

# Toward a calculator for the Georgi-Machacek model

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K. Hartling, K. Kumar, H.E.L., 1404.2640 and work in progress

## Outline

Introduction: why triplets?

The Georgi-Machacek model

The decoupling limit

Indirect constraints

Outlook: toward a calculator

## Introduction: why triplets?

Isospin-triplet scalars appear in models for a variety of reasons:

- to generate neutrino mass via  $\bar{L}_L^c \Delta L_L$  operator (type-2 seesaw)
- to enhance the  $h \rightarrow \gamma\gamma$  width ( $H^{++}$  in the loop)
- because they show up in some nice BSM models (3-3-1 models, some little Higgs models, left-right symmetric models)
- because they lead to an interesting benchmark model for properties measurements of the SM-like Higgs boson

## Introduction: why triplets?

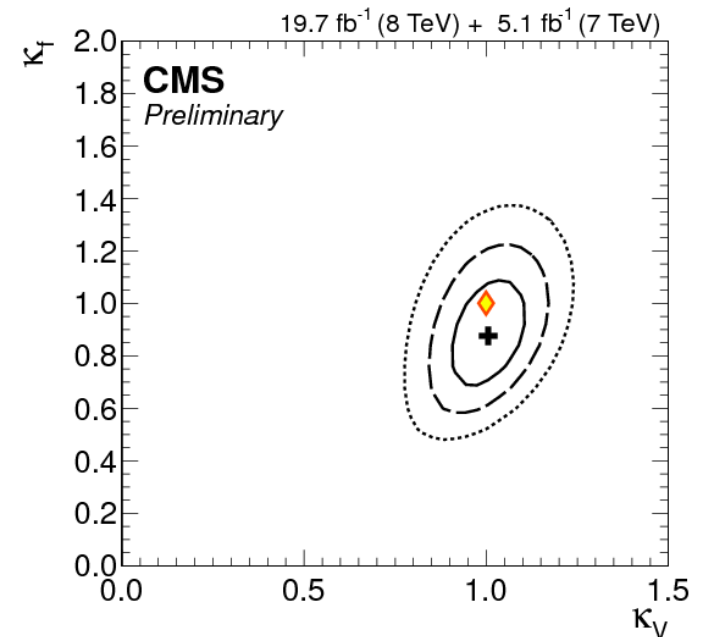
Consider the  $hWW$  coupling:

- SM:  $i\frac{g^2v}{2}g_{\mu\nu}$  ( $v \simeq 246$  GeV)
- 2HDM:  $i\frac{g^2v}{2}g_{\mu\nu} \sin(\beta - \alpha)$
- SM + singlet:  $i\frac{g^2v}{2}g_{\mu\nu} \cos \alpha$  ( $h = \phi \cos \alpha - s \sin \alpha$ )

Extended Higgs sector with isospin doublets or singlets always have  $hVV$  couplings **less than or equal to** those in the SM.

- SM + some multiplet  $X$ :  $i\frac{g^2vX}{2}g_{\mu\nu} \cdot 2 \left[ T(T + 1) - \frac{Y^2}{4} \right]$  ( $Q = T^3 + Y/2$ )

The only way to **enhance** the  $hWW$  coupling above its SM value is through a scalar with **isospin  $\geq 1$**  that has a **non-negligible vev** and **mixes into the observed Higgs  $h$** .  $\Rightarrow$  triplets benchmark



## Introduction: why triplets?

Enhancement of (all) the  $h$  couplings is also interesting because it can hide a non-SM contribution to the Higgs BRs.

LHC measures **rates** in particular final states:

$$\text{Rate} = \frac{\sigma_{\text{SM}} \Gamma_{\text{SM}}}{\Gamma_{\text{SM}}^{\text{tot}}} \rightarrow \frac{\kappa^2 \sigma_{\text{SM}} \cdot \kappa^2 \Gamma_{\text{SM}}}{\kappa^2 \Gamma_{\text{SM}}^{\text{tot}} + \Gamma_{\text{new}}}$$

Rates are identical to SM Higgs predictions if

$$\kappa^2 = \frac{1}{1 - \text{BR}_{\text{new}}}$$

Constraint on  $\Gamma^{\text{tot}}$  (equivalently on  $\kappa$ ) from off-shell  $gg (\rightarrow h^*) \rightarrow ZZ$  assumes no new resonances in  $s$ -channel: rather model-dependent.

To study this further, nice to have a concrete model  $\Rightarrow$  can study effect of heavy  $H^0$  resonance on off-shell  $gg (\rightarrow h^*) \rightarrow ZZ$ .

## The Trouble with Triplets: the $\rho$ parameter

$\rho \equiv$  ratio of strengths of charged and neutral weak currents  $\simeq 1$  to high precision.



$$\rho = \frac{M_W^2}{M_Z^2 \cos \theta_W} = \frac{\sum_k 2[T_k(T_k + 1) - Y_k^2/4]v_k^2}{\sum_k Y_k^2 v_k^2}$$

( $Q = T^3 + Y/2$ , vevs defined as  $\langle \phi_k^0 \rangle = v_k/\sqrt{2}$  for complex reps and  $\langle \phi_k^0 \rangle = v_k$  for real reps)

$\rho = 1$  “by accident” for SM doublet; isospin septet with  $Y = 4$   
(septet: Hisano & Tsumura, 1301.6455; Kanemura, Kikuchi & Yagyu, 1301.7303)

SM + real triplet  $\xi$ :  $\rho > 1$

SM + complex triplet  $\chi$  ( $Y = 2$ ):  $\rho < 1$

**Combine them both:**  $\langle \chi^0 \rangle = v_\chi$ ,  $\langle \xi^0 \rangle = v_\xi$ ; doublet  $\langle \phi^0 \rangle = v_\phi/\sqrt{2}$

$$\rho = \frac{v_\phi^2 + 4v_\xi^2 + 4v_\chi^2}{v_\phi^2 + 8v_\chi^2} = 1 \text{ when } v_\xi = v_\chi$$

## Georgi-Machacek model

Georgi & Machacek, NPB262, 463 (1985)

Chanowitz & Golden, PLB165, 105 (1985)

Enforce  $v_\xi = v_\chi$  using a symmetry.

**Observation:** the SM Higgs potential accidentally preserves a global  $SU(2)_L \times SU(2)_R$  symmetry: bigger than the gauged  $SU(2)_L \times U(1)_Y$  symmetry.

**Consequence:** When  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ , the global symmetry breaks via  $SU(2)_L \times SU(2)_R \rightarrow SU(2)_c$ : “custodial symmetry”. Ensures  $\rho = 1$  at tree level; violated by hypercharge and  $m_t \neq m_b$ .

**Idea:** Construct by hand a scalar potential for the triplets that preserves  $SU(2)_L \times SU(2)_R$

## Georgi-Machacek model

Georgi & Machacek, NPB262, 463 (1985)

Chanowitz & Golden, PLB165, 105 (1985)

Assemble the real + complex triplets into a **bitriplet** (analogous to the SM Higgs bidoublet) under  $SU(2)_L \times SU(2)_R$ :

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \quad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

VEVs: (preserves the diagonal  $SU(2)_c$  subgroup)

$$\langle \Phi \rangle = \frac{v_\phi}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \langle X \rangle = v_\chi \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$W$  and  $Z$  boson masses constrain

$$v_\phi^2 + 8v_\chi^2 \equiv v^2 \simeq (246 \text{ GeV})^2$$

Gauging hypercharge breaks the  $SU(2)_R$ : divergent radiative correction to  $\rho$  at 1-loop (need a relatively low cutoff scale)

Gunion, Vega & Wudka, PRD43, 2322 (1991)



**Physical spectrum:** Custodial symmetry fixes almost everything!

Bidoublet:  $2 \times 2 \rightarrow 3 + 1$

Bitriplet:  $3 \times 3 \rightarrow 5 + 3 + 1$

Custodial 5-plet  $(H_5^{++}, H_5^+, H_5^0, H_5^-, H_5^{--})$ , common mass  $m_5$

$$H_5^{++} = \chi^{++}, H_5^+ = (\chi^+ - \xi^+)/\sqrt{2}, H_5^0 = \sqrt{2/3}\xi^0 - \sqrt{1/3}\chi^{0,r}$$

Custodial triplet  $(H_3^+, H_3^0, H_3^-)$ , common mass  $m_3$

$$H_3^+ = -\sin\theta_H\phi^+ + \cos\theta_H(\chi^+ + \xi^+)/\sqrt{2}, H_3^0 = -\sin\theta_H\phi^{0,i} + \cos\theta_H\chi^{0,i}; \tan\theta_H = 2\sqrt{2}v_\chi/v_\phi$$

(orthogonal triplet is the Goldstones)

Two custodial singlets  $h^0, H^0$ , masses  $m_h, m_H$ , mixing angle  $\alpha$

$$h^0 = \cos\alpha\phi^{0,r} - \sin\alpha(\sqrt{1/3}\xi^0 + \sqrt{2/3}\chi^{0,r})$$

$$H^0 = \sin\alpha\phi^{0,r} + \cos\alpha(\sqrt{1/3}\xi^0 + \sqrt{2/3}\chi^{0,r})$$

Free parameters:  $m_h, m_H, m_3, m_5, v_\chi, \alpha$ . ( $m_h$  or  $m_H = 125$  GeV)

## Most general scalar potential:

Aoki & Kanemura, 0712.4053

Chiang & Yagyu, 1211.2658; Chiang, Kuo & Yagyu, 1307.7526

Hartling, Kumar & HEL, 1404.2640

$$\begin{aligned} V(\Phi, X) = & \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 \\ & + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) + \lambda_3 \text{Tr}(X^\dagger X X^\dagger X) \\ & + \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b) \\ & - M_1 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) (UXU^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t^a X t^b) (UXU^\dagger)_{ab} \end{aligned}$$

9 parameters, 2 fixed by  $M_W$  and  $m_h \rightarrow$  free parameters are  $m_H, m_3, m_5, v_\chi, \alpha$  plus two triple-scalar couplings.

Dimension-3 terms usually omitted by imposing  $Z_2$  sym. on  $X$ .

These dim-3 terms are essential for the model to possess a decoupling limit!

$(UXU^\dagger)_{ab}$  is just the matrix  $X$  in the Cartesian basis of  $SU(2)$ , found using

$$U = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$$

## Theory constraints

**Perturbative unitarity:** impose  $|\text{Re } a_0| < 1/2$  on eigenvalues of coupled-channel matrix of  $2 \rightarrow 2$  scalar scattering processes. Constrain ranges of  $\lambda_{1-5}$ .

Aoki & Kanemura, 0712.4053

**Bounded-from-belowness of the scalar potential:** consider all combinations of fields nonzero. Further constraints on  $\lambda_{1-5}$ .

Hartling, Kumar & HEL, 1404.2640

**Absence of deeper custodial SU(2)-breaking minima:** numerical check that desired minimum is the deepest (1-dim scan over finite parameter range). Constraints involve all 9 parameters.

Hartling, Kumar & HEL, 1404.2640

(we do not consider situations in which the desired vacuum is metastable)

## Decoupling limit

Fix  $\mu_2^2$  using  $W$  mass:

→ Scalar potential has 3 dimensionful parameters:  $\mu_3^2$ ,  $M_1$ ,  $M_2$ .

Decoupling limit is  $\mu_3^2 \gg v^2$ .

Perturbativity and absence of bad minima constrain  $|M_1|/\sqrt{\mu_3^2} \lesssim 3.3$  and  $|M_2|/\sqrt{\mu_3^2} \lesssim 1.2$ .

$m_H \simeq m_3 \simeq m_5 \simeq \sqrt{\mu_3^2}$  up to relative  $\mathcal{O}(v^2/\mu_3^2)$  corrections.

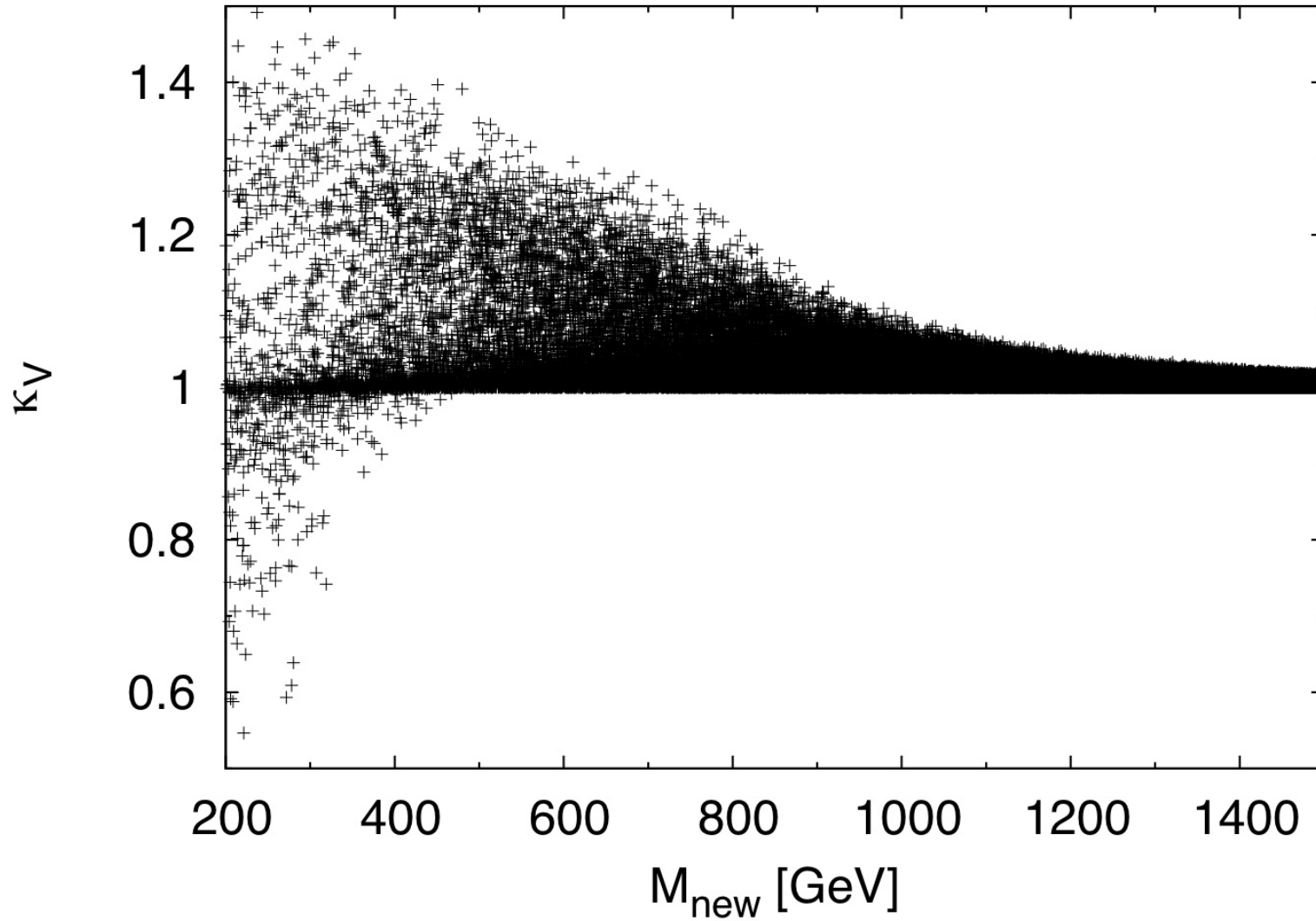
$\sin \theta_H \equiv \frac{2\sqrt{2}v\chi}{v} \simeq \frac{M_1 v}{\sqrt{2}\mu_3^2} \Rightarrow$  Triplet contribution to  $M_W, M_Z$  goes away as  $\mu_3 \rightarrow$  large.

$\sin \alpha \simeq -\frac{\sqrt{3}M_1 v}{2\mu_3^2} \Rightarrow$  Triplet admixture in  $h^0$  goes away as  $\mu_3 \rightarrow$  large.

$hVV$  coupling:  $\kappa_V = \cos \alpha \frac{v_\phi}{v} - \frac{8}{\sqrt{3}} \sin \alpha \frac{v_\chi}{v} \simeq 1 + \frac{3}{8} \frac{M_1^2 v^2}{\mu_3^4} \geq 1!$

$hff$  coupling:  $\kappa_f = \cos \alpha \frac{v}{v_\phi} \simeq 1 - \frac{1}{8} \frac{M_1^2 v^2}{\mu_3^4}$  deviation related to  $\kappa_V!$

Numerical results:  $hVV$  coupling enhancement can be quite large!



$M_{\text{new}} \equiv$  mass of *lightest* new state.

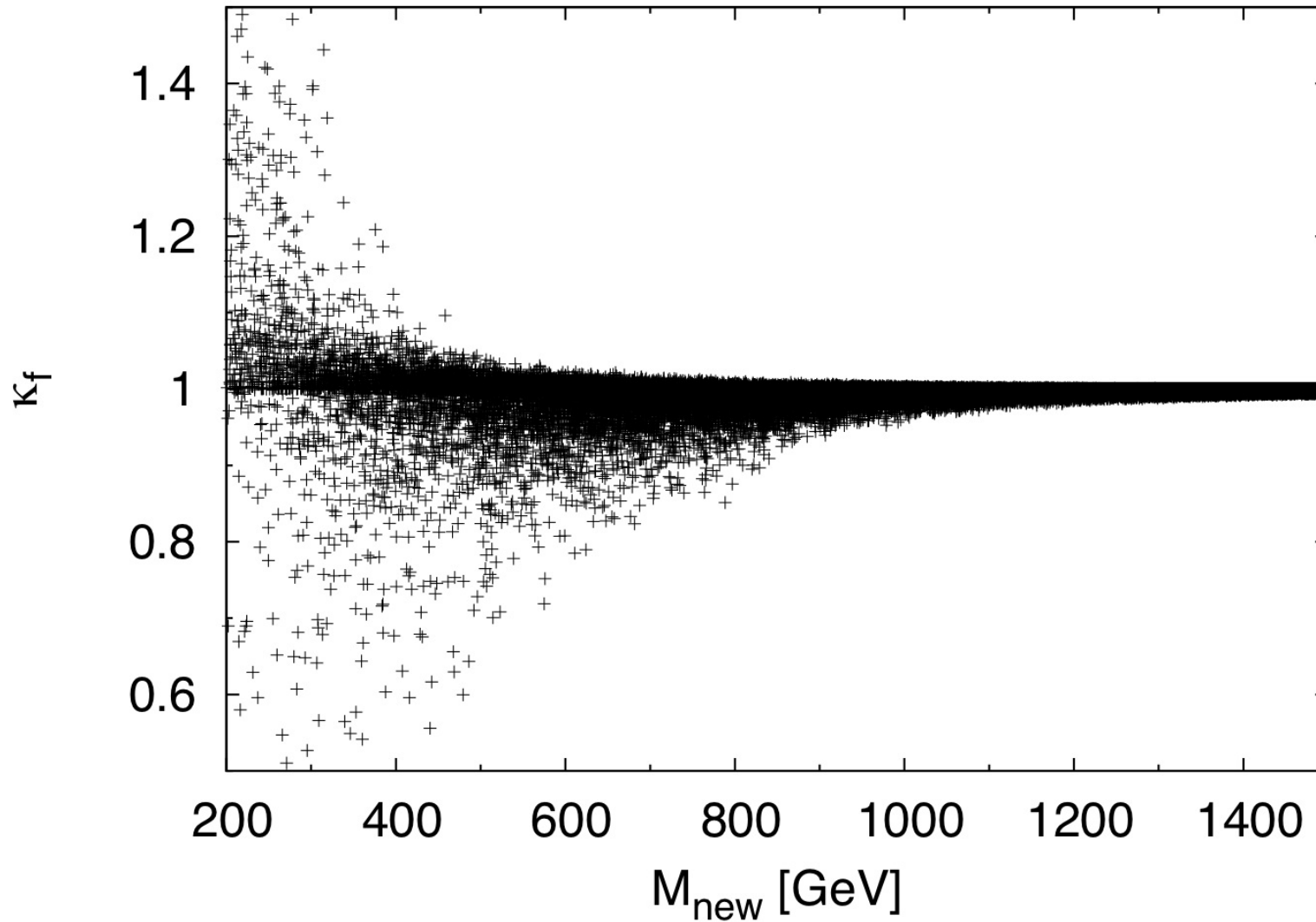
[Hartling, Kumar & HEL, 1404.2640](#)

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Georgi-Machacek model

Multi-Higgs, Lisbon, 2014

Numerical results:  $hff$  coupling typically  $< 1$ ;  $\kappa_f > 1$  possible at low  $M_{\text{new}}$



$M_{\text{new}} \equiv$  mass of *lightest* new state.

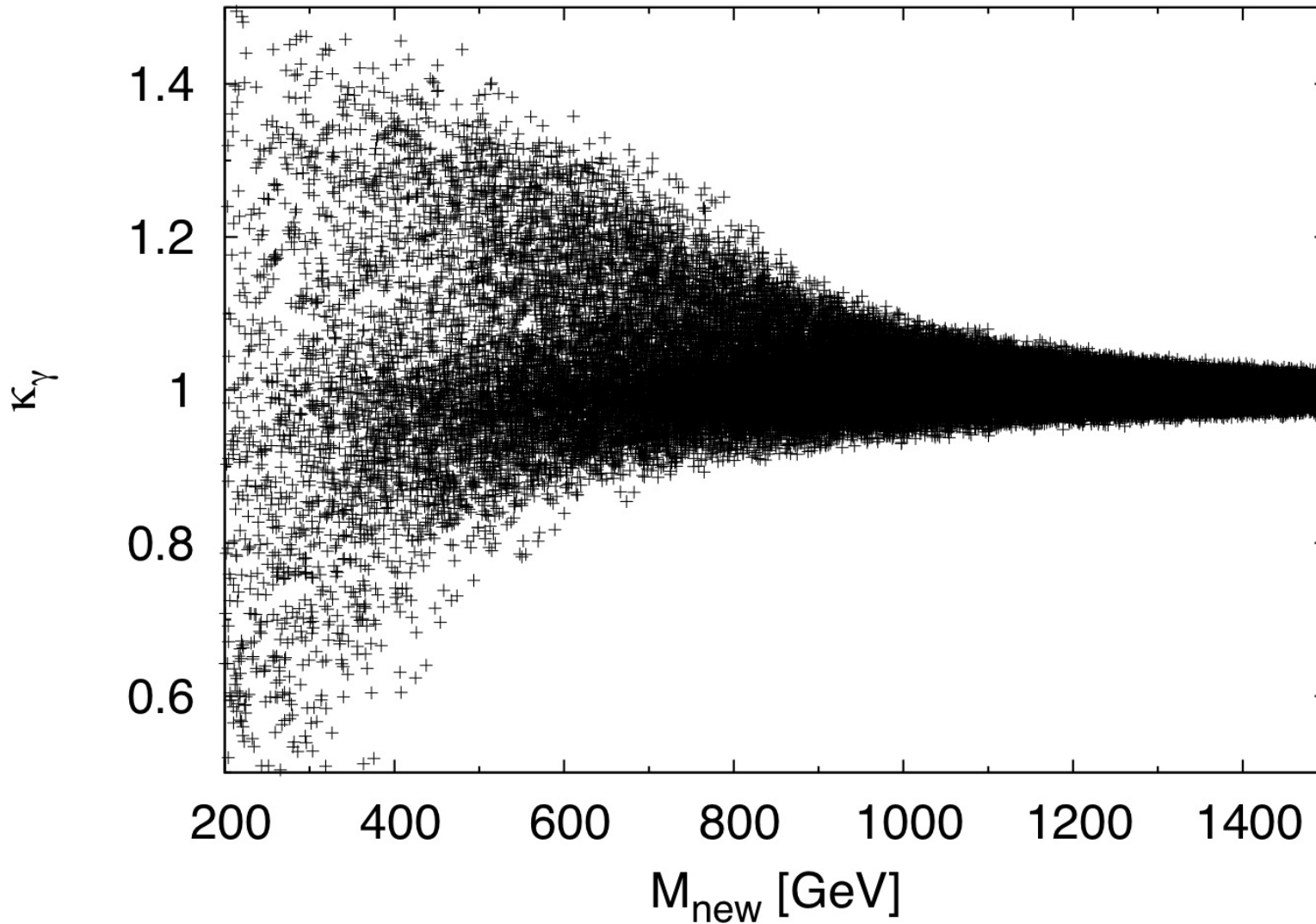
Hartling, Kumar & HEL, 1404.2640

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Numerical results:  $h\gamma\gamma$  coupling contributions from charged scalars in loop



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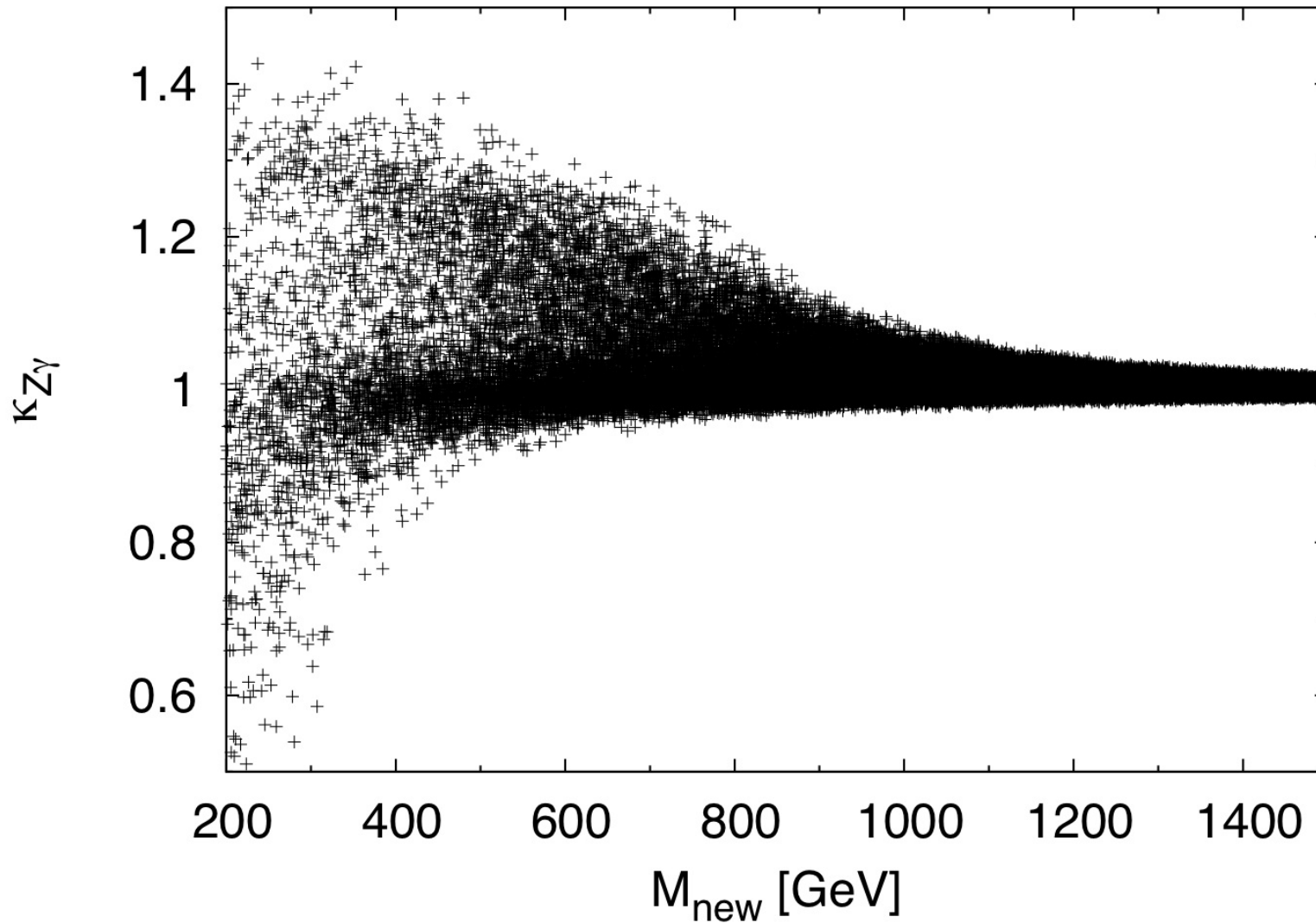
Hartling, Kumar & HEL, 1404.2640

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Numerical results:  $hZ\gamma$  coupling contributions from charged scalars in loop



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Hartling, Kumar & HEL, 1404.2640

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## Decoupling limit: the bottom line

All  $h^0$  coupling deviations decouple like  $M_1^2 v^2 / \mu_3^4$ .

- Slow decoupling  $\sim v^2 / M_{\text{new}}^2$  if  $M_1 \sim \sqrt{\mu_3^2}$ .
- Fast decoupling  $\sim v^4 / M_{\text{new}}^4$  if  $M_1 \sim v$ .

Compare 2HDM:

$\kappa_V \sim 1 + \mathcal{O}(v^4 / M_{\text{new}}^4)$ ,  $\kappa_f \sim 1 + \mathcal{O}(v^2 / M_{\text{new}}^2)$  at tree level. [Gunion & Haber, hep-ph/0207010](#)

1-loop 2HDM corrections  $\rightarrow \delta\kappa_V \sim (\text{loop factor}) \cdot \mathcal{O}(v^2 / M_{\text{new}}^2)$ . [M. Kikuchi, talk on Tuesday](#)

- Relationship in decoupling limit:  $\kappa_V \simeq 1 + 3\epsilon$ ,  $\kappa_f \simeq 1 - \epsilon$ .

$$\epsilon = M_1^2 v^2 / 8\mu_3^4$$

## Indirect constraints

Key observations:

$$(\tan \theta_H = 2\sqrt{2}v_\chi/v_\phi)$$

1) Fermion masses generated by a *single*  $SU(2)_L$  Higgs doublet.

$$h\bar{f}f : \quad -i\frac{m_f \cos \alpha}{v \cos \theta_H}, \quad H\bar{f}f : \quad -i\frac{m_f \sin \alpha}{v \cos \theta_H},$$

$$H_3^0\bar{u}u : \quad \frac{m_u}{v} \tan \theta_H \gamma_5, \quad H_3^0\bar{d}d : \quad -\frac{m_d}{v} \tan \theta_H \gamma_5,$$

$$H_3^+\bar{u}d : \quad -i\frac{\sqrt{2}}{v} V_{ud} \tan \theta_H (m_u P_L - m_d P_R),$$

$$H_3^+\bar{\nu}\ell : \quad i\frac{\sqrt{2}}{v} \tan \theta_H m_\ell P_R \quad (\text{all } H_5 f \bar{f} \text{ couplings} = 0)$$

( $b, \tau$  Yukawas *not* enhanced: nonoblique/ $b$ -phys effects involve couplings  $\sim m_t \tan \theta_H$ )

2)  $H_3^+ H_3^- Z$  coupling is identical to  $H^+ H^- Z$  coupling in 2HDMs due to custodial symmetry.

$\Rightarrow$  Leading nonoblique  $Z$ -pole and  $b$ -physics constraints are the same as those in the Type-I 2HDM, with  $\cot \beta \rightarrow \tan \theta_H$  and  $m_{H^+} \rightarrow m_3!$

## Indirect constraints

$R_b$ : known a long time in GM model; same form as Type-I 2HDM  
HEL & Haber, hep-ph/9909335; Chiang & Yagyu, 0902.4665; Type-I: Grant, hep-ph/9410267

$B_s - \bar{B}_s$  mixing: adapted from Type-I 2HDM

Mahmoudi & Stal, 0907.1791

$b \rightarrow s\gamma$ : adapted from Type-I 2HDM

Barger, Hewett & Phillips, PRD41, 3421 (1990)

$B_s \rightarrow \mu^+ \mu^-$ : adapted from new calculation for Aligned 2HDM by  
Li, Lu & Pich, 1404.5865

Only relevant contribution comes from the  $Z$  penguin:

$$\frac{\bar{B}(B_s^0 \rightarrow \mu^+ \mu^-)}{\bar{B}(B_s^0 \rightarrow \mu^+ \mu^-)_{SM}} = \left| \frac{C_{10}^{SM} + C_{10}^{GM}}{C_{10}^{SM}} \right|^2$$
$$C_{10}^{GM} = \frac{1}{8} \tan^2 \theta_H \frac{m_t^2}{M_W^2} \left[ \frac{R}{1-R} + \frac{R \log R}{(1-R)^2} \right], \quad R = \frac{m_t^2}{m_3^2}$$

## Indirect constraints

We also implement the  $S$ -parameter constraint, marginalizing over the  $T$ -parameter.

Rationale:

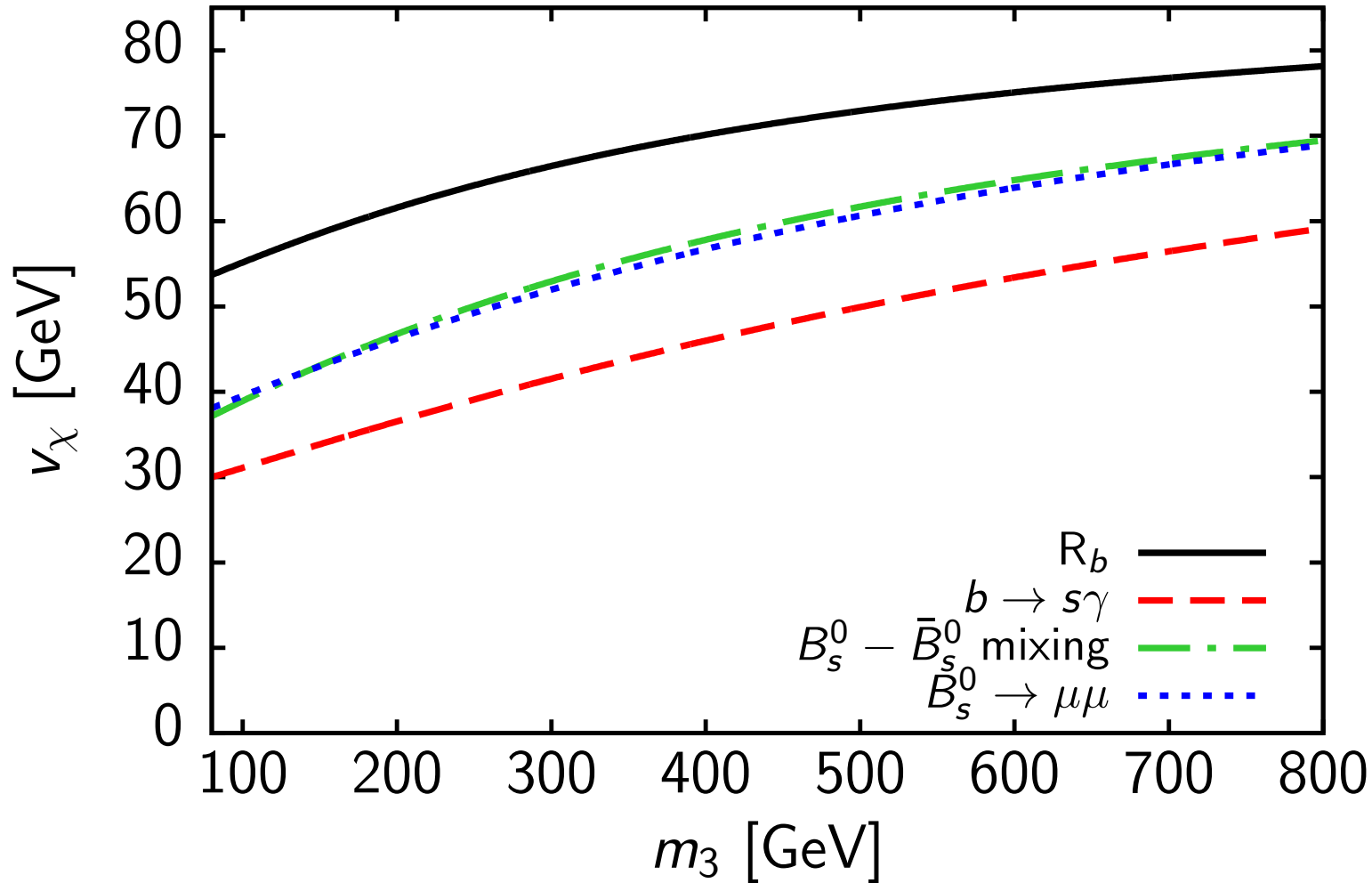
$T$ -parameter is (notoriously) divergent at 1-loop in GM model; to cancel the divergence one must introduce a global- $SU(2)_{R^-}$  violating counterterm. [Gunion, Vega & Wudka, PRD43, 2322 \(1991\)](#)

Introduces a small tree-level breaking of custodial  $SU(2)$

→ small tree-level contribution to  $\rho$  parameter

→ use to cancel a finite piece of the 1-loop contribution to  $T$ .

## Comparison of nonoblique & $b$ -physics constraints



VERY PRELIMINARY!

Hartling, Kumar & HEL, work in progress

(Li, Lu & Pich 1404.5865 identify  $B_s \rightarrow \mu\mu$  constraint as stronger than others, incl.  $b \rightarrow s\gamma$ )

## Aside on numerical results:

$b \rightarrow s\gamma$ : we used [Not yet cross-checked]

- SM calculation from [Misiak et al, hep-ph/0609232](#):

$$\text{BR}(\bar{B} \rightarrow X_s \gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4} \text{ for } E_\gamma > 1.6 \text{ GeV}$$

- current preliminary [HFAG](#) combined experimental result:

$$\text{BR}(\bar{B} \rightarrow X_s \gamma)_{\text{expt}} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4} \text{ for } E_\gamma > 1.6 \text{ GeV}$$

$B_s \rightarrow \mu\mu$ : we used [Cross-checked]

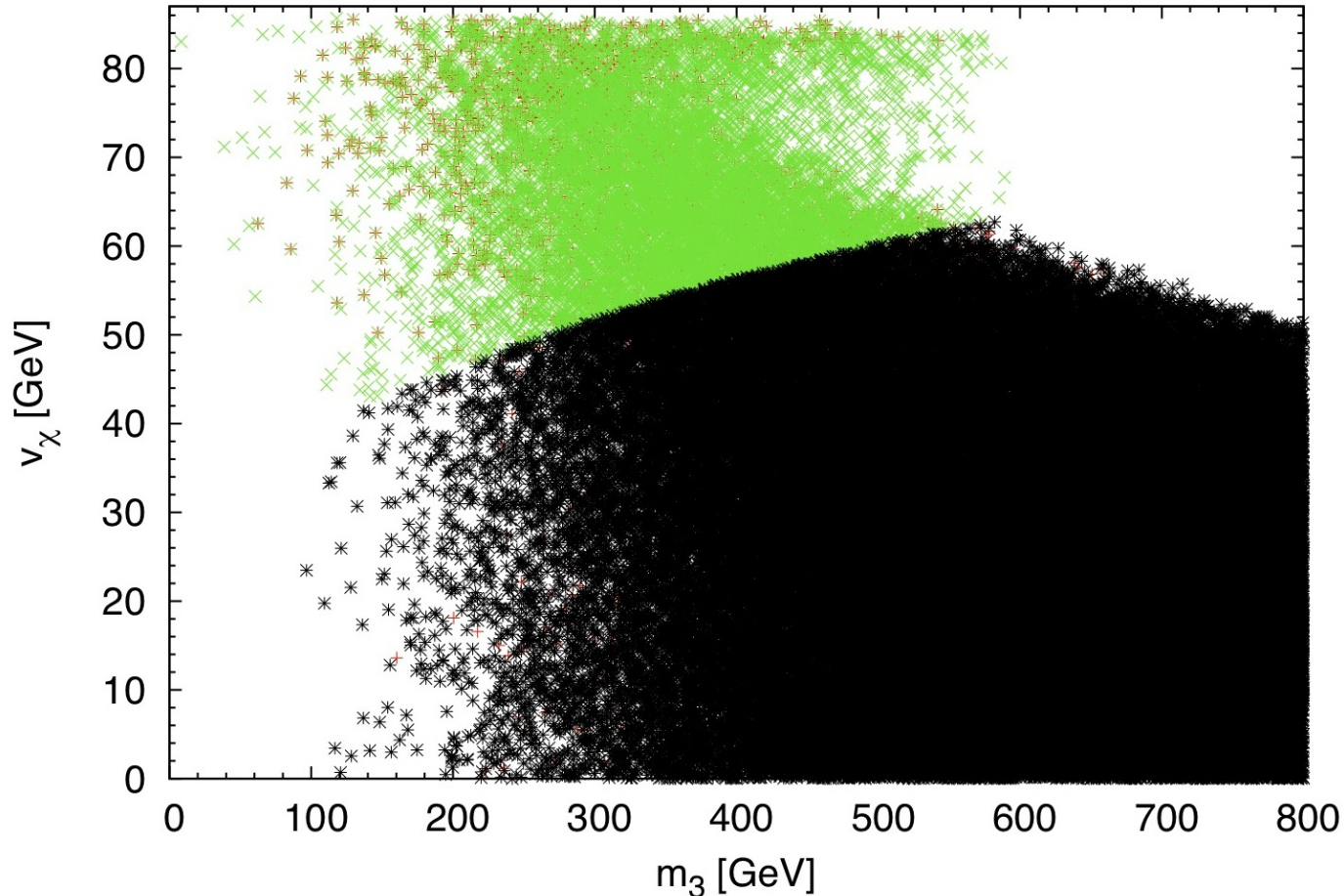
- SM calculation from [Bobeth et al, 1311.0903](#), updated in [Li, Lu & Pich 1404.5865](#) with slightly different  $m_t$ :

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.67 \pm 0.25) \times 10^{-9}$$

- current combined [CMS and LHCb](#) measurement,

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)_{\text{expt}} = (2.9 \pm 0.7) \times 10^{-9}$$

$B_s \rightarrow \mu\mu$  constraint: interplay with decoupling (green points are excluded)



PRELIMINARY-ish

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(Still working on implementing the  $b \rightarrow s\gamma$  constraint in our numerical-scan code: bound on  $v_\chi$  will be about 10 GeV lower.)

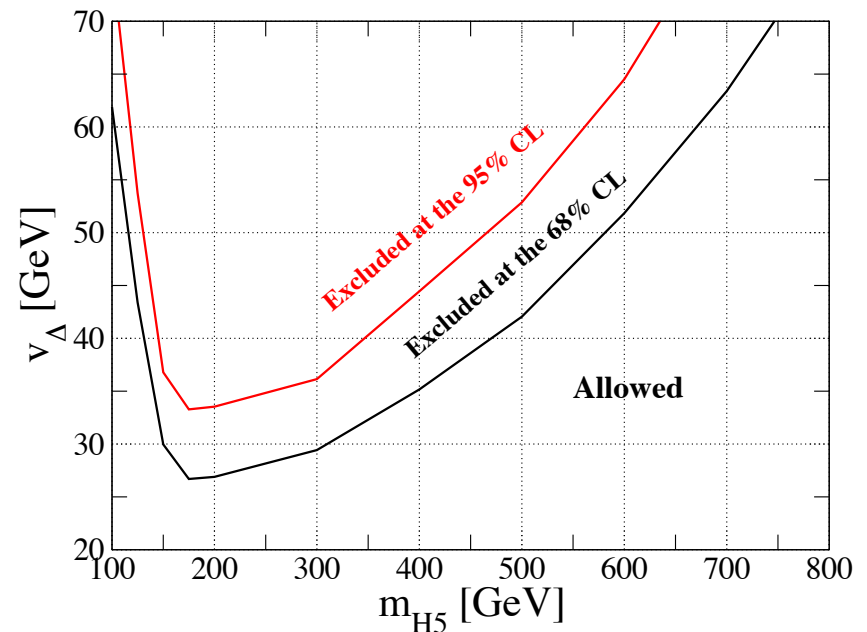
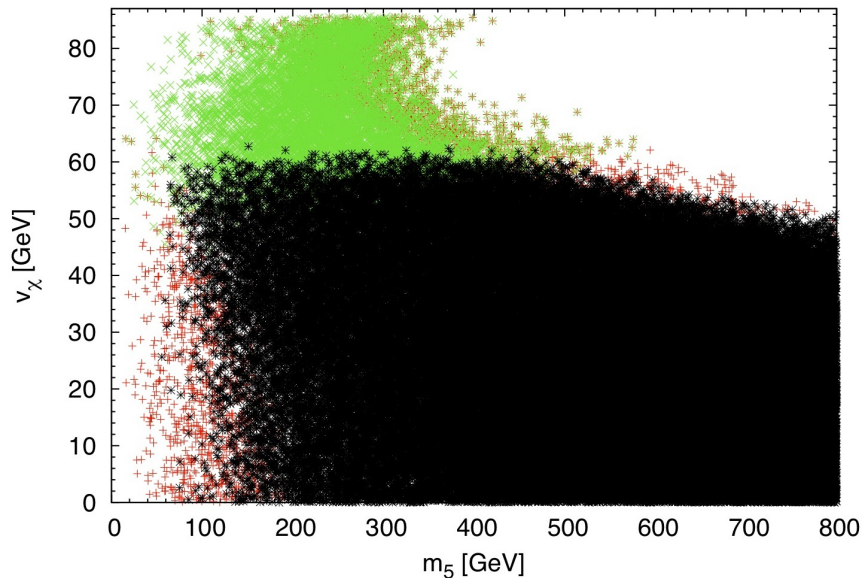
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# Effect of $B_s \rightarrow \mu\mu$ constraint on $m_5$ : compare direct search!

PRELIMINARY-ish ↓



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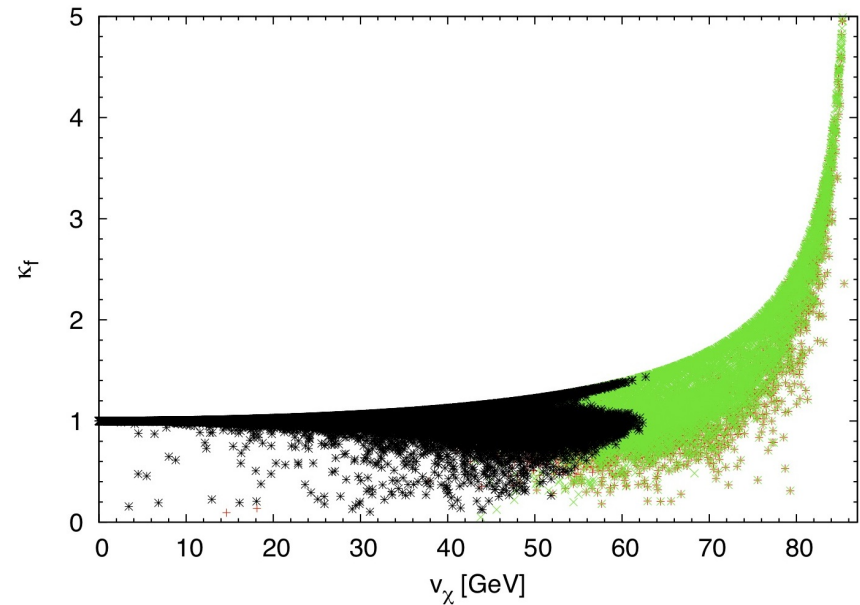
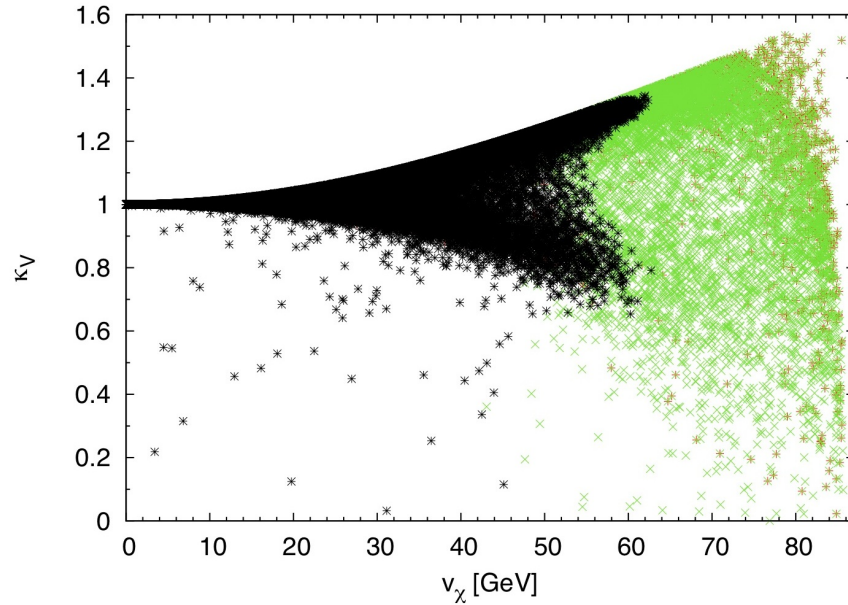
Chiang, Kanemura & Yagyu, 1407.5053

(red points are excluded by  $S$  parameter)

Recasting of ATLAS measurement of like-sign  $W^\pm W^\pm jj$  cross section to constrain VBF  $W^\pm W^\pm \rightarrow H^{\pm\pm} \rightarrow W^\pm W^\pm$ .



Effect of  $B_s \rightarrow \mu\mu$  constraint on  $\kappa_V$  and  $\kappa_f$  (green points are excluded)



PRELIMINARY-ish

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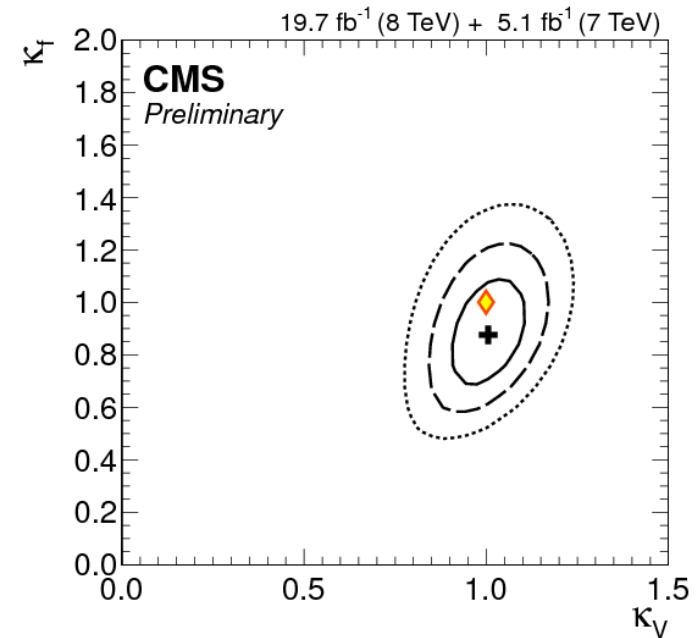
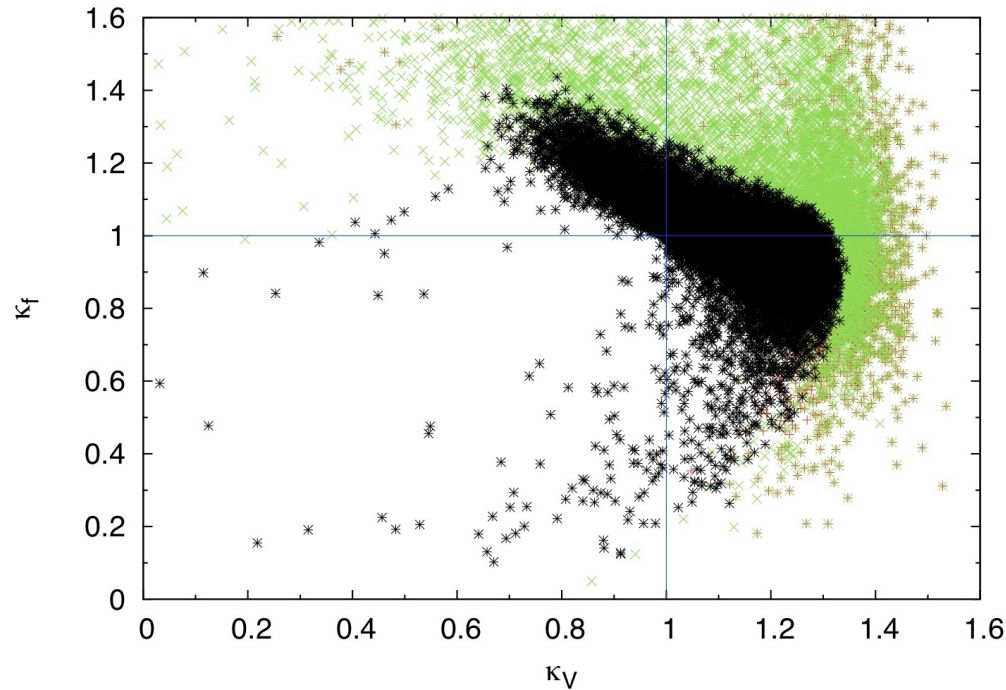
$$\kappa_V = \cos \alpha \frac{v_\phi}{v} - \frac{8}{\sqrt{3}} \sin \alpha \frac{v_\chi}{v}$$

$$\kappa_f = \cos \alpha \frac{v}{v_\phi}$$

The upper bound on  $v_\chi$  imposed by  $B_s \rightarrow \mu\mu$  constrains  $\kappa_V \lesssim 1.35$  and  $\kappa_f \lesssim 1.45$ .

- $b \rightarrow s\gamma$  constraint will tighten this a little.
- Direct search in like-sign  $W^\pm W^\pm jj$  will tighten it a little more.

Effect of  $B_s \rightarrow \mu\mu$  constraint on  $\kappa_V$  and  $\kappa_f$  (green points are excluded)

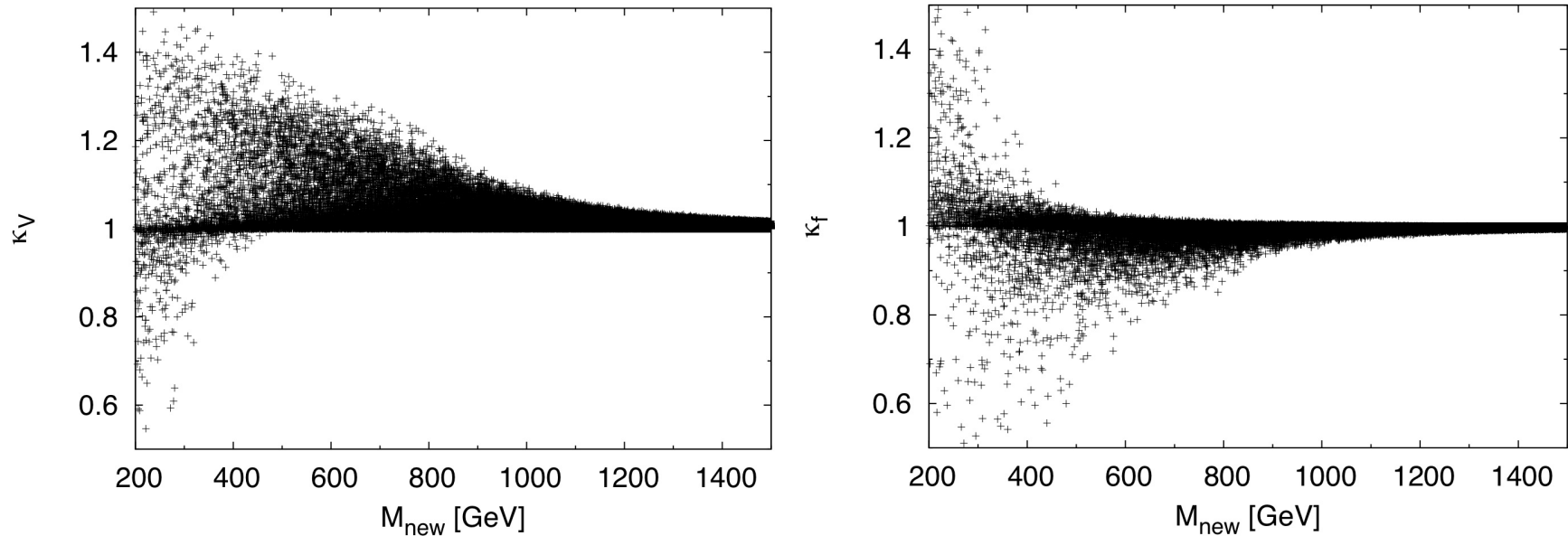


**PRELIMINARY-ish** Hartling, Kumar & HEL, work in progress

Along the line  $\kappa_V = \kappa_f$ , the  $B_s \rightarrow \mu\mu$  measurement constrains  $\kappa_V = \kappa_f \lesssim 1.15$ . ( $b \rightarrow s\gamma$ , like-sign  $WWjj$  will tighten this a little)

All LHC Higgs cross sections can be simultaneously enhanced by up to  $\sim 30\%$   $\Leftrightarrow$  enhancement can be hidden by an unobserved non-SM Higgs decay  $BR_{\text{new}}$  up to  $\sim 25\%$ . (LHC flat direction!)

“Flat direction” region of  $\kappa_V \simeq \kappa_f > 1$  is NOT near decoupling.



$M_{\text{new}} \equiv$  mass of *lightest* new state.

Hartling, Kumar & HEL, 1404.2640

$\kappa_f$  is suppressed near decoupling limit: significant enhancement requires  $M_{\text{new}} \lesssim 400\text{--}500$  GeV.

There are relatively light states to search for in this scenario!

Illustrates the value of a full model implementation.

## Outlook: toward a calculator for the Georgi-Machacek model

GMCALC code:

Hartling, Kumar & HEL, work in progress

- model parameter input set includes  $m_h$
- computes spectrum,  $h^0-H^0$  mixing angle,  $v_\chi$
- implements theory checks (unitarity, bounded-from-below, no alt minima)
- implements  $S$  parameter,  $B_s \rightarrow \mu\mu$  constraints; adding  $b \rightarrow s\gamma$
- outputs spectrum, decay tables
- currently working on implementing QCD and offshell corrections to decay partial widths
- working on suitable format to output production couplings