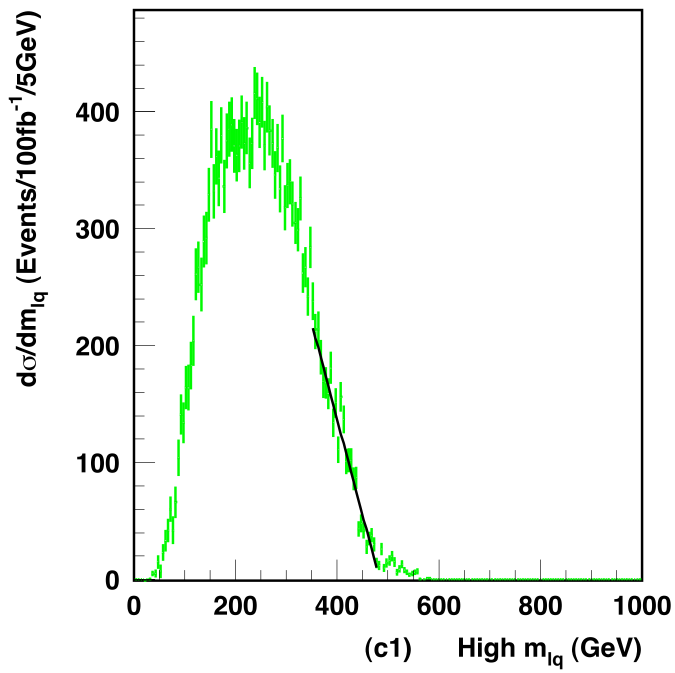
$$\tilde{q} \to \widetilde{N}_2 q \to \tilde{\ell}^{\pm}\ell^{\mp} q \to \widetilde{N}_1 \ell^+ \ell^- q$$

Invariant mass of q and the first lepton emitted (ℓ_1) has an endpoint analogous to the $\ell\ell$ endpoint:

$$M_{q\ell_1}\Big|^{\max} = \left[\frac{\left(m_{\tilde{q}}^2 - m_{\widetilde{N}_2}^2\right)\left(m_{\widetilde{N}_2}^2 - m_{\tilde{\ell}}^2\right)}{m_{\widetilde{N}_2}^2} \right]^{1/2}$$

How to distinguish ℓ_1 from ℓ_2?
$\longrightarrow$ ℓ_1 likely to have higher energy.

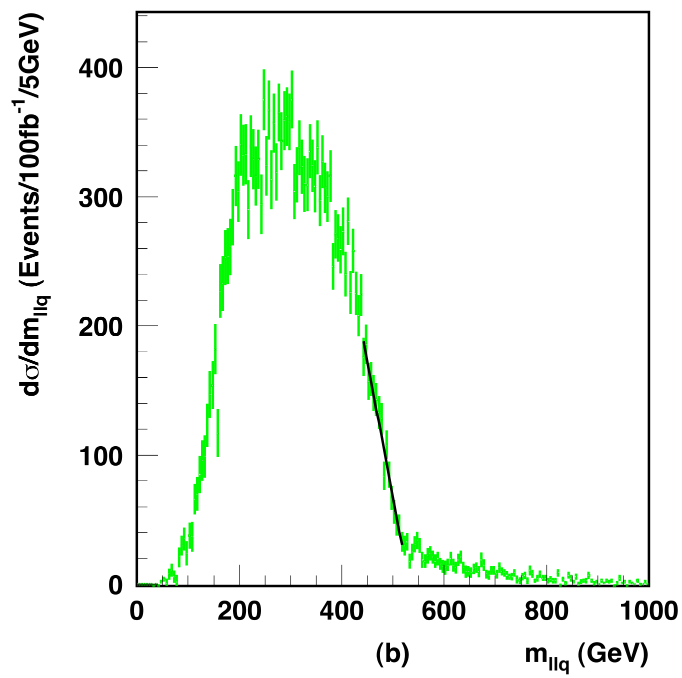
With $M_{q\ell_1}|^{\max}$ and $M_{\ell\ell}|^{\max}$ we have 2 measurements and 4 unknowns.
Not doing better than before... yet.



from Paige, hep-ph/0211017

Decay chain has an extra kinematic invariant:
Invariant mass of $q\ell^+\ell^-$.

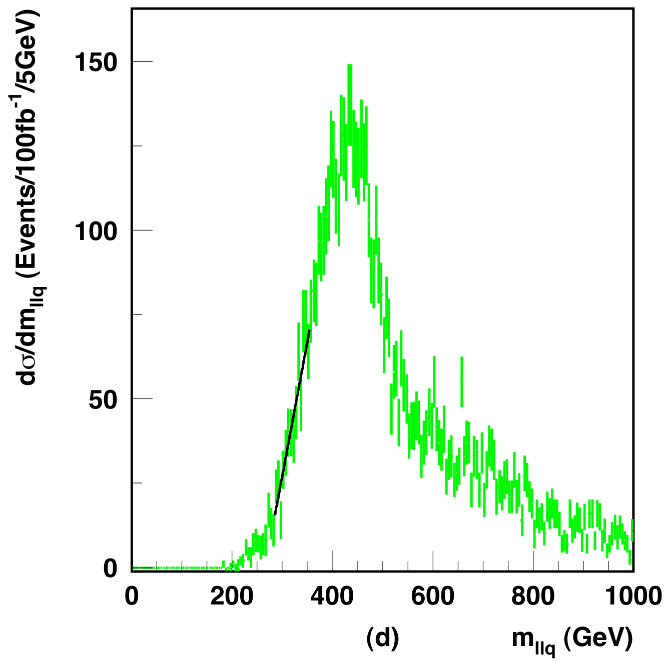
$$M_{q\ell\ell}|^{\max} = \left[\frac{\left(m_{\tilde{q}}^2 - m_{\tilde{N}_2}^2\right)\left(m_{\tilde{N}_2}^2 - m_{\tilde{N}_1}^2\right)}{m_{\tilde{N}_2}^2} \right]^{1/2}$$



from Paige, hep-ph/0211017

3 measurements and 4 unknowns.
Doing better!

There are also lower kinematic edges:



from Paige, hep-ph/0211017

After applying a cut $M_{\ell\ell} > M_{\ell\ell}^{\rm max}/\sqrt{2}$, get a complicated formula for a lower kinematic endpoint for $M_{q\ell\ell}$.

Can also consider the decay chain $\tilde{q} \to \widetilde{N}_2 q \to \widetilde{N}_1 h q$ with $h \to b\bar{b}$

[The Higgs mass can be measured elsewhere]

Then M_{hq} has a threshold (lower kinematic edge)

Get enough measurables to extract all the masses!

Uncertainties from blurring of the kinematic endpoints by backgrounds, wrong jet/lepton combinations, also gluon radiation off the jet at NLO.

Kinematic endpoints: Statistics are not super; we're only making use of the events right near the endpoints.

Can we use the events from the middles of the distributions to do better? Some avenues of research:

Kinematic shapes:

Fit to the whole shape of the invariant mass distributions, not just the endpoint. Helps to deal with background.

Gjelsten, Miller, & Osland, hep-ph/0410303, 0501033

Exact kinematic relations:

Completely solve the kinematics of each SUSY cascade decay.

Each event gives 4-momenta of all decay products except $\widetilde{N}_1$:

4 unmeasured momentum components

Need longer decay chain: at least 5 sparticles $\rightarrow$ 5 mass-shell conditions. E.g.: $\tilde{g} \rightarrow q\tilde{q} \rightarrow qq\widetilde{N}_2 \rightarrow qq\ell\tilde{\ell} \rightarrow qq\ell\ell\widetilde{N}_1$

Each event picks out a 4-dimensional hypersurface in a 5-dimensional mass parameter space.

Overlap multiple events in this hyperspace $\rightarrow$ find a discrete set of solutions from overlap of different hypersurfaces.

Kawagoe, Nojiri, & Polesello, hep-ph/0410160

Superparticle production at e^+e^- collider (ILC)

All (kin. accessible) sparticles can be pair produced in e^+e^-.
Even gluino can be pair-produced via a loop.

Squarks, sleptons: pair production via s-channel Z, γ exchange
$e^+e^- \to Z^*, \gamma^* \to \tilde{\ell}\tilde{\ell}, \tilde{q}\tilde{q}$ $\qquad$ $e^+e^- \to Z^* \to \tilde{\nu}\tilde{\nu}$
Selectrons $\tilde{e}_L\tilde{e}_L$, $\tilde{e}_R\tilde{e}_R$ and electron-sneutrinos $\tilde{\nu}_e\tilde{\nu}_e$:
also have production from t-channel exchange of a virtual neutralino or chargino (respectively)

- $\bar{e}_L e_L \to \tilde{e}_L\tilde{e}_L$: t-channel $\tilde{B}$, $\widetilde{W}^0$
- $\bar{e}_L e_L \to \tilde{\nu}_e\tilde{\nu}_e$: t-channel $\widetilde{W}^\pm$
- $\bar{e}_R e_R \to \tilde{e}_R\tilde{e}_R$: t-channel $\tilde{B}$

e^-e^- collisions isolate t-channel $\tilde{e}^-\tilde{e}^-$ production

Charginos and neutralinos: pair production via s-channel Z, γ exchange
$e^+e^- \to Z^*, \gamma^* \to \tilde{C}_i^+\tilde{C}_i^-$ $\qquad$ $e^+e^- \to Z^* \to \tilde{C}_i^+\tilde{C}_j^-, \widetilde{N}_i\widetilde{N}_j$
Nondiagonal charginos $\tilde{C}_i^+\tilde{C}_j^-$ and neutralinos $\widetilde{N}_i\widetilde{N}_j$:
also have production from t-channel exchange of a virtual electron-sneutrino or selectron (respectively)

Measuring SUSY particle masses at ILC

Imagine we produce $\tilde{\ell}_R \tilde{\ell}_R$ pairs in e^+e^- collisions, and they each decay to $\ell \widetilde{N}_1$. How can we measure their masses?

- Do a threshold scan, and/or
- Use "kinematic endpoints"

Kinematic endpoints: Measure maximum and minimum values of ℓ energies $\rightarrow$ extract $M_{\tilde{\ell}_R}$ and $M_{\widetilde{N}_1}$.

Relativistic kinematics gives the lepton energy in the CM frame:

$$E_\ell^{\rm CM} = \frac{M_{\tilde{\ell}_R}^2 - M_{\widetilde{N}_1}^2}{4 M_{\tilde{\ell}_R}^2} \left(\sqrt{s} + \sqrt{s - 4 M_{\tilde{\ell}_R}^2} \cos\theta^* \right)$$

Max (min) lepton energy corresponds to $\cos\theta^* = 1$ (-1)

[$\cos\theta^*$ is defined in $\tilde{\ell}$ rest frame]

Solve for $M_{\tilde{\ell}_R}$ and $M_{\widetilde{N}_1}$:

$$M_{\tilde{\ell}_R}^2 = \frac{s}{4} \left[1 - \left(\frac{E^{max} - E^{min}}{E^{max} + E^{min}} \right)^2 \right] \qquad M_{\widetilde{N}_1}^2 = M_{\tilde{\ell}_R}^2 \left[1 - \frac{2(E^{max} + E^{min})}{\sqrt{s}} \right]$$

Example: $e^+e^- \to \tilde{\mu}^+_{L,R}\tilde{\mu}^-_{L,R}$, $M_{\tilde{\mu}_R} = 178$ GeV, $M_{\tilde{\mu}_L} = 287$ GeV

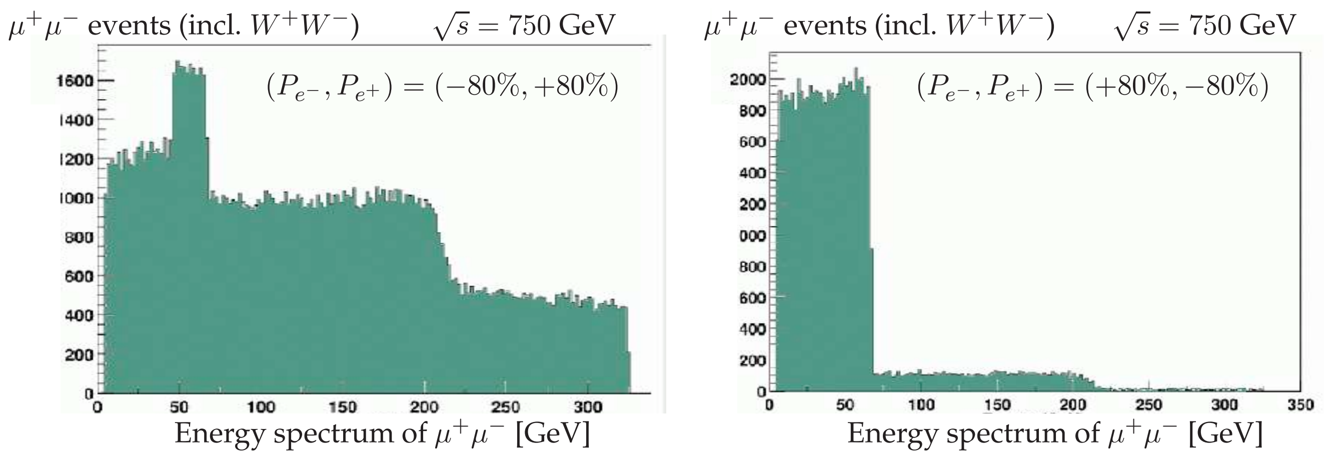


Figure 3.4: Energy spectrum of muons from $\tilde{\mu}_{L,R}$ decays into $\mu\tilde{\chi}^0_1$ final states, including the W^+W^- background decaying into $\mu\nu$ final states in the scenario S3, cf. table 3.1, for two combinations of beam polarizations for $\sqrt{s} = 750$ GeV and $\mathcal{L}_{\rm int} = 500$ fb^{-1} [87].

from Moortgat-Pick et al, hep-ph/0507011

Beam polarization:

(a) $e^-_L e^+_R$: Producing $\tilde{\mu}_L\tilde{\mu}_L$ and $\tilde{\mu}_R\tilde{\mu}_R$ and WW background
(b) $e^-_R e^+_L$: Producing mostly $\tilde{\mu}_R\tilde{\mu}_R$

Eyeballing the muon energy edges at about 50, 65 and 220 GeV:

$\tilde{\mu}_L$: $E^{max} \approx 220$ GeV, $E^{min} \approx 50$ GeV (note drop: pol dep $\to \tilde{\mu}_L$)
$\tilde{\mu}_R$: $E^{max} \approx 65$ GeV, E^{min} not visible!

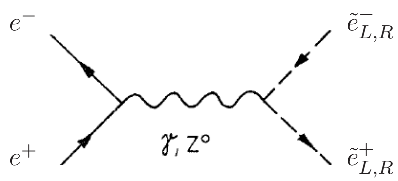
Solve $\to$ $m_{\tilde{\mu}_L} \approx 282$ GeV (compare input 287 GeV)
$m_{\tilde{N}_1} \approx 153$ GeV
$m_{\tilde{\mu}_R} \approx 167$ GeV (compare input 178 GeV)

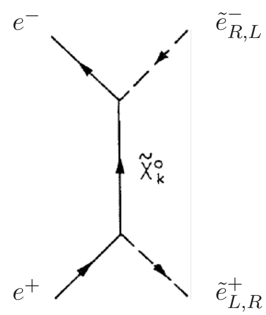
Testing SUSY coupling relations at ILC

SUSY predictions: [Want to test these!]

- Selectrons carry same quantum numbers as electrons
- Electron-selectron-gaugino Yukawa couplings related to electron gauge couplings

Study the production processes $e^+e^- \to \tilde{e}^+_{L,R}\tilde{e}^-_{L,R}$.





s-channel: Can produce $\tilde{e}^+_L\tilde{e}^-_L$ and $\tilde{e}^+_R\tilde{e}^-_R$ through γ, Z couplings.
t-channel: Can produce all 4 combinations: $\tilde{e}^+_L\tilde{e}^-_L$, $\tilde{e}^+_R\tilde{e}^-_R$, $\tilde{e}^+_L\tilde{e}^-_R$, and $\tilde{e}^+_R\tilde{e}^-_L$.

Signal rates depend on:

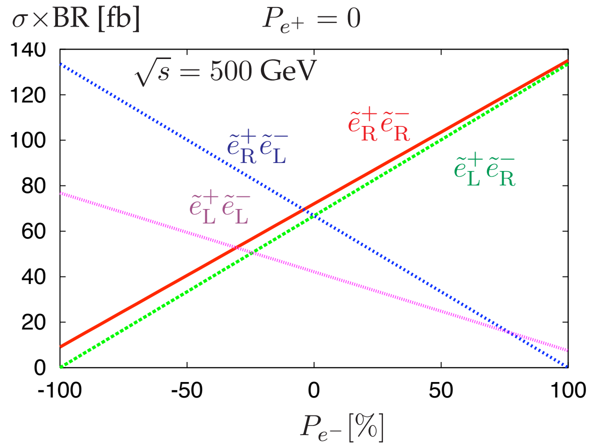
- $\tilde{e}_L$ and $\tilde{e}_R$ masses
- Selectron gauge couplings and $e\tilde{e}\widetilde{N}_i$ Yukawa couplings
- Masses and composition of all 4 $\widetilde{N}_i$ exchanged in t-channel

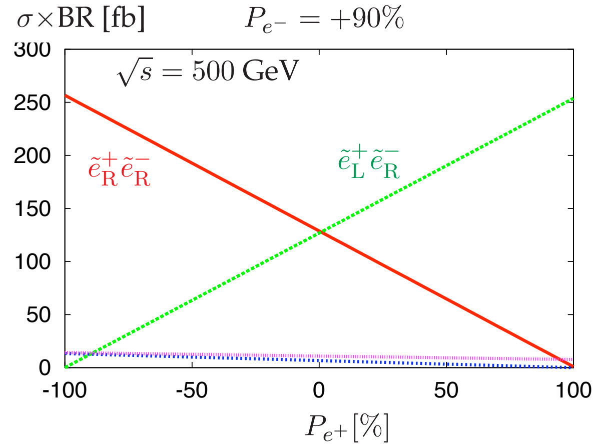
e^+e^- beam polarization is a great asset!

Check quantum numbers of $\tilde{e}_L, \tilde{e}_R$

See if e_L couples to $\tilde{e}_L + \widetilde{N}_i$ and e_R couples to $\tilde{e}_R + \widetilde{N}_i$.

Isolate t-channel process by colliding $\bar{e}_R e_L \to \tilde{e}^+_R \tilde{e}^-_L$, $\bar{e}_L e_R \to \tilde{e}^+_L \tilde{e}^-_R$.





Plots assume $M_{\tilde{e}_L} \approx M_{\tilde{e}_R}$, decay mode $\tilde{e}_{L,R} \to e\widetilde{N}_1$

from Moortgat-Pick et al, hep-ph/0507011

Linear dependence of cross sections with polarization is just due to dialling the luminosity of the relevant polarization component of the beam.

Dial the polarization to separate the four modes.

Check whether $e\tilde{e}\widetilde{N}$ Yukawa $=$ eeV gauge coupling

Want to separately measure $e_L\tilde{e}_L\widetilde{N}_i$ coupling and $e_R\tilde{e}_R\widetilde{N}_i$ coupling

For this we must assume that the neutralino masses and mixing parameters have already been measured!

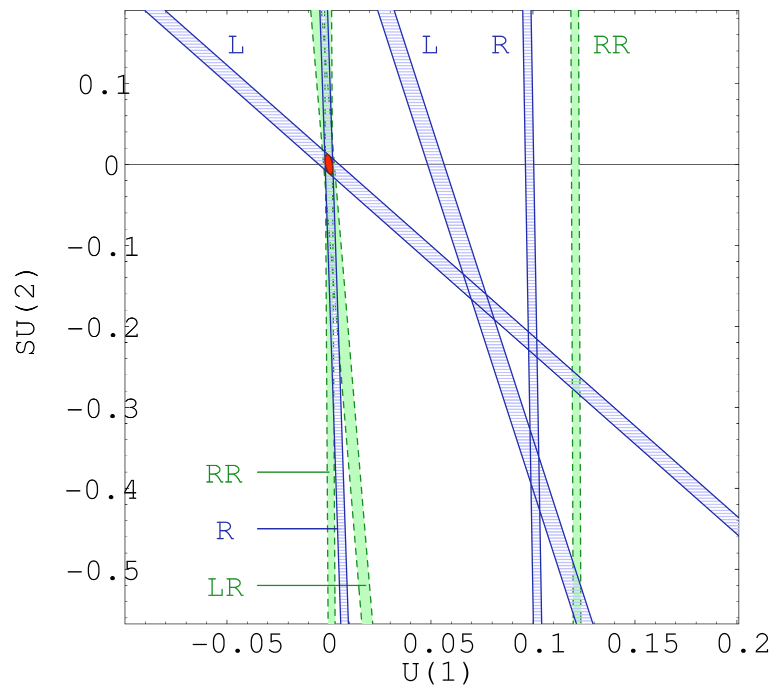
Technique: measure $\tilde{e}\tilde{e}$ production cross sections from polarized initial beams.

- $\bar{e}_R e_R \to \tilde{e}_R\tilde{e}_R, \tilde{e}_L\tilde{e}_L$ via s-channel γ, Z and t-channel $\widetilde{N}_i$
- $\bar{e}_L e_L \to \tilde{e}_R\tilde{e}_R, \tilde{e}_L\tilde{e}_L$ via s-channel γ, Z and t-channel $\widetilde{N}_i$
- $\bar{e}_R e_L \to \tilde{e}^+_R\tilde{e}^-_L$ via t-channel $\widetilde{N}_i$

Fit to extract deviations of Yukawas from SM gauge couplings.

Works even if final-state $\tilde{e}_R$ and $\tilde{e}_L$ cannot be distinguished easily (e.g., close in mass, same decay modes)

(Plot for 90% e^- pol, 60% e^+ pol)



from Moortgat-Pick et al, hep-ph/0507011

Combine LHC and ILC measurements to do better than either one alone

Particle	Mass	"LHC"	"ILC"	"LHC+ILC"
h^0	116.9	0.25	0.05	0.05
H^0	425.0		1.5	1.5
$\tilde{\chi}_1^0$	97.7	4.8	0.05	0.05
$\tilde{\chi}_2^0$	183.9	4.7	1.2	0.08
$\tilde{\chi}_4^0$	413.9	5.1	$3-5$	2.5
$\tilde{\chi}_1^\pm$	183.7		0.55	0.55
$\tilde{e}_R$	125.3	4.8	0.05	0.05
$\tilde{e}_L$	189.9	5.0	0.18	0.18
$\tilde{\tau}_1$	107.9	$5-8$	0.24	0.24
$\tilde{q}_R$	547.2	$7-12$	–	$5-11$
$\tilde{q}_L$	564.7	8.7	–	4.9
$\tilde{t}_1$	366.5		1.9	1.9
$\tilde{b}_1$	506.3	7.5	–	5.7
$\tilde{g}$	607.1	8.0	–	6.5

LHC measures mass *differences* fairly well; big source of uncertainty is LSP mass.

ILC nails down LSP mass; sharpens up measurements of light sparticles.

table from Kilian & Zerwas, hep-ph/0601217

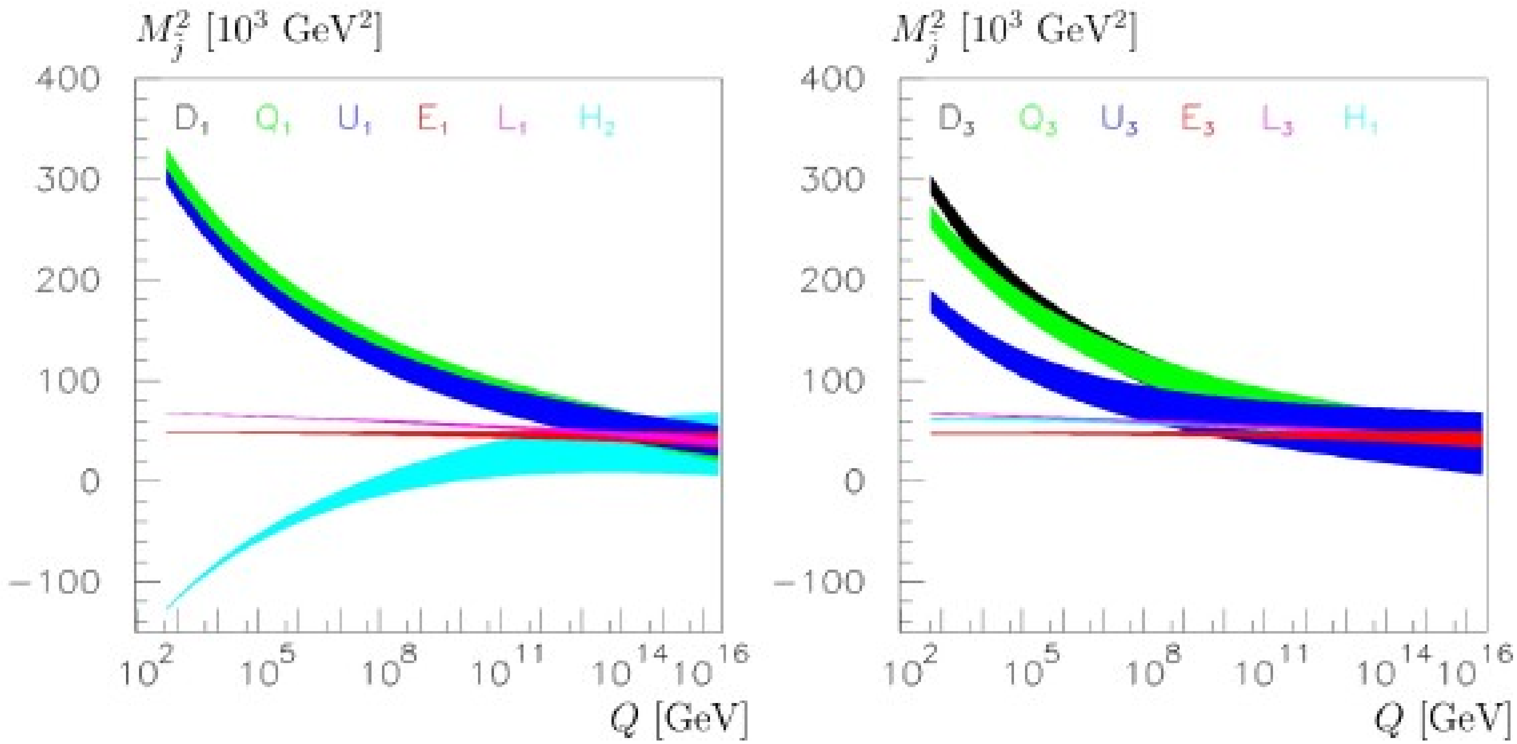
for SUSY reference point SPS1a$'$ [Masses in GeV]

Reconstructing the high-scale theory: "Einstein's Telescope"

The RGEs will let us extrapolate the high-scale physics based on measurements of the EW scale parameters.

Need high precision: experimental uncertainties can be amplified by the RGE running.

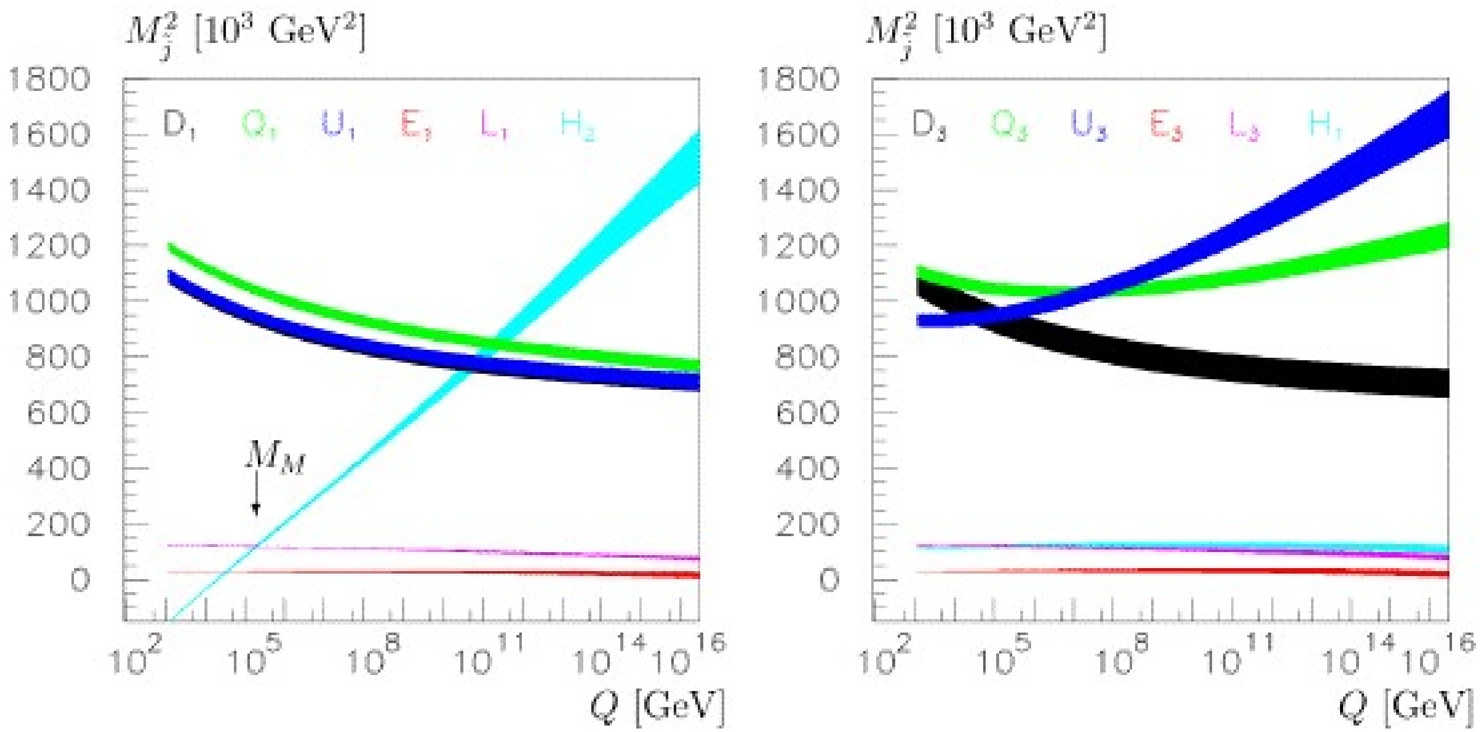
"Constrained MSSM" (CMSSM) model (a.k.a. mSUGRA):



from Blair, Porod & Zerwas, hep-ph/0210058

Run soft-SUSY-breaking parameters up, see if they unify at the high scale!

Contrast "gauge-mediated SUSY breaking" (GMSB) model:



from Blair, Porod & Zerwas, hep-ph/0210058

Soft-SUSY-breaking parameters do not unify:
they are related to beta-functions at the messenger scale M_M.

Hope is to learn about high-scale physics from low-scale SUSY spectrum.

Summary

SUSY is a mathematically beautiful solution to the Hierarchy Problem

It potentially offers insight to the highest energy scales through the pattern of SUSY-breaking masses

Gives a nice dark matter candidate

Discovery prospects good at the LHC

lots of jets, missing p_T

Can do fantastic high-precision studies at ILC

Near-term challenge:

Find more ways to measure masses, couplings, spins at LHC