TRIUMF Summer Institute 2006
Collider and Energy Frontier Physics

Beyond the Standard Model

Lecture 1

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Plan

Lecture 1 Monday July 17

- Why BSM?
- Supersymmetry

Lecture 2 Monday July 17

- Supersymmetry continued: phenomenology

Lecture 3 Wednesday July 19

- Large extra dimensions: ADD
- Universal extra dimensions; particle spins and UED vs. SUSY

Lecture 4 Thursday July 20

- Deconstruction and the Little Higgs
- T-parity

Lecture 5 Friday July 21

- Warped extra dimensions: RS
- RS and Technicolour

Goals of these lectures

I want to give you a flavour of the "landscape" of Beyond the Standard Model physics.

I'll introduce the general classes of BSM models, describe their motivations, features, differences and similarities, and give an overview of the collider phenomenology.

I'll start today by motivating why we go beyond the SM.

Why go Beyond the Standard Model?

All our collider data agrees very well with the Standard Model. So why go beyond?

There are things that the SM cannot explain.

- Dark matter – what is it?
- The universe is so smooth – inflation?
- How did the matter/antimatter balance get skewed?
- Neutrino masses – where do they come from?
- What about a quantum theory of gravity?

These are all important questions. But the driving force behind Beyond the Standard Model physics is the Hierarchy Problem.

This is a sort of theoretical problem with scalars, in particular with the Standard Model Higgs.

The Higgs mechanism in the Standard Model

In the Standard Model, electroweak symmetry is broken by a single scalar Higgs doublet.

- scalar: spin zero
- doublet under SU(2)$_L$

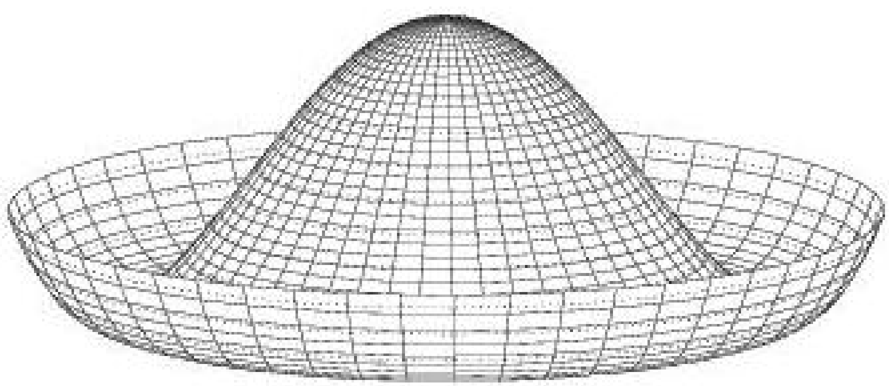
$$H = \begin{pmatrix} G^+ \\ (h+v)/\sqrt{2} + iG^0/\sqrt{2} \end{pmatrix}$$

- G^+ and G^0 are the Goldstone bosons that get "eaten" by the W^+ and Z bosons, giving them mass.
- v is the SM Higgs vacuum expectation value (vev), $v = 2m_W/g \simeq 246$ GeV.
- h is the SM Higgs field, a physical particle.

Electroweak symmetry breaking comes from the Higgs potential:

$$V = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

where $\lambda \sim \mathcal{O}(1)$
and $m^2 \sim \mathcal{O}(M_{\rm EW}^2)$

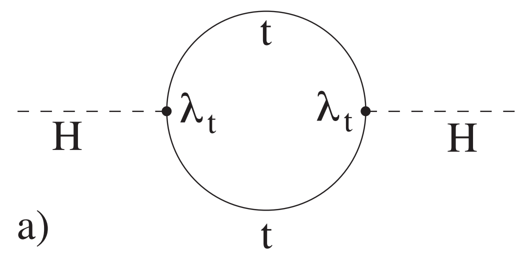


The Hierarchy Problem

The Higgs mass-squared parameter m^2 gets quantum corrections that depend quadratically on the high-scale cutoff of the theory.

Calculate radiative corrections

from, e.g., a top quark loop



Bare $V_0 = -m_0^2 H_0^\dagger H_0 + \lambda_0 (H_0^\dagger H_0)^2$

Renormalized $V = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$

$m^2 = m_0^2 + \Delta m^2, \quad \lambda = \lambda_0 + \Delta\lambda$, etc

$$\text{top loop}: \quad \Delta m^2 = \frac{N_c \lambda_t^2}{16\pi^2}\left[-2\Lambda^2 + 6 m_t^2 \ln(\Lambda/m_t) + \cdots\right]$$

We measure $m^2 \sim \mathcal{O}(M_{\rm EW}^2) \sim 10^4~{\rm GeV}^2$.

Nature sets m_0^2 at the cutoff scale Λ.

If $\Lambda = M_{\rm Pl} = \frac{1}{\sqrt{8\pi G_N}} \sim 10^{18}$ GeV, then $\Delta m^2 \sim -10^{35}~{\rm GeV}^2$!

Nature has to engineer a cancellation to 31 decimal places!

and not just at one loop – must cancel two-, three-, four-, ... loop contributions

Could such a fine-tuned cancellation be only a coincidence?

Looks horrible; there "must" be a physics reason why $m^2 \ll M_{\rm Pl}^2$!

Solutions to the hierarchy problem

How low must the cutoff scale Λ be for the cancellation to be "natural"? Want $|\Delta m^2| \sim 10^4$ GeV^2 $\longrightarrow \Lambda \sim 1$ TeV!
The fine-tuning argument tells us to expect New Physics that solves the hierarchy problem to appear around 1 TeV!

(plus or minus an order of magnitude...)

So what is the New Physics?
There are two main approaches in BSM physics:

1. Make the Higgs composite

2. Use supersymmetry

1. Make the Higgs composite

There are no fundamental scalars that we've discovered.
The only scalar particles that we know of are mesons, composite quark+antiquark bound-states.

Maybe the Higgs is also a composite bound-state of some new fermions. No fundamental scalars → no hierarchy problem.

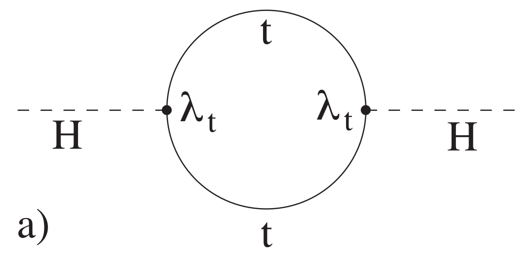
The analogy is QCD:
The strong coupling runs stronger in the infrared (low energies) until QCD confines. m^2 fixed by scale where coupling gets strong
After confinement there is a quark condensate $\langle q\bar{q}\rangle \neq 0$
– analogous to the Higgs vev.
The pions $\pi^\pm$, π^0 actually have the same quantum numbers as the Goldstone bosons $G^\pm$, G^0 that give $W^\pm$ and Z their masses.
– With no Higgs, QCD would break electroweak symmetry and give W and Z masses at the 100 MeV scale!

This approach is called Technicolour – analogous to the strong (colour) interaction. [I'll talk about Technicolour in Lecture 5.]

2. Use supersymmetry

First recall the top loop:

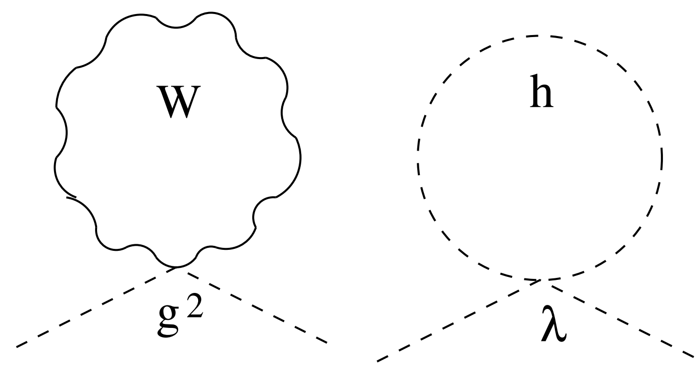
$$\Delta m^2 = \frac{N_c \lambda_t^2}{16\pi^2}\left[-2\Lambda^2 + 6m_t^2 \ln(\Lambda/m_t) + \cdots\right]$$



The minus sign comes because it's a fermion loop.

Loops of bosons (W, Z, Higgs) give positive contributions to Δm^2:
$\Delta m^2 \sim (g^2/16\pi^2)\Lambda^2 + \cdots$

Maybe these cancel the top loop?
No, because the couplings are different.



But: if we introduced new bosons with couplings engineered to cancel the fermion loops and new fermions with couplings engineered to cancel the boson loops then it would work.

This is how Supersymmetry solves the hierarchy problem:

Each SM fermion gets a boson partner (sfermion)

Each SM boson gets a fermion partner (-ino)

The relevant couplings for the Δm^2 cancellation are related by the (super-) symmetry

E.g., the Higgs self-coupling loop:

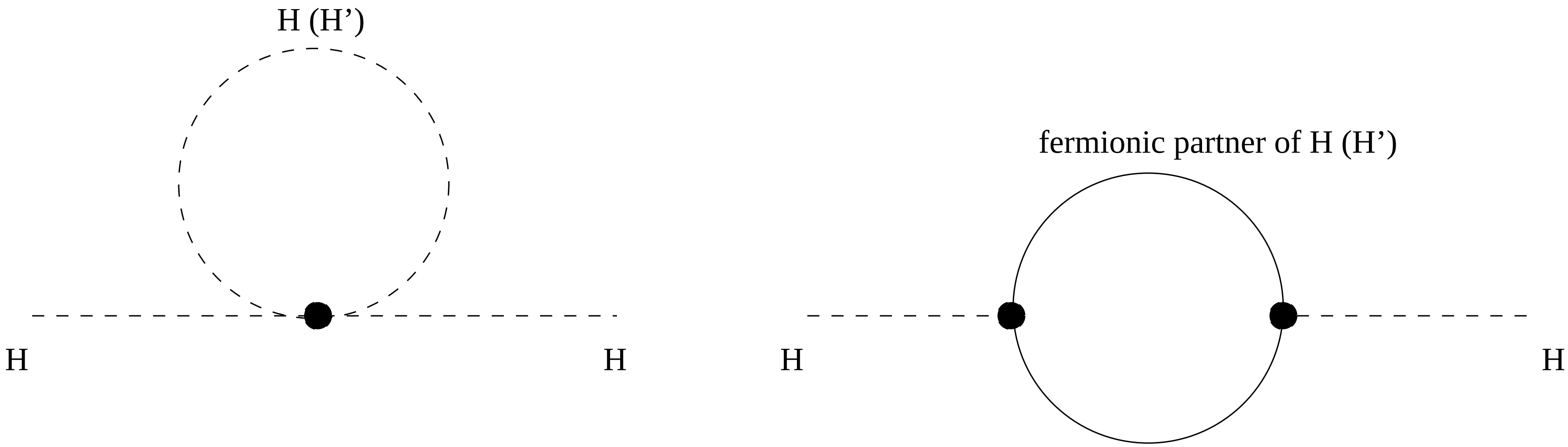


figure from Poppitz, hep-ph/9710274

It's easy to show that it works at one loop.
More difficult to check the two-, three-, ... loops (but it works!).

It's easier to understand the cancellation from a symmetry point of view.

Recall that fermion masses don't have a hierarchy problem:
e.g., fermion self-energy diagram with a gauge boson loop gives

$$\Delta m_f \sim \frac{g^2}{16\pi^2} m_f \log\left(\frac{\Lambda^2}{m_f^2}\right)$$

Notice that $\Delta m_f \propto m_f$.
This is a manifestation of chiral symmetry:
In the limit $m_f = 0$ the system has an extra symmetry: the left- and right-handed components of the fermion are separate objects.
In this limit, radiative corrections cannot give $m_f \neq 0$ – fermion mass is protected by chiral symmetry.

Scalars have no such symmetry protection (in a non-SUSY theory).

But Supersymmetry relates a scalar to a partner fermion:
it links the scalar mass to the fermion mass!
(In unbroken SUSY they are degenerate)
So the scalar mass is also protected by chiral symmetry – the Λ^2 divergences all cancel and only $\log(\Lambda^2/m^2)$ divergences are left.

The Minimal Supersymmetric Standard Model (MSSM)

The MSSM is defined by adding the minimal amount of new particles for a working supersymmetric theory.

Particle content:

Each fermion gets a boson (scalar) partner:

$e_L,\, e_R \leftrightarrow \tilde{e}_L,\, \tilde{e}_R$ "selectrons"

$t_L,\, t_R \leftrightarrow \tilde{t}_L,\, \tilde{t}_R$ "top squarks" (or "stops")

and similarly for the rest of the quarks and leptons

The number of degrees of freedom match:

chiral fermion has 2 d.o.f $\leftrightarrow$ complex (charged) scalar has 2 d.o.f.

Each gauge boson gets a fermionic partner:

$W^\pm \leftrightarrow \widetilde{W}^\pm$ "winos"

$Z,\, \gamma \leftrightarrow \tilde{Z},\, \tilde{\gamma}$ "zino", "photino"

(or $W^0,\, B \leftrightarrow \widetilde{W}^0,\, \tilde{B}$ "neutral wino", "bino")

Again the number of degrees of freedom match:

Transverse gauge boson has 2 d.o.f. (polarizations) $\leftrightarrow$ chiral fermion (not a normal Dirac fermion!) [explanation coming next slide...]

MSSM particle content, continued:

In the MSSM we are forced to expand to two Higgs doublets

Structure of MSSM couplings require a second Higgs to give masses to both up and down type fermions

Since Higgses now have fermionic partners, anomaly cancellation requires two Higgs doublets with opposite hypercharges

Instead of 1 Higgs boson, get 5 d.o.f.: $h^0, H^0, A^0, H^\pm$

Each Higgs boson gets a fermionic partner:

$$H_u = (H_u^+, H_u^0) \leftrightarrow (\widetilde{H}_u^+, \widetilde{H}_u^0)$$

$$H_d = (H_d^0, H_d^-) \leftrightarrow (\widetilde{H}_d^0, \widetilde{H}_d^-)$$ "Higgsinos"

Again the number of degrees of freedom match:

Complex scalar has 2 d.o.f. $\leftrightarrow$ chiral fermion.

Dealing with the chiral fermions:

- Have 4 neutral chiral fermions: $\widetilde{B}$, $\widetilde{W}^0$, $\widetilde{H}_u^0$, $\widetilde{H}_d^0$. These mix and give four Majorana neutralinos $\widetilde{N}_i$ or $\tilde{\chi}_i^0$.
- Have 4 charged chiral fermions: $\widetilde{W}^\pm$, $\widetilde{H}_u^+$, $\widetilde{H}_d^-$. These pair up (and mix) and give two Dirac charginos $\widetilde{C}_i$ or $\tilde{\chi}_i^\pm$.

Summary: the particle content of the MSSM

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$ $\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$ $\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	" " " " $\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$ $\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$ $\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	" " " " $\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \ \widetilde{W}^0\ \ \widetilde{H}_u^0\ \ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \ \widetilde{H}_u^+\ \ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	" "
gravitino/ goldstino	3/2	−1	$\widetilde{G}$	" "

... plus the usual SM quarks, leptons, and gauge bosons.

If Supersymmetry were an exact symmetry, the SUSY particles would be degenerate with their SM partners.

Clearly they are not $\longrightarrow$ SUSY must be broken.

Most general set of SUSY-breaking terms $\rightarrow > 100$ new parameters [specific SUSY-breaking-mediation models $\rightarrow \mathcal{O}(5-10)$ new params]

Let's do some phenomenology!

The Higgs sector is an easy place to start.

MSSM has 2 Higgs doublets: $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$.
Study the most general CP-conserving two-Higgs-doublet model.

Two complex doublets = 8 d.o.f.
$\rightarrow$ 3 longitudinal gauge bosons $(G^0, G^\pm)$ + 5 physical states:

- one CP-odd neutral scalar A^0
- two CP-even neutral scalars h^0, H^0
- a charged Higgs pair $H^\pm$ (2 d.o.f.)

The two Higgs doublets have vacuum expectation values (vevs)

$$\langle H_u^0 \rangle = v_u/\sqrt{2}, \qquad \langle H_d^0 \rangle = v_d/\sqrt{2}.$$

Sum of squares fixed by W mass:

$$v_u^2 + v_d^2 = v_{SM}^2 = 4m_W^2/g^2 \simeq (246 \text{ GeV})^2.$$

Ratio is a free parameter (a key parameter in SUSY!):

$$v_u/v_d \equiv \tan\beta.$$

Physical states defined in terms of H_u, H_d by mixing angles α, β:

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} \sqrt{2}\mathrm{Im}H_u^0 \\ \sqrt{2}\mathrm{Im}H_d^0 \end{pmatrix}$$

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix}$$

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \sqrt{2}\mathrm{Re}H_u^0 - v_u \\ \sqrt{2}\mathrm{Re}H_d^0 - v_d \end{pmatrix}$$

The Higgs masses, the mixing angle α, and the 3-Higgs and 4-Higgs couplings are determined by the scalar potential.
The MSSM contains a constrained two Higgs doublet model:
The Higgs potential is of a special form, determined by SUSY.

The MSSM Higgs potential, at tree level:

$$\begin{aligned} V &= \left(|\mu|^2 + m_{H_u}^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) \\ &+ \left[b\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \text{h.c.}\right] \\ &+ \frac{1}{8}\left(g^2 + g'^2\right)\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 \\ &+ \frac{1}{2}g^2 \left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 \end{aligned}$$

Dimensionful terms: $(|\mu|^2 + m_{H_{u,d}}^2)$, b set the mass-squared scale.

μ terms come from F-terms: SUSY-preserving

$m_{H_{u,d}}^2$ and b terms come directly from soft SUSY breaking

Dimensionless terms: fixed by the gauge couplings g and g'

D-term contributions: SUSY-preserving

The scalar potential fixes the tree-level Higgs masses:
(all these get modified by radiative corrections)

$$\begin{aligned} m^2_{A^0} &= 2b/\sin 2\beta \\ m^2_{H^\pm} &= m^2_{A^0} + m^2_W \\ m^2_{h^0,H^0} &= \frac{1}{2}\left(m^2_{A^0} + m^2_Z \mp \sqrt{(m^2_{A^0} + m^2_Z)^2 - 4m^2_Z m^2_{A^0} \cos^2 2\beta}\right) \end{aligned}$$

By convention, h^0 is lighter than H^0.

The mixing angle α between Re(H_u^0, H_d^0) and (h^0, H^0) is given by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m^2_{A^0} + m^2_Z}{m^2_{H^0} - m^2_{h^0}} \qquad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m^2_{A^0} - m^2_Z}{m^2_{H^0} - m^2_{h^0}}$$

The m_W^2 and m_Z^2 factors come from g^2v^2 and $(g^2 + g'^2)v^2$ terms – they are due to the g^2 and g'^2 in the scalar potential.
SUSY relates gauge couplings to couplings in the scalar potential!

The scalar potential fixes the tree-level Higgs masses:
(all these get modified by radiative corrections)

$$\begin{aligned} m_{A^0}^2 &= 2b/\sin 2\beta \\ m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2 \\ m_{h^0,H^0}^2 &= \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4 m_Z^2 m_{A^0}^2 \cos^2 2\beta} \right) \end{aligned}$$

By convention, h^0 is lighter than H^0.

- A^0, H^0 and $H^\pm$ masses can be arbitrarily large: they grow with $b/\sin 2\beta$.
- h^0 mass is bounded from above: $m_{h^0} < |\cos 2\beta| m_Z \leq m_Z$ (!!)

This is already ruled out by LEP!

The MSSM would be dead if not for the large radiative corrections to m_{h^0}.

The largest correction comes from top and stop loops:

$$\Delta(m_{h^0}^2) \simeq \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \; \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

Revised bound (full 1-loop + dominant 2-loop): $m_{h^0} \lesssim 135$ GeV.

The scalar potential fixes the tree-level Higgs masses:
(all these get modified by radiative corrections)

$$
\begin{aligned}
m^2_{A^0} &= 2b/\sin 2\beta \\
m^2_{H^\pm} &= m^2_{A^0} + m^2_W \\
m^2_{h^0,H^0} &= \frac{1}{2}\left(m^2_{A^0} + m^2_Z \mp \sqrt{(m^2_{A^0} + m^2_Z)^2 - 4m^2_Z m^2_{A^0}\cos^2 2\beta}\right)
\end{aligned}
$$

By convention, h^0 is lighter than H^0.

How many free parameters are there? From the Higgs potential:

- b term
- $(|\mu|^2 + m^2_{H_u})$
- $(|\mu|^2 + m^2_{H_d})$

One combination determines $v_u^2 + v_d^2 = v_{\rm SM}^2$ – already known from the W mass. That leaves two free parameter combinations:

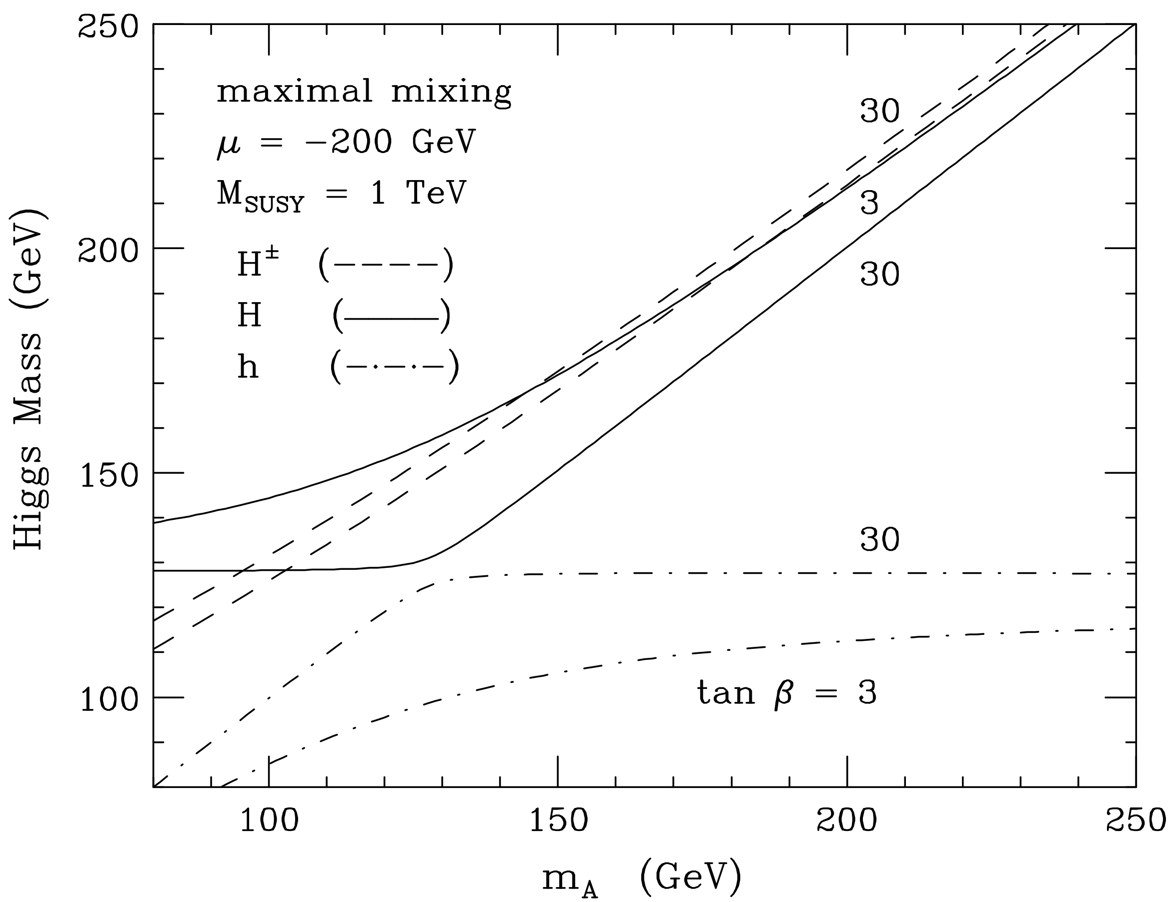
usually chosen to be m_{A^0} and $\tan\beta$.

This only works at tree level. Once radiative corrections are included, other SUSY parameters enter into the Higgs sector. E.g., h^0 mass correction:

$$\Delta(m^2_{h^0}) \simeq \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \;\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

Higgs masses as a function of m_A

for $\tan\beta$ small (3) and large (30)



from Carena & Haber, hep-ph/0208209

For large m_A:

m_h asymptotes

m_{H^0} and m_{H^+} become increasingly degenerate with m_A

Higgs couplings to SM fermions

The Yukawa-coupling Lagrangian is:

$$\begin{aligned}\mathcal{L} &= -y_t \bar{u}_3 Q H_u + y_b \bar{d}_3 Q H_d + y_\tau \bar{e}_3 L H_d \\ &= -y_t \left(\bar{t} t H_u^0 - \bar{t} b H_u^+ \right) + y_b \left(\bar{b} t H_d^- - \bar{b} b H_d^0 \right) + y_\tau \left(\bar{\tau} \nu_\tau H_d^- - \bar{\tau} \tau H_d^0 \right)\end{aligned}$$

Using $v_u = \sqrt{2}\langle H_u^0 \rangle = v_{\rm SM} \sin\beta$ and $v_d = \sqrt{2}\langle H_d^0 \rangle = v_{\rm SM} \cos\beta$ and $m_W = g v_{\rm SM}/2$, we can solve for the Yukawa couplings in terms of the fermion masses:

$$y_t = \frac{g m_t}{\sqrt{2} m_W \sin\beta} \qquad y_b = \frac{g m_b}{\sqrt{2} m_W \cos\beta} \qquad y_\tau = \frac{g m_\tau}{\sqrt{2} m_W \cos\beta}$$

If $\tan\beta \gg 1$ then y_b and y_τ get enhanced.

Couplings of the Higgs mass eigenstates:

$$g_{h^0 \bar{t} t} = \frac{g m_t}{2 m_W} \frac{\cos\alpha}{\sin\beta} \qquad g_{H^0 \bar{t} t} = \frac{g m_t}{2 m_W} \frac{\sin\alpha}{\sin\beta} \qquad g_{A^0 \bar{t} t} = \frac{i g m_t}{2 m_W} \cot\beta \gamma^5$$

$$g_{h^0 \bar{b} b} = -\frac{g m_b}{2 m_W} \frac{\sin\alpha}{\cos\beta} \qquad g_{H^0 \bar{b} b} = \frac{g m_b}{2 m_W} \frac{\cos\alpha}{\cos\beta} \qquad g_{A^0 \bar{b} b} = \frac{i g m_b}{2 m_W} \tan\beta \gamma^5$$

$$g_{H^+ \bar{t}_R b_L} = \frac{g m_t}{\sqrt{2} m_W} \cot\beta \qquad g_{H^+ \bar{t}_L b_R} = \frac{g m_b}{\sqrt{2} m_W} \tan\beta$$

An interesting limit occurs for $m_{A^0} \gg m_Z$: the decoupling limit.
In the decoupling limit:

- $m_{A^0} \simeq m_{H^0} \simeq m_{H^\pm} \gg m_Z$
- m_{h^0} saturates its upper bound; $m_{h^0} \simeq m_Z |\cos 2\beta|$ at tree level
- $\alpha \simeq \beta - \pi/2$:
- A^0, H^0, $H^\pm$ live together in one linear combination of H_u, H_d
- h^0 lives in the other linear combination of H_u, H_d, together with the Goldstone bosons G^0, $G^\pm$ and the vev $v_{\rm SM} = \sqrt{v_u^2 + v_d^2}$
- The couplings of h^0 become the same as the couplings of the Standard Model Higgs

Then the Higgs sector looks like:

- One light SM-like Higgs h^0
- A multiplet A^0, H^0, $H^\pm$ of heavy Higgses that don't affect the low-energy physics very much.

Need high-precision measurements of the h^0 couplings to distinguish it from the SM!

The details:

$\cos(\beta - \alpha)$ goes to zero in the limit $m_{A^0} \gg m_Z$:

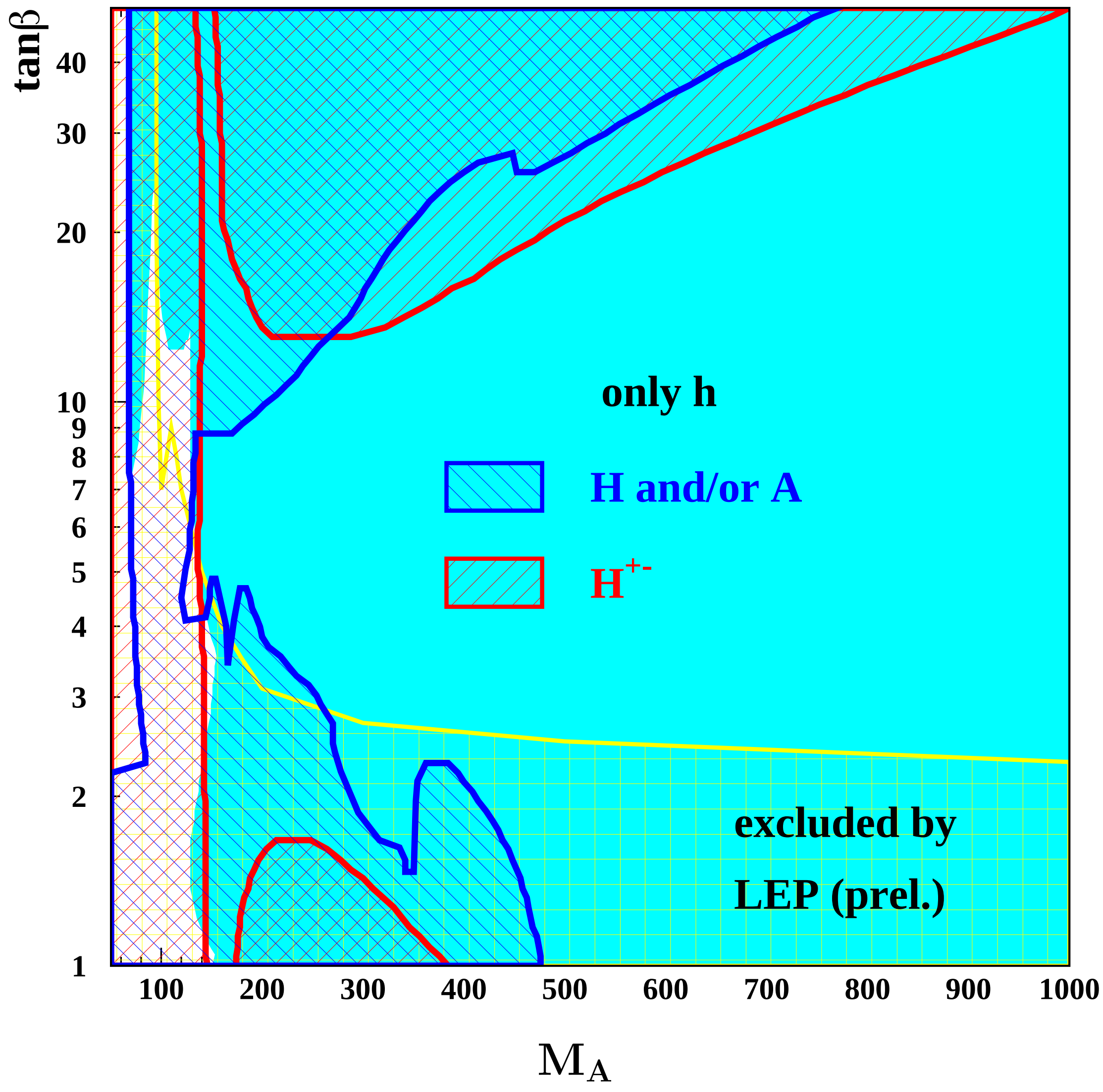
$$\cos(\beta - \alpha) \simeq \frac{1}{2} \sin 4\beta \frac{m_Z^2}{m_{A^0}^2}$$

The h^0 couplings can be rewritten in a useful form in terms of $\cos(\beta - \alpha)$:

$$\begin{aligned} g_{h^0\bar{t}t} &= \frac{g m_t}{2 m_W} [\sin(\beta - \alpha) + \cot\beta \cos(\beta - \alpha)] \\ g_{h^0\bar{b}b} &= \frac{g m_b}{2 m_W} [\sin(\beta - \alpha) - \tan\beta \cos(\beta - \alpha)] \\ g_{h^0 W^+ W^-} &= g m_W \sin(\beta - \alpha) \\ g_{h^0 ZZ} &= \frac{g m_Z}{\cos\theta_W} \sin(\beta - \alpha) \end{aligned}$$

These all approach their SM values in the limit $\cos(\beta - \alpha) \to 0$.

Search for all the MSSM Higgs bosons at LHC



ATLAS, 300 fb^{-1}, $m_h^{\rm max}$ scenario. From Haller, hep-ex/0512042

The Supersymmetric partners

I'll take a phenomenologist's approach: study the spectrum, then go back and discuss SUSY-breaking models.

Recall the particle content:

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	$+1$	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	-1	$\tilde{u}_L\ \tilde{u}_R\ \tilde{d}_L\ \tilde{d}_R$ $\tilde{s}_L\ \tilde{s}_R\ \tilde{c}_L\ \tilde{c}_R$ $\tilde{t}_L\ \tilde{t}_R\ \tilde{b}_L\ \tilde{b}_R$	" " " " $\tilde{t}_1\ \tilde{t}_2\ \tilde{b}_1\ \tilde{b}_2$
sleptons	0	-1	$\tilde{e}_L\ \tilde{e}_R\ \tilde{\nu}_e$ $\tilde{\mu}_L\ \tilde{\mu}_R\ \tilde{\nu}_\mu$ $\tilde{\tau}_L\ \tilde{\tau}_R\ \tilde{\nu}_\tau$	" " " " $\tilde{\tau}_1\ \tilde{\tau}_2\ \tilde{\nu}_\tau$
neutralinos	1/2	-1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	-1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$
gluino	1/2	-1	$\tilde{g}$	" "
gravitino/ goldstino	3/2	-1	$\widetilde{G}$	" "

Squarks and sleptons

In general, all sfermions with the same charge/colour can mix:

$(\widetilde{u}_L, \widetilde{c}_L, \widetilde{t}_L, \widetilde{u}_R, \widetilde{c}_R, \widetilde{t}_R) \rightarrow$ 6×6 mass-squared matrix
$(\widetilde{d}_L, \widetilde{s}_L, \widetilde{b}_L, \widetilde{d}_R, \widetilde{s}_R, \widetilde{b}_R) \rightarrow$ 6×6 mass-squared matrix
$(\widetilde{e}_L, \widetilde{\mu}_L, \widetilde{\tau}_L, \widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R) \rightarrow$ 6×6 mass-squared matrix
$(\widetilde{\nu}_e, \widetilde{\nu}_\mu, \widetilde{\nu}_\tau) \rightarrow$ 3×3 mass-squared matrix

This would be a disaster for flavour-changing neutral currents! (E.g., $K^0 - \overline{K}^0$ mixing.)

Let's ignore inter-generational mixing for now. [more on this later]

What about $\widetilde{f}_L$–$\widetilde{f}_R$ mixing?

This is chirality violation – can only arise via the fermion mass.

$\widetilde{f}_L$–$\widetilde{f}_R$ mixing matrix:

$$\widehat{M}^2_{\widetilde{f}} \equiv \begin{pmatrix} M^2_{L_f} & m_f X_f \\ m_f X_f & M^2_{R_f} \end{pmatrix}$$

$$\begin{aligned} M^2_{L_f} &= M^2_{\widetilde{Q},\widetilde{L}} + m_f^2 + \cos 2\beta m_Z^2 (T_3^f - Q_f s_W^2) \\ M^2_{R_f} &= M^2_{\widetilde{U},\widetilde{D},\widetilde{E}} + m_f^2 + \cos 2\beta m_Z^2 Q_q s_W^2 \\ X_t &= A_t - \mu \cot\beta \qquad\qquad X_{b,\tau} = A_{b,\tau} - \mu \tan\beta. \end{aligned}$$

Some features of the sfermion mass matrix:

$$\widehat{M}^2_{\widetilde{f}} \equiv \begin{pmatrix} M^2_{L_f} & m_f X_f \\ m_f X_f & M^2_{R_f} \end{pmatrix}$$

• m_f in off-diagonal terms: mixing is only significant for the 3rd generation.

$$\begin{aligned} M^2_{L_f} &= M^2_{\widetilde{Q},\widetilde{L}} + m_f^2 + \cos 2\beta m_Z^2 (T_3^f - Q_f s_W^2) \\ M^2_{R_f} &= M^2_{\widetilde{U},\widetilde{D},\widetilde{E}} + m_f^2 + \cos 2\beta m_Z^2 Q_q s_W^2 \\ X_t &= A_t - \mu \cot\beta \qquad\qquad X_{b,\tau} = A_{b,\tau} - \mu \tan\beta. \end{aligned}$$

• $\tan\beta$ in $X_{b,\tau}$: down-type sfermion mixing becomes important at large $\tan\beta$.

Diagonalize mass-squared matrix: get mass eigenstates ($m^2_{\widetilde{f}_1} < m^2_{\widetilde{f}_2}$ by convention)

$$\begin{pmatrix} \widetilde{f}_1 \\ \widetilde{f}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\widetilde{f}} & \sin\theta_{\widetilde{f}} \\ -\sin\theta_{\widetilde{f}} & \cos\theta_{\widetilde{f}} \end{pmatrix} \begin{pmatrix} \widetilde{f}_L \\ \widetilde{f}_R \end{pmatrix}$$

Neutralinos

Higgsinos and electroweak gauginos mix with each other because of the effects of electroweak symmetry breaking.

Neutral gauginos $\widetilde{B}$, $\widetilde{W}^0$ and neutral Higgsinos $\widetilde{H}_u^0$, $\widetilde{H}_d^0$ mix: form four neutral mass eigenstates called neutralinos, $\widetilde{N}_{1,2,3,4}$ or $\tilde{\chi}^0_{1,2,3,4}$.

- Convention: $m_{\widetilde{N}_1} < m_{\widetilde{N}_2} < m_{\widetilde{N}_3} < m_{\widetilde{N}_4}$
- The lightest neutralino $\widetilde{N}_1$ is usually (assumed to be) the LSP.

Neutralino mass matrix:

$$\widehat{M}_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix}$$

Abbreviations: $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, $s_W = \sin\theta_W$, $c_W = \cos\theta_W$

Neutralino sector has 4 free parameters: M_1, M_2, μ, and $\tan\beta$.

Gaugino-Higgsino mixing is controlled by electroweak symmetry breaking [the m_Z terms in the off-diagonal blocks]

If $m_Z \ll |\mu \pm M_1|, |\mu \pm M_2|$ then the gaugino-Higgsino mixing is small. Then (assuming also $M_1 < M_2 < |\mu|$):

$$\begin{aligned} \widetilde{N}_1 &\approx \tilde{B}, & m_{\widetilde{N}_1} &\approx M_1 + \cdots \\ \widetilde{N}_2 &\approx \widetilde{W}^0, & m_{\widetilde{N}_2} &\approx M_2 + \cdots \\ \widetilde{N}_3, \widetilde{N}_4 &\approx \frac{1}{\sqrt{2}} \left(\widetilde{H}_u^0 \pm \widetilde{H}_d^0 \right), & m_{\widetilde{N}_3}, m_{\widetilde{N}_4} &\approx |\mu| + \cdots \end{aligned}$$

Charginos

Winos $\widetilde{W}^+$, $\widetilde{W}^-$ and charged Higgsinos $\widetilde{H}_u^+$, $\widetilde{H}_d^-$ mix:
form two charged mass eigenstates called charginos, $\tilde{C}_{1,2}$ or $\tilde{\chi}^\pm_{1,2}$.
- Convention: $m_{\widetilde{C}_1} < m_{\widetilde{C}_2}$

Chargino mass terms:

$$\mathcal{L} = -\frac{1}{2}\left(\psi^\pm\right)^T \widehat{M}_{\widetilde{C}}\psi^\pm + \text{h.c.}$$

where $\psi^\pm = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$.
The mass matrix is (in 2×2 block form):

$$\widehat{M}_{\widetilde{C}} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix}, \qquad \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} s_\beta m_W \\ \sqrt{2} c_\beta m_W & \mu \end{pmatrix}$$

The chargino mass matrix is diagonalized by *two* unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$, one acting on the + charged states and one acting on the − charged states:

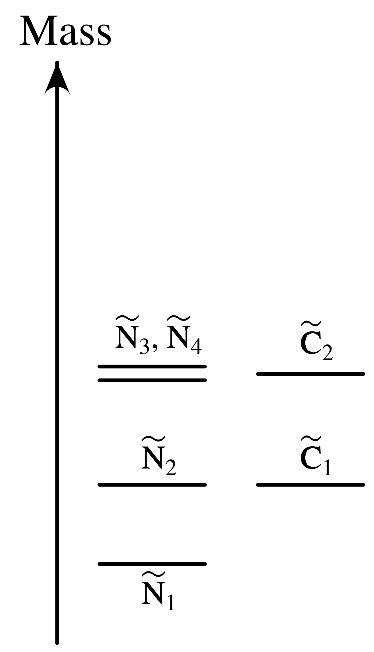
$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix} \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}$$

Wino-Higgsino mixing is controlled by electroweak symmetry breaking [the m_W terms in the off-diagonal entries]

If $m_W \ll |\mu \pm M_2|$ then the wino-Higgsino mixing is small. Then (assuming also $M_2 < |\mu|$):

$$\tilde{C}_1^{\pm} \approx \widetilde{W}^{\pm}, \qquad m_{\tilde{C}_1} \approx M_2$$

$$\tilde{C}_2^{+} \approx \widetilde{H}_u^{+}, \quad \tilde{C}_2^{-} \approx \widetilde{H}_d^{-}, \qquad m_{\tilde{C}_2} \approx |\mu|$$



Note that in this case:
- the neutral wino $\widetilde{N}_2$ is roughly degenerate with the charged wino $\tilde{C}_1$
- the neutral Higgsinos $\widetilde{N}_3$, $\widetilde{N}_4$ are roughly degenerate with the charged Higgsino $\tilde{C}_2$
- the bino $\widetilde{N}_1$ is by itself

Gluino

There is only one gluino (the superpartner of the gluon) and it doesn't mix with anything.

It is a colour-octet Majorana fermion.

Its mass is $M_{\widetilde{g}} = M_3$. [up to radiative corrections]

Summary: the particle content of the MSSM

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$ $\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$ $\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	" " " " $\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$ $\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$ $\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	" " " " $\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \ \widetilde{W}^0\ \ \widetilde{H}_u^0\ \ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \ \widetilde{H}_u^+\ \ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	" "
gravitino/ goldstino	3/2	−1	$\widetilde{G}$	" "

... plus the usual SM quarks, leptons, and gauge bosons.

Phenomenological problems of the MSSM

There are several features that happen "by accident" in the Standard Model that must be engineered in the MSSM.

Proton (non-)decay

SM: baryon and lepton number are conserved (perturbatively)

MSSM: can write down renormalizable gauge-invariant interactions that violate baryon and lepton number.

Small flavour-changing neutral currents

SM: GIM mechanism

MSSM: can write down renormalizable gauge-invariant flavour-violating couplings for squarks and sleptons that give large flavour-changing effects.

CP violation appears to come only from phase of the CKM matrix

SM: CKM matrix is the only possible source of CP violation (aside from $\theta_{\rm QCD}\ldots$)

MSSM: can have potentially large new sources of CP violation from complex couplings of the SUSY partners

Solutions to these problems drive the SUSY-breaking models.

Proton (non-)decay

Baryon- and lepton-number violating terms can be forbidden by R-parity, under which SUSY partners are odd.

SUSY partners can be produced only in pairs; lightest SUSY particle is stable $\rightarrow$ missing energy signatures.

Small flavour-changing neutral currents

CP violation

SUSY-breaking models try to keep SUSY breaking "flavour-blind", so that the only flavour dependence comes from the CKM matrix.

Minimal Supergravity (mSUGRA); Gauge-mediated SUSY breaking (GMSB); etc. Tend to have squarks/sleptons degenerate at the high scale $\rightarrow$ patterns in low-energy mass spectrum.

Next lecture:

SUSY phenomenology