Neutralino annihilation beyond leading order

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• V. Barger, W.-Y. Keung, HEL, G. Shaughnessy, A. Tregre, Phys. Lett. B633 (2006) 98-105 [hep-ph/0510257]
• V. Barger, W.-Y. Keung, HEL, G. Shaughnessy, *work in progress*

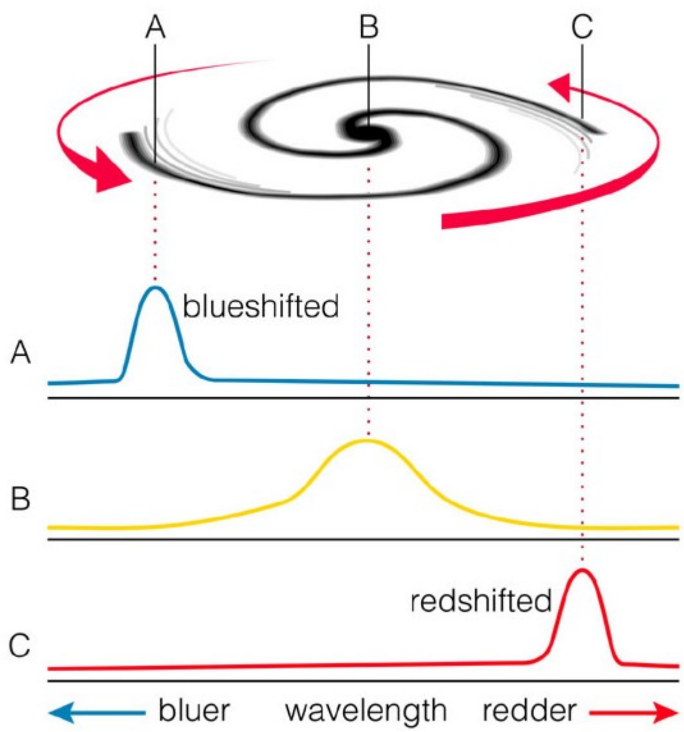
Outline

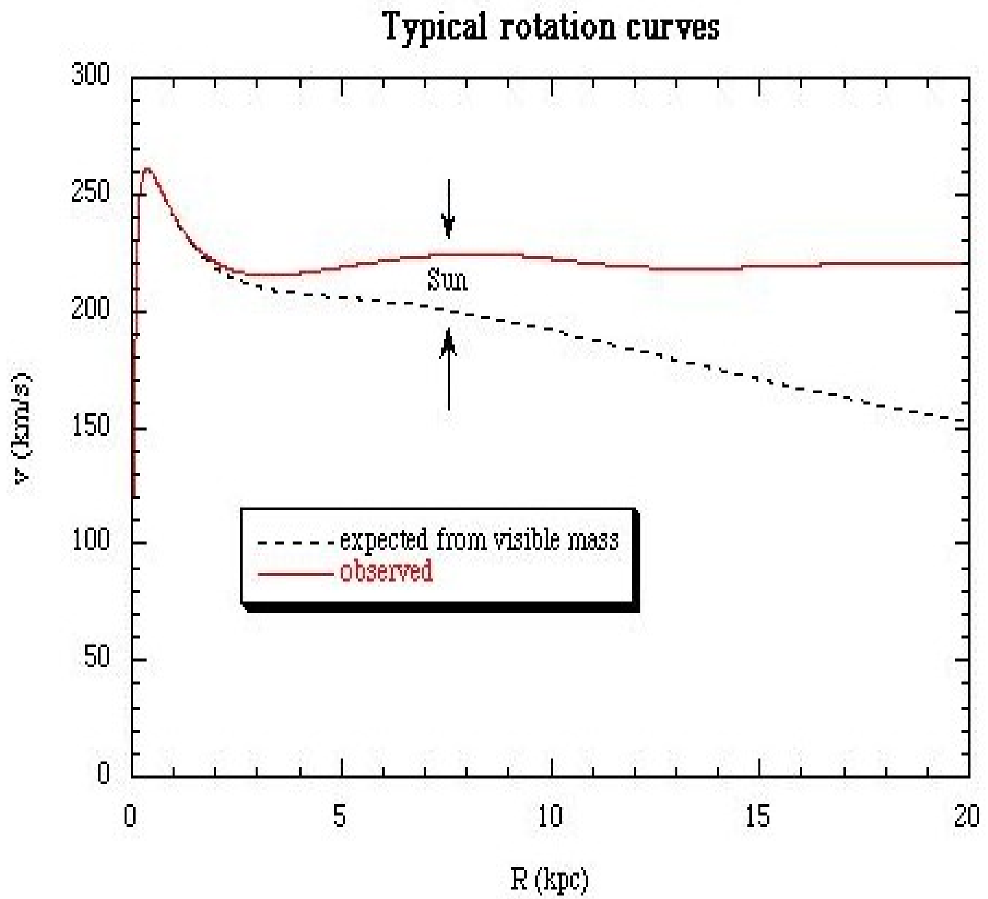
- Dark matter – what we know
- Supersymmetric dark matter: neutralino annihilation
  - Dealing with Majorana particles
  - Why go beyond leading order
- QCD corrections to neutralino annihilation
  - Annihilation to gluons
  - Using the Adler-Bardeen theorem
- Results and future directions

Dark Matter

How do we know about dark matter?
→ Purely through its gravitational effects.

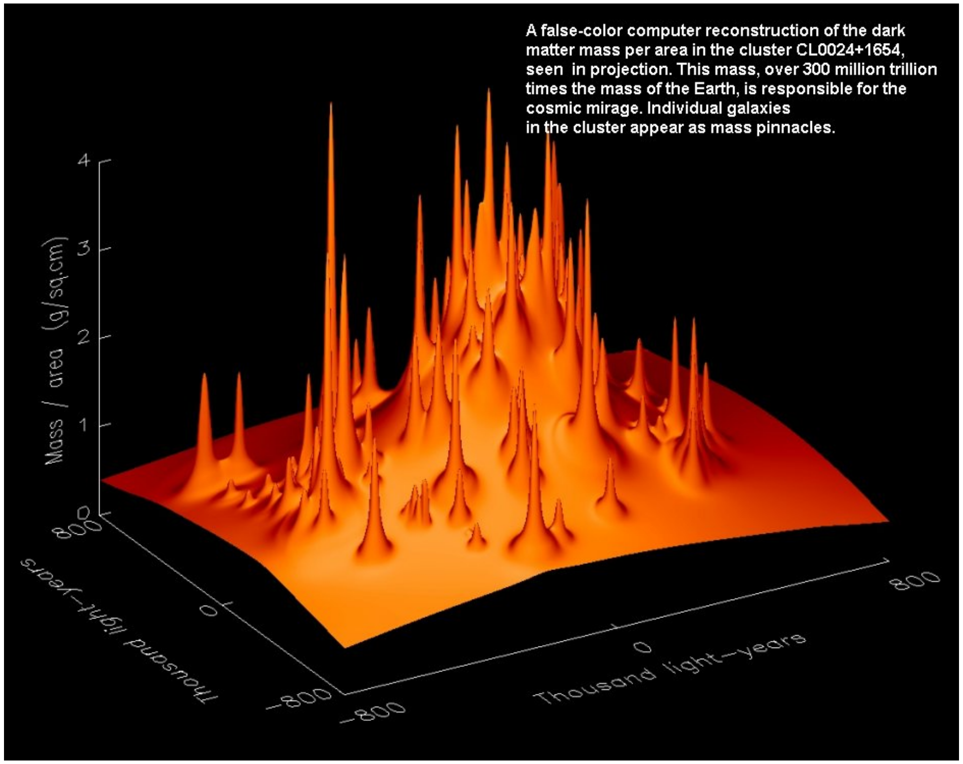
"Weigh the stars" by their gravitational effects:





More matter in our galaxy than what we see in stars, gas, dust, etc

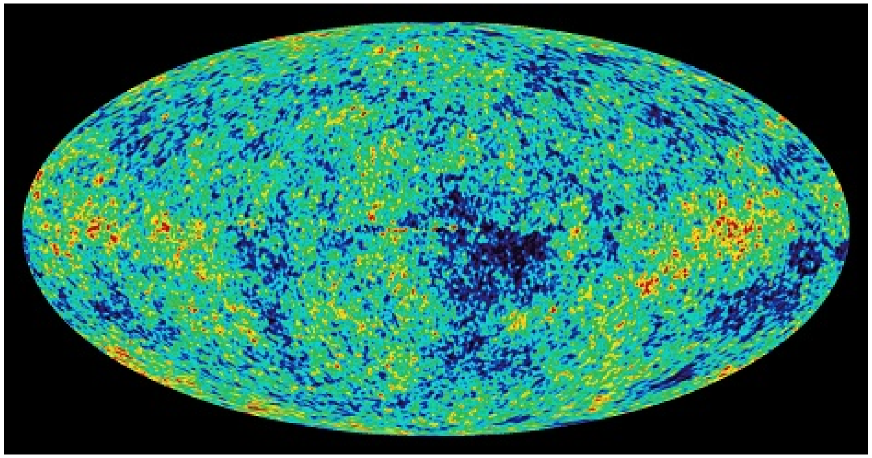
On a larger scale, bending of light by matter lets us reconstruct the distribution of mass in galaxy clusters.
Gravitational lensing of background galaxies by foreground cluster:

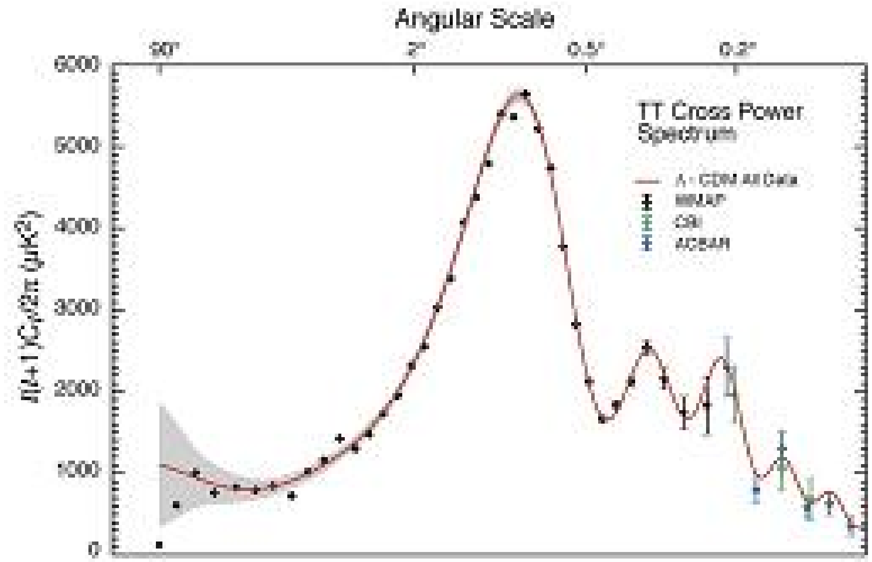


Reconstruct mass distribution of cluster:
mass per unit area along line of sight.
Right plot is for cluster CL0024+1654; vertical axis is g/cm^2

Again, much more mass than we see in stars, gas, dust, etc in the cluster:
broad mass distribution even outside of individual galaxies

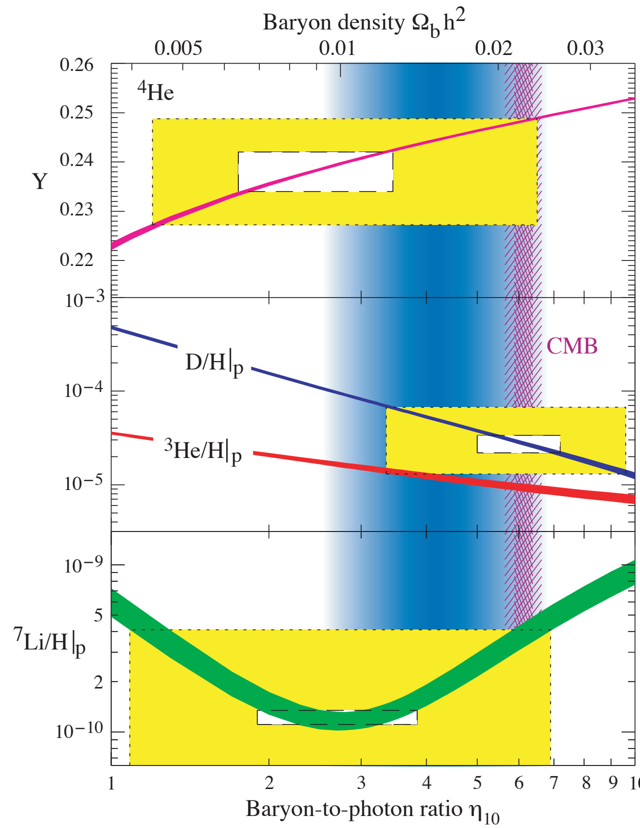
- Temperature fluctuations in the Cosmic Microwave Background





Measure total matter density and baryon density separately!

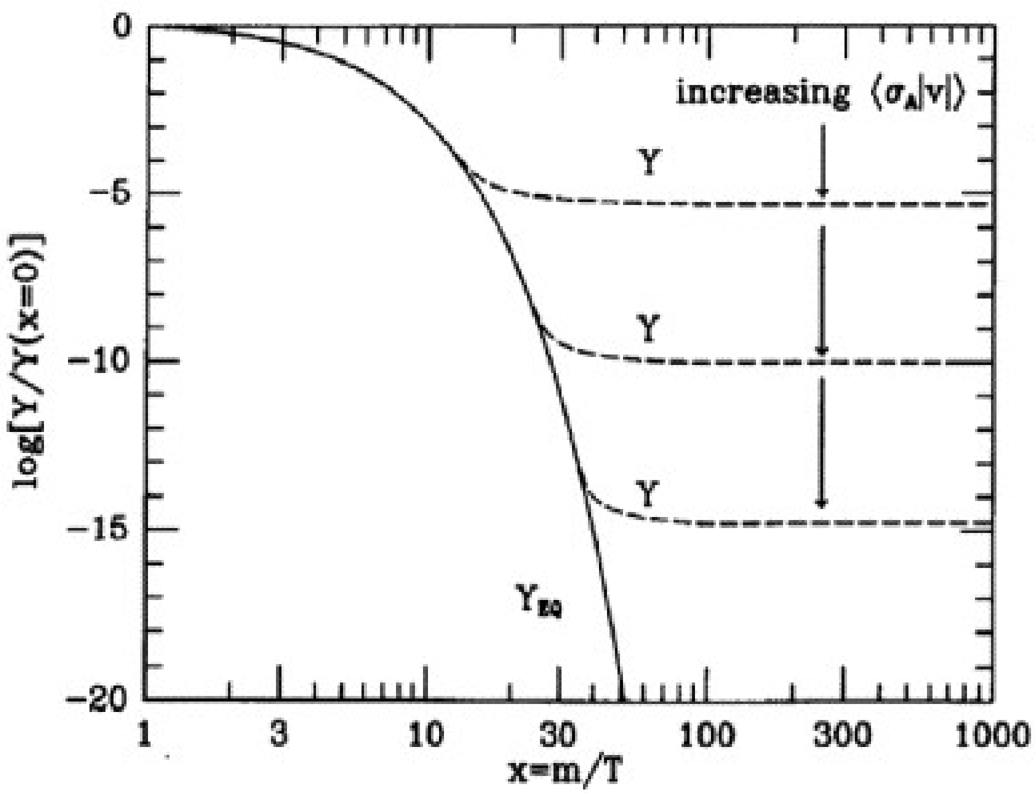
Consistent with big bang nucleosynthesis →



- Baryons 4%
- Nonbaryonic dark matter 23%
  known to ±10% precision!
- Dark energy 73%

Thermal production of dark matter

Early universe:
DM particles in thermal equilibrium with ordinary SM particles.
Pair production $\leftrightarrow$ pair annihilation in balance.
Universe cools and expands...
DM particle abundance drops as annihilation outpaces pair production.
Annihilation continues until DM particle density becomes so low that DM particles cannot find each other.
$\rightarrow$ "freeze-out" of DM relic density:
determined by annihilation cross section.



[Kolb & Turner]

What is the dark matter?

There are many possibilities:

- Weakly interacting massive particles (WIMPs)
  The "most popular" option – weak-scale mass and weak-strength annihilation naturally gives correct relic abundance.
  - Supersymmetry (SUSY) with R-parity: lightest SUSY particle (LSP)
  - Extra dimensions with conserved KK-parity: lightest KK particle
  - Little Higgs model with conserved T-parity: lightest T-odd particle
  - Technicolor with conserved technibaryon number: lightest technibaryon

- Axions
  Extremely light, nonrelativistic. Searches ongoing using microwave cavities
  Nonthermal production mechanism

- Superheavy weakly-interacting particles ("WimpZillas")
  Nonthermal production mechanism required

- Modified gravity?
  (Not clear whether this can be made to work)

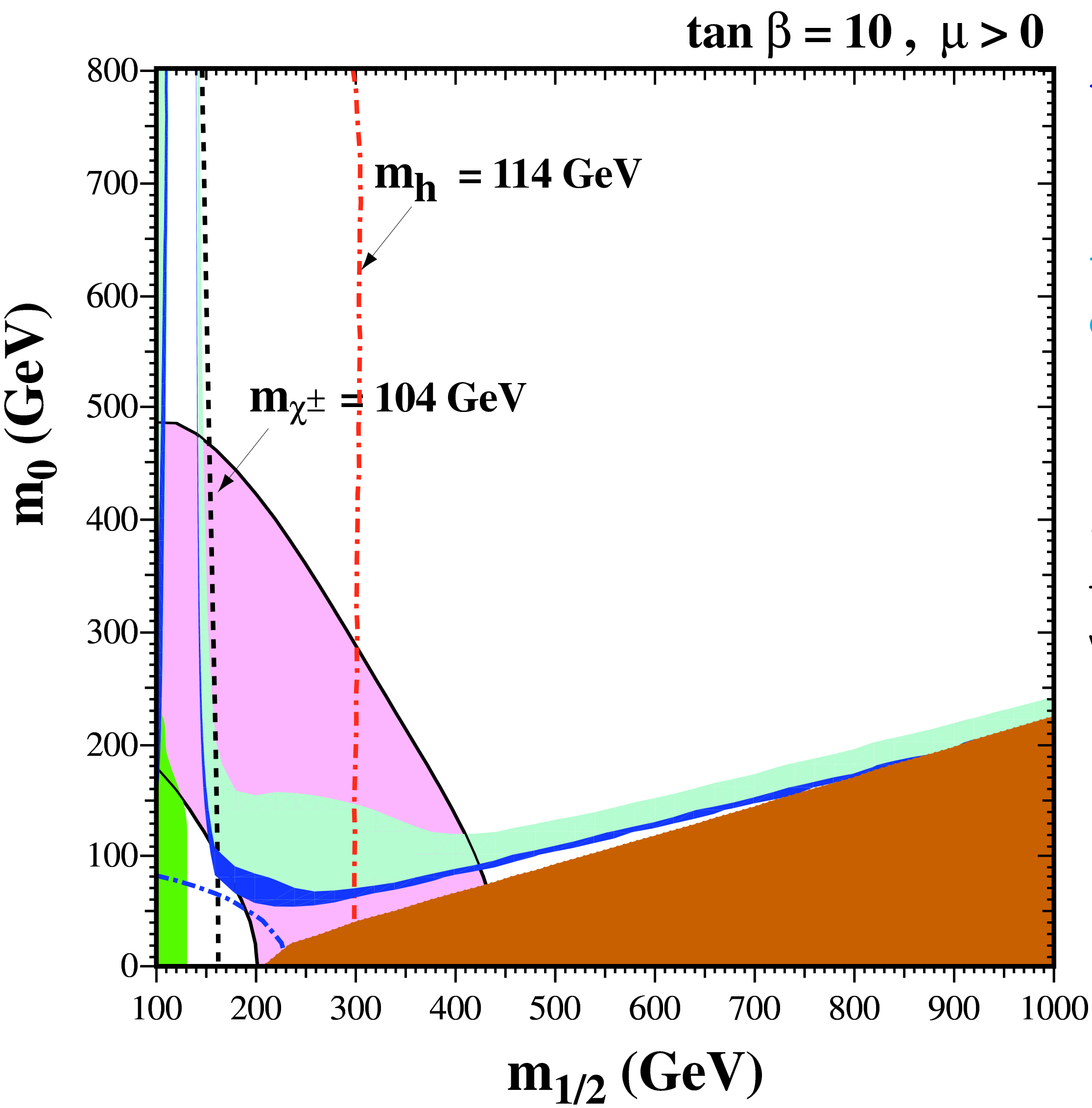
This talk: focus on supersymmetry
Consider lightest neutralino (χ) as the LSP.

How do we get the appropriate (measured) relic abundance?
Need a neutralino with the right combination of mass and annihilation cross section.

Many ways χ can annihilate:
multiple regions of parameter space that give the right relic abundance.

Neutralino annihilation

mSUGRA is general enough to illustrate most of the possibilities



Thin blue area gives correct relic abundance

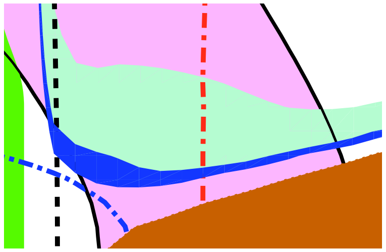
Thicker cyan region: from older, lower-precision cosmological measurements

Brown triangle at the bottom is where $\tilde{\tau}_1$ becomes lighter than χ: excluded because DM would be electrically charged.

- Below the allowed region, relic abundance is too small.
- Above the allowed region, relic abundance is too large.

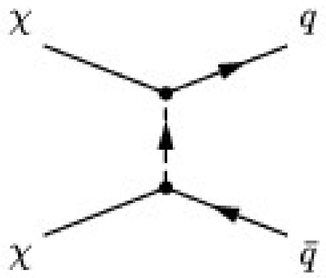
Trick is getting large enough annihilation cross section.

Focus on the "bulk region"



- χ is mostly bino, and relatively light
- Sfermions are relatively light

Main annihilation process is $\chi\chi \to f\bar{f}$ through a t-channel sfermion



A complication: χ is a Majorana particle.
The incoming $\chi\chi$ are identical fermions.
Wavefunction must be antisymmetric under interchange of the two χ's.

- $^1S_0$:

Initial state has zero orbital angular momentum (symmetric) and zero net spin (antisymmetric).

– When $\chi\chi$ are at rest, can only get this state.

Need to produce a SM fermion-antifermion pair in same $^1S_0$ state.

- If SM fermion is massless: Can only produce $f_L\bar{f}_L$ or $f_R\bar{f}_R$

E.g., left-handed electron and right-handed positron.

Back-to-back: net spin is 1! Need to flip a helicity in order to get $^1S_0$ state.
But: massless fermion $\to$ can never flip the helicity. Cross section is zero!

- If SM fermion is not massless:

– Can produce $f_L\bar{f}_L$ or $f_R\bar{f}_R$ and then flip one of the spins, at the cost of a factor of m_f in the amplitude.
– Can produce $f_L\bar{f}_R$ or $f_R\bar{f}_L$ directly, again at the cost of a factor of m_f in the amplitude.
The cross section is suppressed by a m_f^2/m_χ^2 factor.

Details coming two slides from now....

• $^3P_0$, $^3P_1$:
Initial state has one unit of orbital angular momentum (antisymmetric) and one unit of spin (symmetric).

– Can get this state only when $\chi\chi$ have nonzero relative velocity.

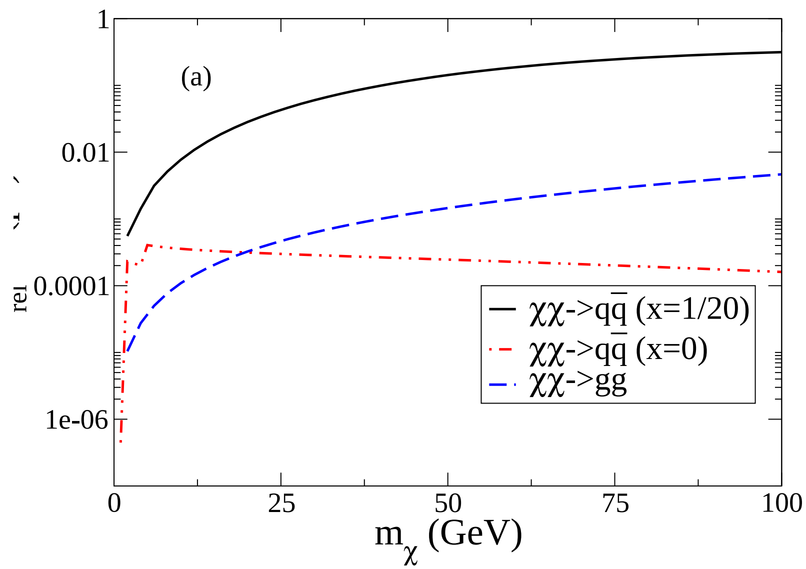
Can produce a SM fermion-antifermion pair in same $^3P_1$ state without a helicity flip: cross section not suppressed by SM fermion mass!

• In the early universe, the $\chi\chi$ relative velocity is small, but not tiny.

– $^3P_1$ wave annihilation can often dominate cross section.

• At the present day, the $\chi\chi$ relative velocity is $v \sim 10^{-3}c$: very small.

– Neutralino annihilation in the galactic halo is controlled by the $^1S_0$ wave only.



← $\chi\chi \to q\bar{q}$ in early universe dominated by p-wave contribution

← $\chi\chi \to q\bar{q}$ in the halo today dominated by s-wave contribution

Detail of the ${}^1S_0$ state: combining Majorana spinors

Let's work in the Dirac-Pauli representation.
The zero-momentum spinors are: and γ-matrices:

$$u^{(1)} = \sqrt{2m}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad u^{(2)} = i\sqrt{2m}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad \gamma^0 = \begin{pmatrix}I & 0\\0 & -I\end{pmatrix}$$

$$v^{(1)} = \sqrt{2m}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \qquad v^{(2)} = i\sqrt{2m}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \qquad \gamma^5 = \begin{pmatrix}0 & I\\I & 0\end{pmatrix}$$

(The phase of spin (2) is chosen for later convenience)
We'll need $\bar{v} = v^\dagger \gamma^0$:

$$\bar{v}^{(1)} = \sqrt{2m}\,(0\ 0\ 0\ -1) \qquad \bar{v}^{(2)} = i\sqrt{2m}\,(0\ 0\ 1\ 0)$$

Now compute $u^{(s_1)}\bar{v}^{(s_2)} - u^{(s_2)}\bar{v}^{(s_1)}$:

$$u^{(1)}\bar{v}^{(1)} - u^{(1)}\bar{v}^{(1)} = u^{(2)}\bar{v}^{(2)} - u^{(2)}\bar{v}^{(2)} = 0$$

$$(a) \qquad u^{(1)}\bar{v}^{(2)} - u^{(2)}\bar{v}^{(1)} = 2im\begin{pmatrix}0 & I\\0 & 0\end{pmatrix}$$

$$(b) \qquad u^{(2)}\bar{v}^{(1)} - u^{(1)}\bar{v}^{(2)} = -2im\begin{pmatrix}0 & I\\0 & 0\end{pmatrix}$$

Note (a) and (b) are not linearly independent! Go to the diagonal state:

$$\frac{1}{\sqrt{2}i}\left[(a) - (b)\right] = 2\sqrt{2}m\begin{pmatrix}0 & I\\0 & 0\end{pmatrix}$$

Can write this in a Lorentz-invariant way.
First note that, in the Dirac-Pauli representation:

$$\frac{1}{2}\left(1+\gamma^0\right) = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$
$$\frac{1}{2}\left(1+\gamma^0\right)\gamma^5 = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}$$

So the properly normalized spin-zero $\chi\chi$ state is

$$2\sqrt{2}m_\chi \cdot \frac{1}{2}\left(1+\gamma^0\right)\gamma^5$$

The four-momentum of the $\chi\chi$ state is $P \equiv p_1 + p_2 = (2m_\chi, \vec{0})$ for the two neutralinos at rest. Dotting this into γ^μ singles out the γ^0 component as desired.

So we get a Lorentz-invariant expression for the zero-velocity $\chi\chi$ spinors:

$$u^{(s_1)}\bar{v}^{(s_2)} - u^{(s_2)}\bar{v}^{(s_1)} = \sqrt{2}\left(m_\chi + \not{P}/2\right)\gamma^5$$

What kind of annihilation cross sections does this give us?
Consider $\chi\chi \to f\bar{f}$, with sfermion $\widetilde{f}_L$ or $\widetilde{f}_R$ exchange (scalar couplings).
First ignore $\widetilde{f}_L$–$\widetilde{f}_R$ mixing:

$$\mathcal{M} \sim \bar{v}_{\bar{f}} P_{R,L} u_\chi \bar{v}_\chi P_{L,R} u_f = \bar{v}_{\bar{f}} P_{R,L} \sqrt{2}\,(m_\chi + \not{P}/2)\,\gamma^5 P_{L,R} u_f$$

The m_χ term gives zero because of the helicity projection operators.
The $\not{P}$ term involves the total centre-of-mass momentum P, which can be written as $P = p_f + p_{\bar{f}}$.
Using the Dirac equation and keeping track of the γ^5's gives

$\not{p}_f \to -m_f \qquad \not{p}_{\bar{f}} \to -m_f$

These two terms add giving an amplitude proportional to m_f
$\to$ The cross section is suppressed by m_f^2/m_χ^2.

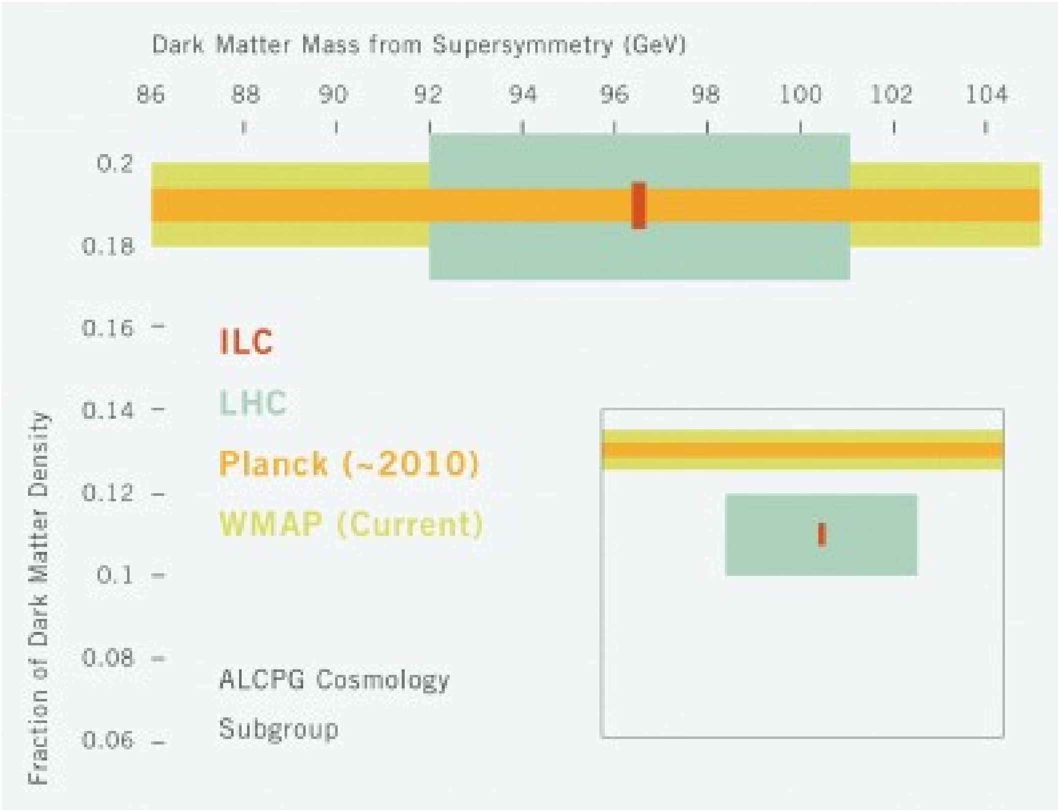
Note that allowing $\widetilde{f}_L$–$\widetilde{f}_R$ mixing lifts the helicity restriction that kills off the m_χ term.
However, $\widetilde{f}_L$–$\widetilde{f}_R$ mixing is due to terms proportional to m_f in the sfermion mass-squared matrix, so the mixing angle itself contains an m_f.
Thus the cross section is still suppressed by m_f^2/m_χ^2.

This is very similar to 2-body decay of pseudoscalar mesons – like $B^0_{d,s}$

$B^0 \to \ell^+\ell^-$: BR $\propto m_\ell^2/m_B^2$: $\sim 10^{-9}$ for $B_s \to \mu^+\mu^-$
$B^0 \to \ell^+\ell^-\gamma$: BR $\sim 10^{-6}$.
Radiating an extra photon lifts the helicity suppression!

So if we want to lift the m_f suppression in $\chi\chi$ annihilation, we should think about radiating a hard photon or gluon: calculate beyond $2 \to 2$ leading order.

Another reason for high-precision calculations: testing cosmology at colliders



← The cartoon version

If we can measure the masses and couplings that go into the $\chi\chi$ annihilation cross section, we can predict the DM abundance.

Then we can check if our collider physics accounts for the cosmologically observed DM.

Measure SUSY masses/couplings, calculate χ relic abundance from thermal production + freeze-out

Different possibilities:

- Prediction is spot-on: we understand the universe all the way back to DM freeze-out!
- Prediction is too low: there must be another species of DM or another source of χ production that we don't yet know about!
- Prediction is too high: maybe χ is only metastable: decayed into something lighter (e.g., gravitino?) which is the true DM!

What must we measure to test neutralino dark matter at colliders?

Need to measure: (for χ in "bulk region")
- χ mass: number density $\leftrightarrow$ mass density; annihilation kinematics
- χ composition: couplings to fermion-sfermion
- Squark and slepton masses: t-channel exchange in annihilation diagram
- Squark and slepton L-R mixings: χ-sfermion-fermion couplings

Need to be able to calculate the relic abundance from the collider inputs at high enough precision to match cosmological measurements
$\rightarrow$ May need to calculate beyond leading order.
QCD corrections to some processes can be large, several tens of percent.

With these measurements in hand, we can also predict:
- Neutralino-nucleon scattering cross section
- Present-day ($v \simeq 0$) annihilation cross section and branching fractions

Use indirect detection of DM...
- gamma rays from DM annihilation in galactic centre
- neutrinos from DM annihilation inside the sun

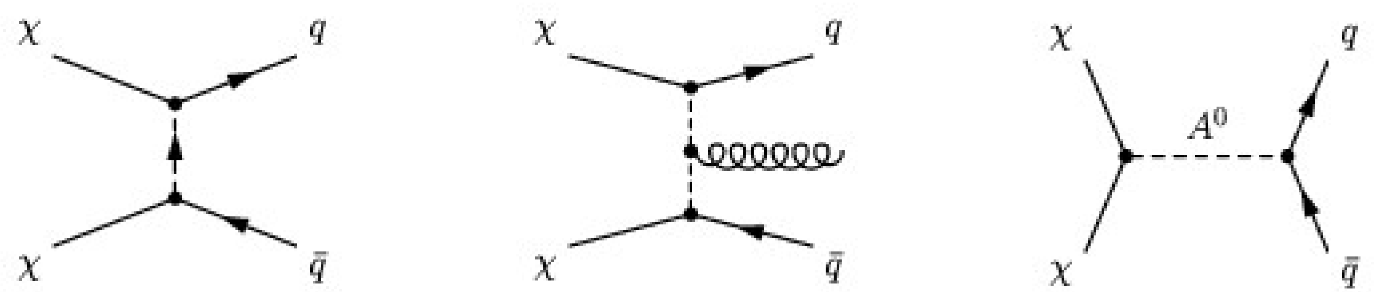
... and direct detection of DM...
- scattering of DM off detectors on earth

... to test our understanding of the particle properties of the DM and to learn about the density profile of the galactic halo.
(use particle physics to learn more about astrophysics)

Again, need to calculate the cross sections with high enough precision.

Neutralino annihilation

Some typical diagrams:



etc.

Focus on $\chi\chi$ annihilation through squark exchange.
The first diagram above can be reduced to an effective vertex described by a dimension-six operator suppressed by the squark mass $\widetilde{M}$:

$$\mathcal{L} = \frac{c}{\widetilde{M}^2}\mathcal{O}_6, \qquad\qquad \mathcal{O}_6 = (\bar{\chi}\gamma_\mu\gamma_5\chi)\,(\bar{q}\gamma^\mu\gamma_5 q)$$

This is valid in the limit $m_\chi \ll \widetilde{M}$.
In the zero-velocity limit the neutralinos behave like a pseudoscalar, and $\mathcal{O}_6$ is related to the divergence of the axial vector current of the quarks:

$$\mathcal{O}_6 \to \left[\bar{\chi}\frac{i\gamma_5}{2m_\chi}\chi\right]\left[\partial_\mu\left(\bar{q}\gamma^\mu\gamma_5 q\right)\right]$$

- If $m_q = 0$, the axial vector current is conserved at tree level, $\partial_\mu\left(\bar{q}\gamma^\mu\gamma_5 q\right) = 0$.
  This is the m_f^2/m_χ^2 suppression showing up.
- There are two ways to lift this suppression:
  (1) go to dimension-eight (or higher).
  (2) Go beyond leading order to include the anomalous triangle diagram.

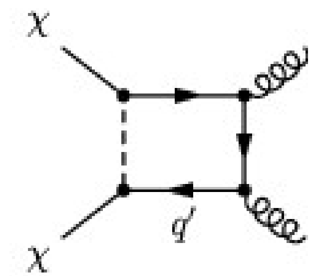
Let's look first at including the anomalous triangle diagram.
This is the well-known Partially Conserved Axial Current (PCAC):
$\partial_\mu (\bar{q}\gamma^\mu\gamma_5 q) \neq 0$ due to the anomaly, even when $m_q = 0$.
The anomaly condition reads: (including m_q and only QCD interactions)

$$\partial_\mu (\bar{q}\gamma^\mu\gamma_5 q) = 2m_q \bar{q} i\gamma_5 q + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \widetilde{G}^{(a)\mu\nu}$$

Neglecting m_q, we can write the zero-velocity dimension-six $\chi\chi$ annihilation amplitude in the form

$$\mathcal{L}_{\rm eff} = \left(\frac{c/m_\chi}{2\widetilde{M}^2} \right) (\bar{\chi} i\gamma_5 \chi) \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \widetilde{G}^{(a)\mu\nu}$$

This is $\chi\chi$ annihilation into gluons.



Expression describes one massless quark running around the loop.

Still working in $\widetilde{M}^{-2}$ approximation for squark propagator.

Calculation first done for $\chi\chi \to \gamma\gamma$ [Rudaz 1989; Bergstrom 1989]
Easy to extend to $\chi\chi \to gg$ [Flores, Olive, Rudaz 1989]
$m_{q'} = 0$ result: (sum is over 5 light quarks; top decouples)

$$v_{\rm rel}\sigma(\chi\chi \to gg) = \frac{m_\chi^2}{64\pi^5} \left(\sum_{q'} A_{q'} \right)^2 \qquad A_{q'} = \frac{g_s^2}{2\sqrt{2}} \left[\frac{g_r^2}{M^2_{\tilde{q}'_R}} + \frac{g_\ell^2}{M^2_{\tilde{q}'_L}} \right]$$

Full $m_{q'}$, $\widetilde{M}$ dependence also known [Drees, Jungman, Kamionkowski, Nojiri 1993]

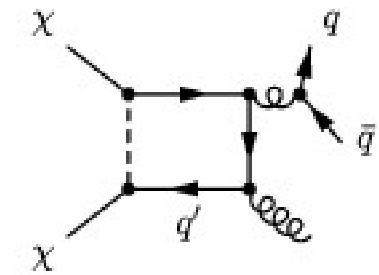
What about beyond leading order?

$\chi\chi \to gg$ is order α_s^2: large scale dependence at leading order.
Set scale $\mu = 2m_\chi$, vary between $\mu/2 \ldots 2\mu$: $v\sigma$ varies by $\pm 16\%$.

At NLO, must include:

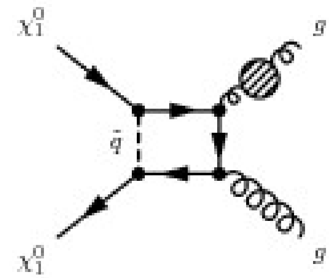
(1) gluon splitting into quark or gluon pairs

(2) radiation of a 3rd gluon off of the internal q' line



(3) virtual corrections: gluons crossing the box, gluons connecting the box to a gluon leg

(4) renormalization; e.g., gluon propagator bubbles containing quarks and gluons



(*) radiating the 3rd gluon off the internal squark line contributes first at order $1/\widetilde{M}^4$ in the amplitude – ignore in our approximation.

The calculation is big and ugly. Are there any tricks we can use?
Luckily the answer is yes!

Remember that in the zero-velocity limit, $\chi\chi \to gg$ is related to the anomaly equation:

$$\partial_\mu(\bar{q}'\gamma^\mu\gamma_5 q') = 2m_{q'}\bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

- Neglecting the $m_{q'}$ term relates the axial vector current divergence to the two-gluon operator:

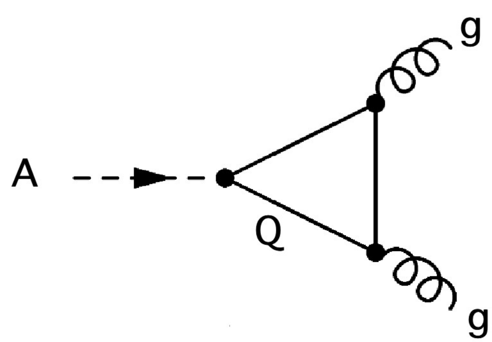
$$\partial_\mu(\bar{q}'\gamma^\mu\gamma_5 q') = \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

- If we take the opposite limit, $m_{q'} \gg m_\chi$, then the anomaly equation relates a pseudoscalar coupling to the same two-gluon operator:

$$0 = 2m_{q'}\bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

(the term in the left-hand side becomes negligible in the $m_{q'} \gg m_\chi$ limit)

This describes pseudoscalar decay through a heavy quark triangle:



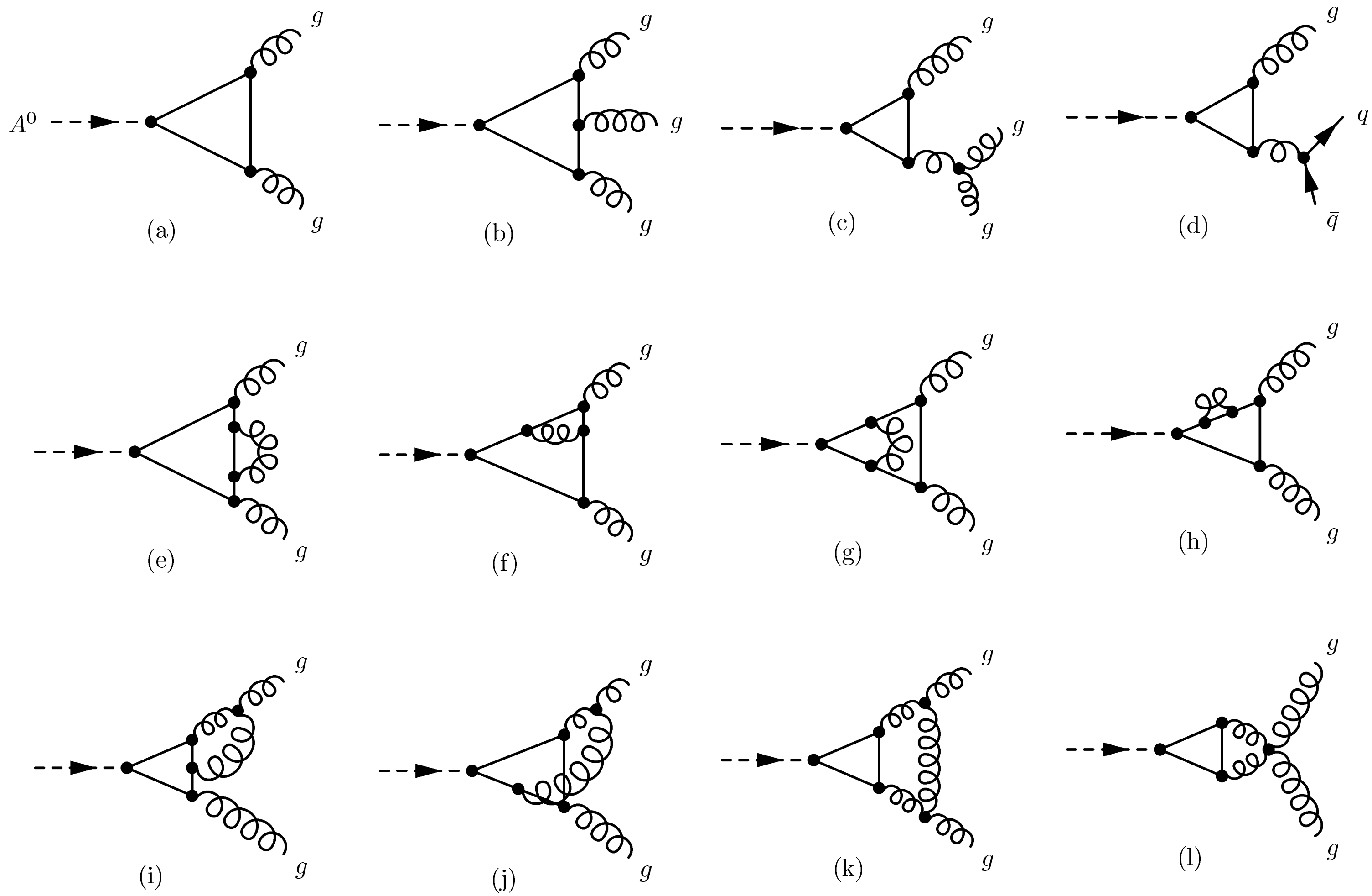
in the limit $m_Q \gg m_A$.

This helps us because of the Adler-Bardeen theorem.

The Adler-Bardeen theorem tells us that the anomaly equation holds to all orders in α_s.

Should be able to relate $\chi\chi \to gg$ at NLO to $A \to gg$ at NLO.

$A \to gg$ at NLO calculated by Spira, Djouadi, Graudenz, Zerwas (1995):



$$\Gamma_{\rm NLO}(A \to gg) \;=\; \Gamma_{\rm LO}(A \to gg) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{97}{4} - \frac{7}{6}N_f + \frac{33 - 2N_f}{6}\log\frac{\mu^2}{4m_\chi^2}\right)\right]$$

The NLO correction is multiplicative.

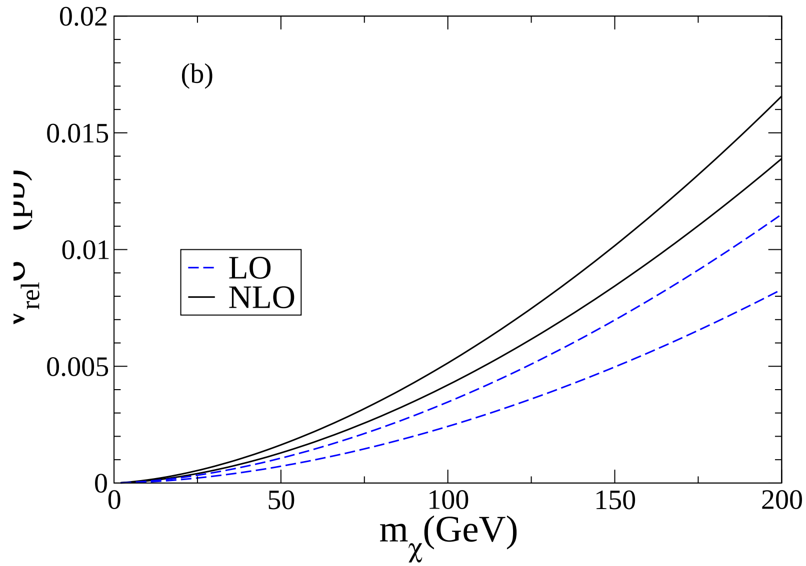
In the large-m_Q limit, it does not depend on m_Q – can take the result over to $\chi\chi \to gg$ using the Adler-Bardeen theorem and the anomaly equation.

(This no longer works at NNLO because $\log(m_Q/m_A)$ dependence begins to creep in – ask me for details at the end)

So we get $\chi\chi \to gg$ at NLO "for free":

$$\begin{aligned}\sigma_{\rm NLO}(\chi\chi \to gg) &= \sigma_{\rm LO}(\chi\chi \to gg) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{97}{4} - \frac{7}{6}N_f + \frac{33-2N_f}{6}\log\frac{\mu^2}{4m_\chi^2}\right)\right] \\ &= \sigma_{\rm LO}(\chi\chi \to gg)\,[1+0.62]\end{aligned}$$

where the last line is for $\mu = 2m_\chi$, $m_\chi = 100$ GeV, and $N_f = 5$.



← NLO: scale uncertainty ±9%

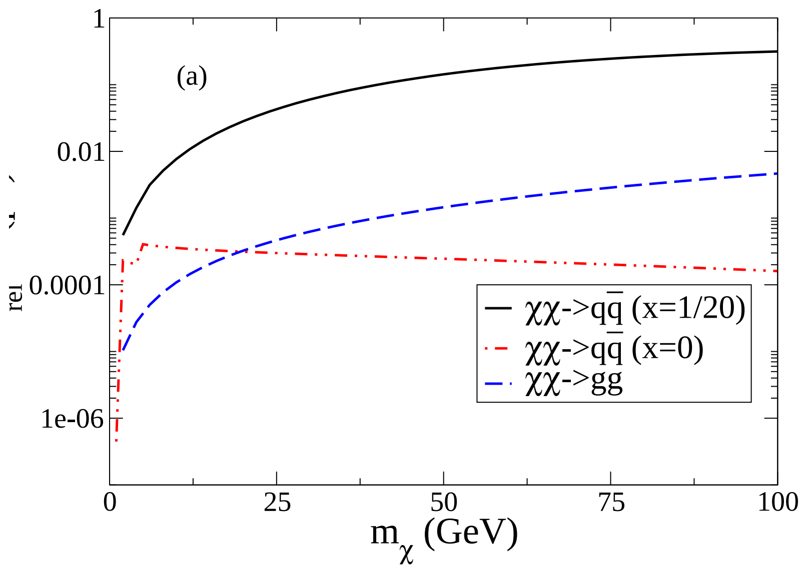
← LO: scale uncertainty ±16%

$\chi\chi \to gg$ cross section is increased by some 60% at NLO.

Where is this useful?

- Early universe: not particularly.
  - $\chi\chi \to gg$ is typically only a small contribution to the total annihilation cross section.
  - Since total cross section controls relic density, corrections to $\chi\chi \to gg$ are not very important.

- Present day: yes!
  - $\chi\chi \to gg$ can be the dominant annihilation mode.
  - Corrections are important for total annihilation cross section and branching fractions – can affect indirect detection rates (gamma rays, neutrinos).



← $\chi\chi \to q\bar{q}$ in early universe

← $\chi\chi \to gg$ (includes NLO)

← $\chi\chi \to q\bar{q}$ today

Leading QCD corrections to p-wave $\chi\chi \to q\bar{q}$ were calculated in [Flores, Olive, Rudaz 1989] – not a huge effect

Indirect detection of dark matter

"Indirect detection" means looking for the $\chi\chi$ annihilation byproducts.
Annihilation rate is proportional to neutralino number density squared.

- WIMP annihilation in the galactic centre

High number density – but depends strongly on the astrophysics model for the core of the halo.
Annihilation byproducts: gamma rays are easiest to see.
Electrons/positrons/hadrons get deflected by galactic magnetic fields; neutrinos hard to detect (but see later)

Look for signal coming from gravitational centre of the galaxy.
EGRET/GLAST; ground-based Cherenkov telescopes
Gamma ray spectrum:

- $\chi\chi \to \gamma\gamma$ gives monochromatic γ-ray line at energy equal to m_χ: smoking gun for DM annihilation. Get m_χ for free.
- Other annihilation processes give γ's from final-state radiation (FSR) or from the hadronic shower. Spectrum depends on annihilation branching fractions and m_χ.

What matters is not the total annihilation rate, but the cross section that results in γ's.

- Our QCD corrections do not affect monochromatic $\sigma(\chi\chi \to \gamma\gamma)$.
- Our QCD corrections increase $\sigma(\chi\chi \to gg)$ by $\sim$ 60%: effect on γ's from hadronization.
- gg are neutral: no FSR photons. But $q\bar{q}g$ final state appears at NLO; can give FSR off the quarks.

• WIMP annihilation in the sun
WIMPs get gravitationally captured in the sun
Neutralino density builds up in the centre of the sun until the total annihilation rate equals the capture rate

• Equilibrium total annihilation rate is fixed by the capture rate
– Capture rate depends on WIMP-nucleon scattering cross section (must scatter to lose energy and get gravitationally captured) and local WIMP density (well constrained by galactic rotation curve).

• Annihilation byproducts: neutrinos are the only thing we can see
The sun is opaque to electrons/positrons/hadrons/photons.

Look for high-energy neutrinos coming from the sun
Amanda/IceCube; Antares/Nestor

Neutrino spectrum:

• Good source is $\chi\chi \to W^+W^-$ with semileptonic W decays, or $\chi\chi$ annihilation to heavy quarks followed by semileptonic decays, or $\chi\chi \to \tau^+\tau^-$.

What matters is the fraction of the total annihilation rate that produces neutrinos.

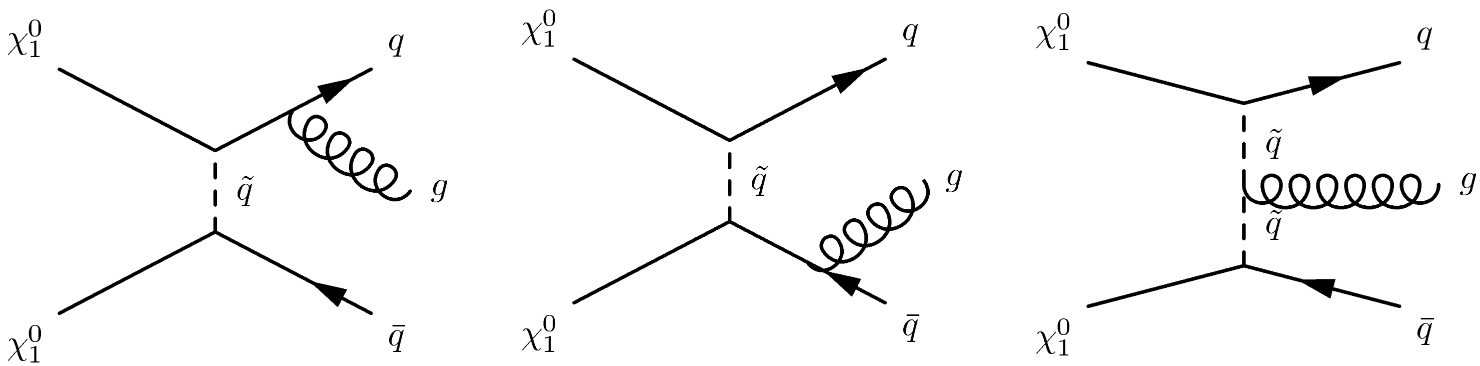
• Our QCD corrections increase $\chi\chi \to gg$, which is a large fraction of the total annihilation rate $\to$ reduces fractions that produce neutrinos.

• $q\bar{q}g$ final state appears at NLO and can give neutrinos from semileptonic decays of heavy $q\bar{q}$ – lower energy than 2-body $\chi\chi \to q\bar{q}$ due to extra gluon.

Further directions: the dimension-eight amplitude

Remember there were two ways to lift the m_f^2/m_χ^2 suppression:
(1) using the anomaly
(2) going to dimension-eight.

The dimension-eight amplitude was calculated for $\chi\chi \to f\bar{f}\gamma$ in [Flores, Olive, Rudaz 1989]

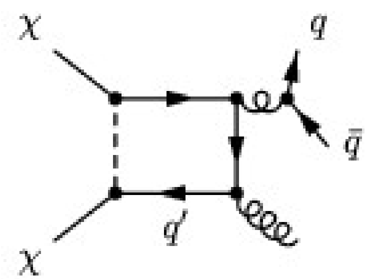


The full calculation was done in [Drees, Jungman, Kamionkowski, Nojiri 1993]

Not yet computed: interference term between

(1) dimension-eight $\chi\chi \to q\bar{q}g$, and

(2) dimension-six $\chi\chi \to q\bar{q}g$ through the box with gluon splitting to $q\bar{q}$



- Interference term is order α_s^2 – same order as $\chi\chi \to gg$ leading order
- Interference term is order $1/\widetilde{M}^6$ – more suppressed than $\chi\chi \to gg$ but less suppressed than pure dimension-eight cross section.

This is work in progress.

Conclusions

- We've entered the era of precision cosmology – needing to match that precision from the theory side motivates calculation at higher orders.

- Neutralinos are Majorana fermions – at zero velocity this leads to suppression of $\chi\chi \to f\bar{f}$ by m_f^2/m_χ^2. Processes that lift the m_f^2/m_χ^2 can have a big impact on present-day annihilation rates.

- We calculated NLO QCD corrections to $\chi\chi \to gg$ by using the Adler-Bardeen theorem and known NLO QCD corrections to $A^0 \to gg$:
  about a +60% effect.

- Calculation of interference term between $\chi\chi \to g^* g \to q\bar{q}g$ and dimension-8 $\chi\chi \to q\bar{q}g$ in progress.
  Same α_s order as $\chi\chi \to gg$; order $1/\widetilde{M}^6$ in squark mass.

- Implications for indirect detection still need to be worked out.

Backup slides

Detail on $A^0 \to gg$ and the Adler-Bardeen theorem

Study the NNLO QCD corrections to $A^0 \to gg$,
following [Chetyrkin, Kniehl, Steinhauser, Bardeen 1998]

Start with the bare Yukawa Lagrangian for interactions of A^0 with quarks:

$$\mathcal{L} = -\frac{A}{v}\left[\sum_{i=1}^{n_l} m_{q_i}^0 \bar{q}_i^0 i\gamma_5 q_i^0 + m_t^0 \bar{t}^0 i\gamma_5 t^0\right]$$

Taking the limit $m_t \to \infty$ and setting $m_{q_i} = 0$ for the light quarks, we can write this as a combination of pseudoscalar operators:

$$\mathcal{L} = -\frac{A}{v}\left[C_1^0 O_1^0 + C_2^0 O_2^0 + \cdots\right]$$

where

$$O_1^0 = G_{\mu\nu}^{0,a}\widetilde{G}^{0,a\mu\nu}, \qquad O_2^0 = \partial_\mu J_5^{0,\mu}, \qquad \text{with} \qquad J_5^{0,\mu} = \sum_{i=1}^{n_l} \bar{q}_i^0 \gamma^\mu \gamma_5 q_i^0$$

Now these must be renormalized.

- $J_5^{0,\mu}$ is the colour-singlet axial-vector current, which is renormalized multiplicatively; $\partial_\mu J_5^{0,\mu}$ likewise is renormalized multiplicatively.
- $G_{\mu\nu}^{0,a}\widetilde{G}^{0,a\mu\nu}$ mixes under renormalization: $\partial_\mu J_5^{0,\mu}$ feeds into $G_{\mu\nu}^{0,a}\widetilde{G}^{0,a\mu\nu}$ at one loop because you can close the quark loop.

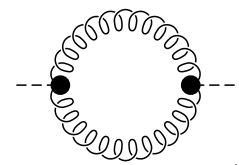
So we'll get:

$$\mathcal{L} = -\frac{A}{v}\left[C_1 O_1 + C_2 O_2 + \cdots\right], \qquad O_1 = Z_{11} O_1^0 + Z_{12} O_2^0, \qquad O_2 = Z_{22} O_2^0.$$

Now let's calculate $A \to gg$. The decay is the imaginary part of the $A \to A$ amplitude, which is described by correlators $\langle O_i O_j \rangle$:

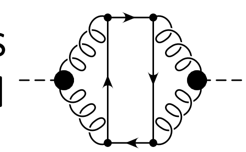
$$\Gamma(A \to gg) = \frac{\sqrt{2} G_F}{M_A} \left[C_1^2 \mathrm{Im}\langle O_1 O_1 \rangle + 2 C_1 C_2 \mathrm{Im}\langle O_1 O_2 \rangle + C_2^2 \langle O_2 O_2 \rangle \right]$$

- $\langle O_1 O_1 \rangle$ first appears at order α_s^0.
  - Diagram $\longrightarrow$



- $\langle O_1 O_2 \rangle$ first appears at order α_s^1.
  - Need to radiate a gluon from $q\bar{q}$ in O_2 and split a gluon into quarks in O_1.
- $\langle O_2 O_2 \rangle$ first appears at order α_s^2.
  - Kinematics kills $\langle O_2 O_2 \rangle$ at leading order for $m_q = 0$. Need to make two boxes and connect the gluons.

- C_1 starts at order α_s^1, since $A G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$ is generated by the top loop.
- C_2 starts at order α_s^2, since $A \partial_\mu J_5^\mu$ is generated at two loops by attaching a quark line to the gluons that were generated by the top loop.



$\chi\chi \to gg$:
- LO: want $C_1^2 \mathrm{Im}\langle O_1 O_1 \rangle$ at leading α_s^2 order.
  - This is just LO $A \to gg$.
- NLO: want $C_1^2 \mathrm{Im}\langle O_1 O_1 \rangle$ at NLO, α_s^3.
  - This is just NLO $A \to gg$.
- NNLO: want $C_1^2 \mathrm{Im}\langle O_1 O_1 \rangle$ at NNLO, α_s^4.
  - Cannot get this simply from $A \to gg$, since $C_1 C_2 \langle O_1 O_2 \rangle$ also contributes at this order.