TRIUMF Summer Institute 2006
Questions for "Beyond the Standard Model" lectures 1, 2 and 3
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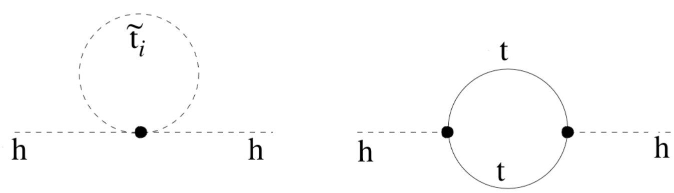
(a) is a quick "comprehension question", designed to check that the material is understood.
(b) is a "calculational question" meant to be marked.

1. (Lecture 1, Monday July 17)

   (a) Convince yourself that the numbers of degrees of freedom match between the SM particles and their SUSY partners.

   (b) Show that the quadratically divergent part of the h^0 mass correction from the top quark loop is cancelled by the contribution from the top squark loops.



The Feynman rules for the vertices are:

$$h^0 t\bar{t} : \qquad i g_{htt}$$
$$h^0 h^0 \tilde{t}_i \tilde{t}_i : \qquad i\lambda_{hh\tilde{t}_i\tilde{t}_i},$$

where the relevant couplings are:

$$\begin{aligned} g_{htt} &= -\frac{g m_t \cos\alpha}{2 m_W \sin\beta} \\ \lambda_{hh\tilde{t}_L\tilde{t}_L} &= \frac{g^2}{2}\left[-\frac{m_t^2}{m_W^2}\frac{\cos^2\alpha}{\sin^2\beta} + \cdots\right] \\ \lambda_{hh\tilde{t}_R\tilde{t}_R} &= \frac{g^2}{2}\left[-\frac{m_t^2}{m_W^2}\frac{\cos^2\alpha}{\sin^2\beta} + \cdots\right] \\ \lambda_{hh\tilde{t}_L\tilde{t}_R} &= 0. \end{aligned}$$

[The ellipsis in $\lambda_{hh\tilde{t}_L\tilde{t}_L}$ and $\lambda_{hh\tilde{t}_R\tilde{t}_R}$ represent electroweak terms (not proportional to m_t) that are cancelled among other sets of loops; you may neglect them.] You will need to rewrite the four-scalar couplings in terms of the top squark mass eigenstates, $\tilde{t}_1 = \cos\theta_t \tilde{t}_L + \sin\theta_t \tilde{t}_R$, $\tilde{t}_2 = -\sin\theta_t \tilde{t}_L + \cos\theta_t \tilde{t}_R$. To show that the cancellation happens, you don't need to actually compute the loop integrals; it's enough to neglect all masses and the external momentum and write the loop integrals in the (divergent) form $\int d^4p/p^2$, where p is the momentum running around the loop.

2. (Lecture 2, Monday July 17)

(a) The renormalization group equation for the soft-SUSY-breaking top squark mass-squared parameter is

$$16\pi^2 \frac{d}{dt} m_{Q_3}^2 \quad = \quad X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2,$$

where $t = \ln(Q/Q_0)$. Convince yourself that the positive terms $(X_t + X_b)$ in this equation cause $m_{Q_3}^2$ to decrease as it's run down to lower energies, and that conversely the negative terms cause $m_{Q_3}^2$ to increase as it's run down.

(b) Consider slepton pair production at the ILC:

$$e^+ e^- \to \widetilde{\ell}_R \widetilde{\ell}_R \to \ell^+ \widetilde{N}_1 \ell^- \widetilde{N}_1.$$

Use conservation of relativistic momentum and energy to derive the formula for the energy of one of the leptons in the centre-of-mass frame (set m_ℓ to zero):

$$E_\ell^{\mathrm{CM}} = \frac{M_{\widetilde{\ell}_R}^2 - M_{\widetilde{N}_1}^2}{4 M_{\widetilde{\ell}_R}^2} \left(\sqrt{s} + \sqrt{s - 4 M_{\widetilde{\ell}_R}^2} \cos\theta^* \right),$$

where $\cos\theta^*$ is the angle, in the $\widetilde{\ell}$ rest frame, between the direction of the $\widetilde{\ell}$ motion and the emission angle of its daughter lepton. This formula gives the maximum $[\cos\theta^* = 1]$ and minimum $[\cos\theta^* = -1]$ lepton energies (endpoints) in terms of the SUSY particle masses.

3. (Lecture 3, Wednesday July 19)

(a) Count up the number of degrees of freedom of the "SM partners" in the first KK level and compare it to the number of degrees of freedom of the corresponding SUSY partners. [**Hint:** The numbers are different.]

(b) Consider pair production of massive fermions f^+f^- and massive scalars $\phi^+\phi^-$ in e^+e^- annihilation via photon exchange (neglect Z exchange for simplicity and assume that f and ϕ are colourless). Show that the total cross section for fermion pair production is

$$\sigma(e^+e^- \to f^+f^-) = \left(\frac{4\pi\alpha^2}{3s}\right)\frac{\beta(3-\beta^2)}{2} \tag{1}$$

and that the total cross section for scalar pair production is

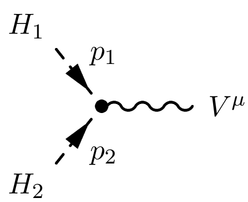
$$\sigma(e^+e^- \to \phi^+\phi^-) = \left(\frac{4\pi\alpha^2}{3s}\right)\frac{\beta^3}{4} \tag{2}$$

where $\beta = |\vec{p}|/E = (1 - 4m^2/s)^{1/2}$ is the speed of either of the final-state particles in the centre-of-mass frame, m is the mass of the final-state fermion or scalar, and s is the square of the centre-of-mass energy. The important thing is to get the β dependence of the cross section near threshold, where β is small; this shows how quickly the cross section "turns on" when you cross the threshold. Notice also that far above threshold, when $\beta \simeq 1$, the cross section for scalar pairs is four times smaller than the cross section for fermion pairs.

The coupling of the scalar to the photon is described by the Lagrangian

$$\mathcal{L} = -ie\left[\phi^\dagger(\partial_\mu\phi) - (\partial_\mu\phi^\dagger)\phi\right]A^\mu,$$

which leads to the Feynman rule



$$= ig_{H_1H_2V}(p_1 - p_2)^\mu$$

with coupling $g_{H_1H_2V} = g_{\phi^+\phi^-\gamma} = -e$ (all particles and momenta incoming).