Distinguishing extended Higgs models using coupling patterns

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Based on V. Barger, H.L., and G. Shaughnessy, arXiv:0901.nnnn

LHC will measure rates ($\sigma\times$ BR).

Fit $\longrightarrow$ extract Higgs couplings. [Dührssen et al; Tilman's talk; Ketevi's talk]

1) Look for a deviation from the Standard Model:
- Procedure is well defined
- "Reach" for 2σ exclusion, 5σ discovery (of a deviation) has been studied in a number of BSM Higgs models

2) Next step, if a deviation is detected, is to determine which model.
- Do parameter fits to "usual suspects." MSSM, Type-II 2HDM, ...
- But consistency $\neq$ discovery! How do we identify *all* models that are allowed or excluded by the data?

Need a strategy.

Strategy:

Our (extracted) observables are the Higgs couplings.

Each model makes a prediction for all couplings, as a function of the model parameters.

free model parameters $\leq$ # observables:
each model predicts a characteristic pattern of coupling relations.

Approach:
- Map out the "footprint" of every possible model in (multidimensional) observable space.
- Non-overlapping footprints mean models can be distinguished in principle.
- Uncertainties in parameter fit determine how well in practice.

"- Map out the "footprint" of every possible model in (multidimensional) observable space."

That's a tall order... let's start modestly.

Our approach: [V. Barger, H.L., and G. Shaughnessy, arXiv:0901.nnnn]

- Consider a single neutral CP-even Higgs state h and study its couplings. Ignore possibility of CP violation. Ignore what we may or may not know about other Higgs sector particles.

- Consider only models containing SU(2) doublets and singlets.

- Require natural flavour conservation: restricts possible forms of Yukawa Lagrangian.

Subject to these restrictions, we can:
- make a complete catalogue of models;
- identify which ones are distinguishable in principle; and
- give explicit procedures to distinguish one from the other.

Natural flavour conservation

Philosophy: absence of large Higgs-mediated flavour-changing neutral currents is due to symmetry structure of model, not tuning of parameters.

[Glashow & Weinberg, PRD15, 1958 (1977); Paschos, PRD15, 1966 (1977)]

SM: $\mathcal{L} \supset -Y_{ij}\bar{q}_{Ri}\Phi^\dagger Q_{Lj} \to -Y_{ij}v\,\bar{q}_{Ri}q_{Lj} - Y_{ij}\,h\bar{q}_{Ri}q_{Lj}$
Diagonalizing the fermion mass matrix $Y_{ij}v$ automatically diagonalizes the Higgs coupling matrix Y_{ij}: no FCNCs.

Two doublets: $\mathcal{L} \supset -Y_{1,ij}\bar{q}_{Ri}\Phi_1^\dagger Q_{Lj} - Y_{2,ij}\bar{q}_{Ri}\Phi_2^\dagger Q_{Lj}$
Mass term: $M_{ij} = Y_{1,ij}v_1 + Y_{2,ij}v_2$. Diagonalizing M_{ij} does not necessarily diagonalize Y_1 and Y_2: Higgs-mediated FCNCs.

FCNCs can be avoided if the mass matrix in each sector of fermions (up-type quarks, down-type quarks, or charged leptons) comes from coupling to exactly one Higgs doublet.

Examples:

Type-I 2HDM:

- One doublet Φ_f couples (and gives mass) to fermions; other doublet Φ_0 does not.
- Pattern can be enforced by Z_2 symmetry: $\Phi_0 \to -\Phi_0$, all other fields invariant (softly broken in Higgs potential).

Type-II 2HDM:

- One doublet Φ_u gives mass to up-type quarks; other doublet Φ_d gives mass to down-type quarks and charged leptons.
- Pattern can be enforced by Z_2 symmetry: $\Phi_u \to -\Phi_u$, $u_{Ri} \to -u_{Ri}$, all other fields invariant (again softly broken in Higgs potential).
- This pattern enforced in MSSM by holomorphicity of superpotential.

Note all 3 generations of fermions (of each sector) get their mass from the same Higgs.

Imposing natural flavour conservation divides all possible multi-doublet/singlet models into 5 classes.

1) Fermion masses from one doublet. Φ_f couples to all 3 sectors of fermions; any other doublets in the model do not couple to fermions.

2) Fermion masses from two doublets. There are 3 ways to assign the couplings:

a) Φ_u gives mass to up-type quarks; Φ_d gives mass to down-type quarks and charged leptons (Type-II 2HDM);

b) Φ_u gives mass to up-type quarks and charged leptons; Φ_d gives mass to down-type quarks (flipped 2HDM);

c) Φ_q gives mass to up- and down-type quarks; Φ_ℓ gives mass to charged leptons (lepton-specific 2HDM).

Any other doublets in the model do not couple to fermions.

3) Fermion masses from three doublets. Φ_u gives mass to up-type quarks; Φ_d gives mass to down-type quarks; Φ_ℓ gives mass to charged leptons. Any other doublets in the model do not couple to fermions.

Observables

Notation: "barred couplings" are normalized to their SM values:
$\bar{g}_x \equiv g_x / g_x^{SM}$ (coupling of h to $x\bar{x}$) compare Tilman's notation $g_x/g_x^{SM} = (1+\delta_x)$

Couplings to fermions: natural flavour conservation implies barred couplings are the same for all 3 generations within a fermion sector: $\bar{g}_u = \bar{g}_c = \bar{g}_t$. Same for d, s, b; same for e, μ, τ.

Models containing only Higgs doublets and/or singlets: custodial symmetry implies $\bar{g}_W = \bar{g}_Z$.

Will not consider loop-induced couplings hgg, $h\gamma\gamma$, $hZ\gamma$: other new physics can run in the loop; alternatively other dim-6 ops from higher-scale physics can have a big effect.

On the other hand, these loop induced couplings are the only place where we can get at the relative signs of the tree-level (dim-4) couplings. These signs are usually important for "solving" the model.

4 primary observables: $\bar{g}_W$, $\bar{g}_u$, $\bar{g}_d$, $\bar{g}_\ell$.

Framework

Define $h = \sum_i a_i \phi_i$ where $\phi_i \equiv \phi_i^{0,r}$ is the properly normalized real neutral component of doublet Φ_i or singlet S_i. $a_i \equiv \langle h | \phi_i \rangle$.

- Ignore CP violation: a_i are real.
- Normalization: $\sum_i a_i^2 = 1$.

W and Z mass generation: the vev is shared among the doublets.
Ignore singlet vevs: they do not affect h couplings.
Define $b_i \equiv v_i / v_{SM}$ (real and positive).

- Normalization: $\sum_i b_i^2 = 1$ to give correct W and Z masses.

Sum runs over doublets only.

$\sum_i b_i^2 = 1$ can also be seen as a normalization condition:
Define "Higgs basis" such that Φ_v carries v_{SM}: $\phi_v = \sum_i b_i \phi_i$
Then $b_i = \langle \phi_i | \phi_v \rangle$ and $\sum_i b_i^2 = 1$ is the normalization condition for ϕ_v.

Higgs couplings

Couplings to W or Z pairs:

$g_W^h = g_W^{SM} \langle h | \phi_v \rangle$ or $\bar{g}_W = \langle h | \phi_v \rangle$.

Inserting a complete set of states, $\bar{g}_W = \sum_i \langle h | \phi_i \rangle \langle \phi_i | \phi_v \rangle = \sum_i a_i b_i$.

Sum runs over doublets only; $b_i \equiv 0$ for singlets.

Couplings to fermions:

$\mathcal{L}_{Yuk} \supset -y_f \bar{f}_R \Phi_f^\dagger F_L +$h.c. which gives $m_f = y_f v_f / \sqrt{2} = y_f b_f v_{SM} / \sqrt{2}$.

$g_f^h = (y_f/\sqrt{2}) \langle h | \phi_f \rangle = (m_f / v_{SM})(a_f / b_f) = g_f^{SM} (a_f / b_f)$

So $\bar{g}_f = a_f / b_f = \langle h | \phi_f \rangle / \langle \phi_v | \phi_f \rangle$.

Decoupling limit: $\bar{g}_W = \bar{g}_f = 1$ when $h = \phi_v$.

Key feature 1: fermion couplings to h $\bar{g}_f = \langle h|\phi_f\rangle / \langle \phi_v|\phi_f\rangle$

1) Fermion masses from one doublet: $\bar{g}_u = \bar{g}_d = \bar{g}_\ell$

2) Fermion masses from two doublets:
  a) Type-II 2HDM–like: $\bar{g}_d = \bar{g}_\ell \neq \bar{g}_u$
  b) Flipped 2HDM–like: $\bar{g}_u = \bar{g}_\ell \neq \bar{g}_d$
  c) Lepton-specific 2HDM–like: $\bar{g}_u = \bar{g}_d \neq \bar{g}_\ell$

3) Fermion masses from three doublets: $\bar{g}_u \neq \bar{g}_d \neq \bar{g}_\ell$

Key feature 2: relation between $\bar{g}_W$ and the $\bar{g}_f$ $\bar{g}_W = \sum_i \langle h|\phi_i\rangle \langle \phi_i|\phi_v\rangle$

Sheds light on relation between ϕ_v and ϕ_f: are there extra doublets that do not couple to fermions?

Fermion masses from one doublet

1. SM

2. SM + singlet(s)

3. 2HDM-I (the SM plus a doublet)

4. 2HDM-I + singlet(s)

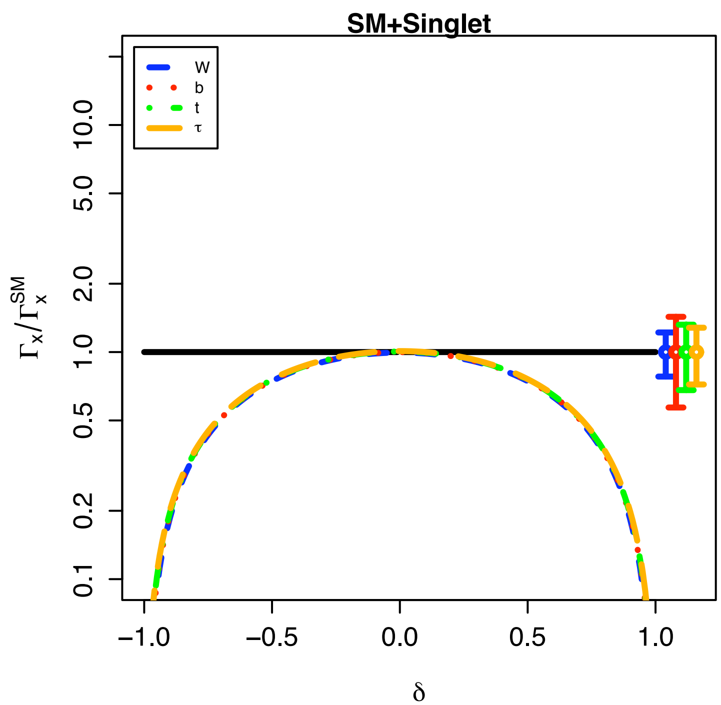
5. 2HDM-I + extra doublet(s)

SM + singlet(s)

Field content: 1 doublet Φ_f, 1 singlet S. $h = a_f \phi_f + a_s S$

Constraints: $b_f^2 = 1$; $a_f^2 + a_s^2 = 1 \rightarrow a_f = \sqrt{1 - a_s^2} \equiv \sqrt{1 - \delta^2}$.

Couplings: $\bar{g}_W = a_f b_f = \sqrt{1 - \delta^2}$, $\bar{g}_f = a_f / b_f = \sqrt{1 - \delta^2}$



Key signature: $\bar{g}_W = \bar{g}_f \leq 1$.

Inverse relations: $a_f = \bar{g}_W = \bar{g}_f$, $a_s = \sqrt{1 - a_f^2}$.

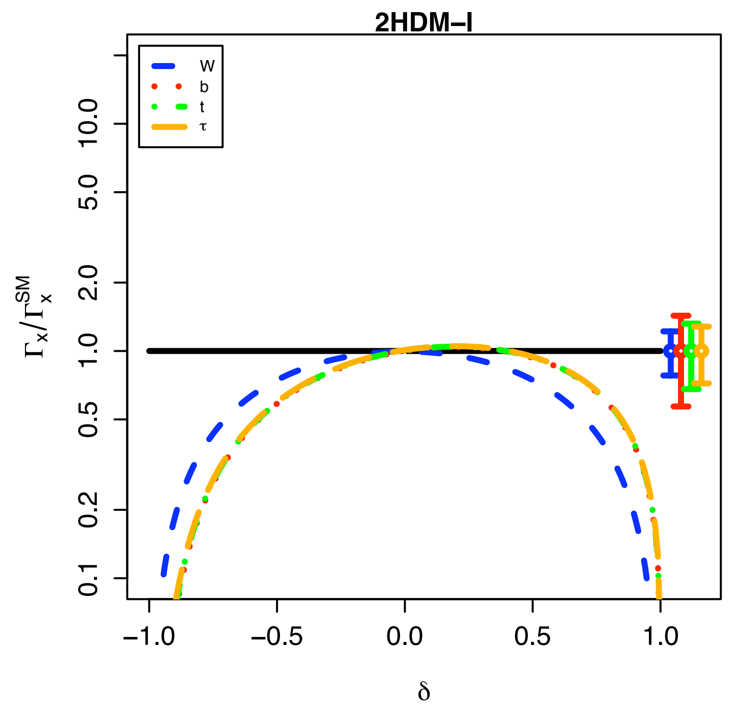
Multiple singlets: $a_s^2 \rightarrow \sum a_{s_i}^2$. No change in any h couplings. Can't determine number of singlets from h couplings.

2HDM-I

Field content: 1 doublet Φ_f couples to fermions; 2nd doublet Φ_0 does not.

Constraints: $a_f^2 + a_0^2 = 1$; $b_f^2 + b_0^2 = 1$ $\qquad h = a_f \phi_f + a_0 \phi_0$

Couplings: $\bar{g}_W = a_f b_f + a_0 b_0$; $\qquad \bar{g}_f = a_f / b_f$



Key signature: $\bar{g}_W \neq \bar{g}_f$;
$\bar{g}_u = \bar{g}_d = \bar{g}_\ell \equiv \bar{g}_f$.

Notation: $\tan\beta \equiv v_f/v_0 = b_f/b_0$,
$\delta \equiv \cos(\beta - \alpha) = a_f b_0 - a_0 b_f$.

$$\bar{g}_W = \sqrt{1-\delta^2}$$
$$\bar{g}_f = \sqrt{1-\delta^2} + \cot\beta\ \delta$$

Plot: $\tan\beta = 5$

2HDM-I

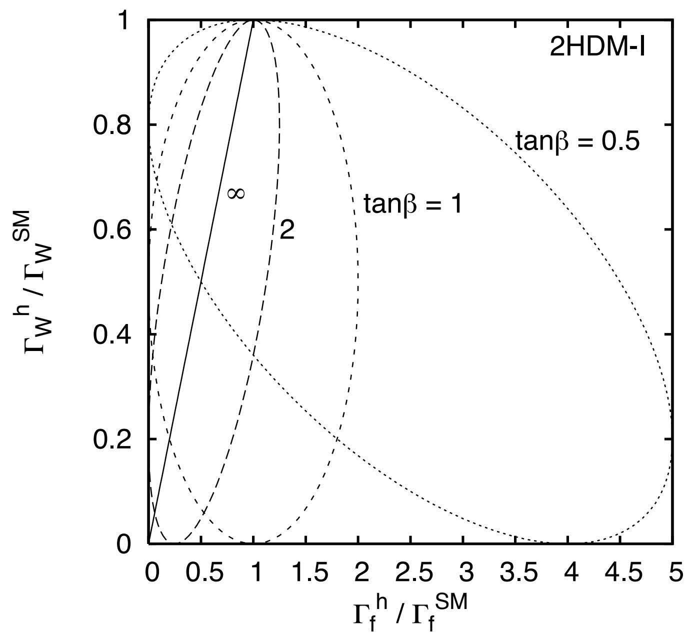
Inverse relations:

$$b_f = \left[\frac{1-\bar{g}_W^2}{1+\bar{g}_f^2-2\bar{g}_W\bar{g}_f}\right]^{1/2}, \qquad b_0 = \sqrt{1-b_f^2}$$

$$a_f = b_f\bar{g}_f\,, \qquad a_0 = \frac{\bar{g}_W - b_f^2\bar{g}_f}{\sqrt{1-b_f^2}}$$

Get a full, unique solution if relative signs of $\bar{g}_W$ and $\bar{g}_f$ are known.

If relative signs are not known, solution is 2-fold degenerate.

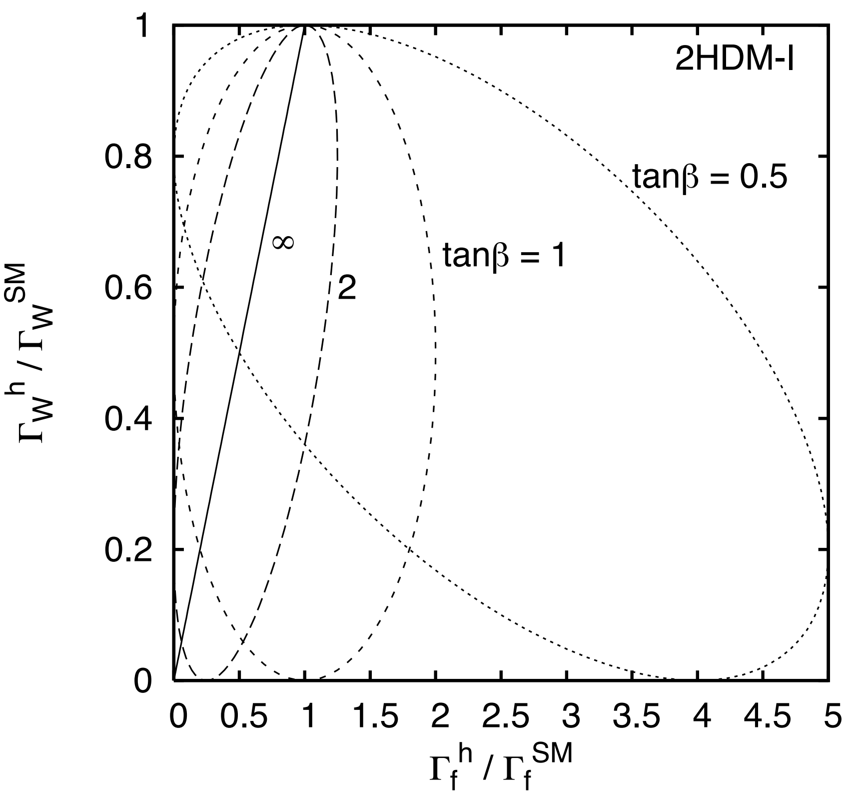


2HDM-I

Note "footprints":

- 2HDM-I populates the plane.

- SM + singlet(s) collapses to $\tan\beta \to \infty$ line (corresponds to $b_0 = 0$).



2HDM-I + singlet(s)

Constraints: $a_f^2 + a_0^2 + a_s^2 = 1$; $b_f^2 + b_0^2 = 1$

$h = a_f \phi_f + a_0 \phi_0 + a_s S$

Couplings: $\bar{g}_W = a_f b_f + a_0 b_0$; $\bar{g}_f = a_f / b_f$

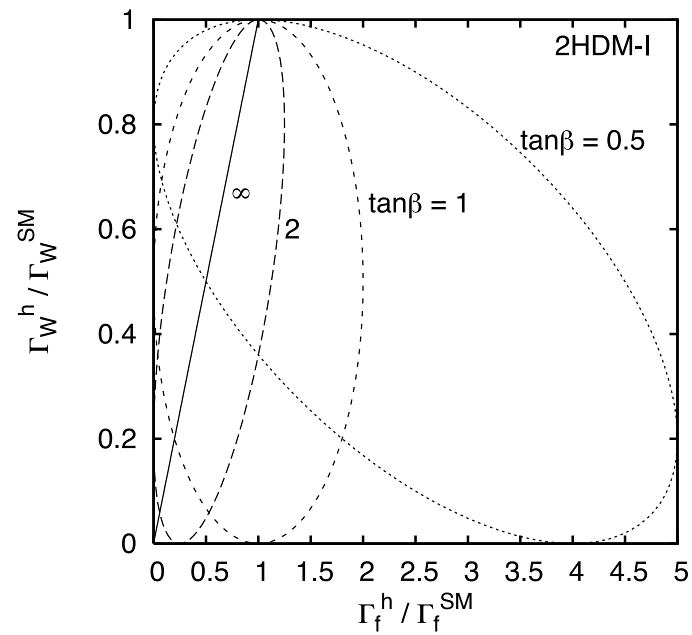
Multiple singlets:
$a_0^2 \to \sum a_{0i}^2$

5 parameters but only 4 equations: no unique solution!

Parameterize singlet mixing: $\xi \equiv 1 - a_s^2 = a_f^2 + a_0^2$

$$\bar{g}_W = \sqrt{\xi}\,\sqrt{1-\delta^2}$$
$$\bar{g}_f = \sqrt{\xi}\left[\sqrt{1-\delta^2} + \cot\beta\ \delta\right]$$

Compare 2HDM-I:
- Footprints are the same.
- Can't tell the models apart based on h couplings.
- Inverse relations will give a solution but it will be wrong.



2HDM-I + extra doublet(s)

$$h = a_f \phi_f + \sum_i a_{0i} \phi_{0i} = a_f \phi_f + a'_0 \phi'_0, \qquad a_f^2 + a_0'^2 = 1.$$

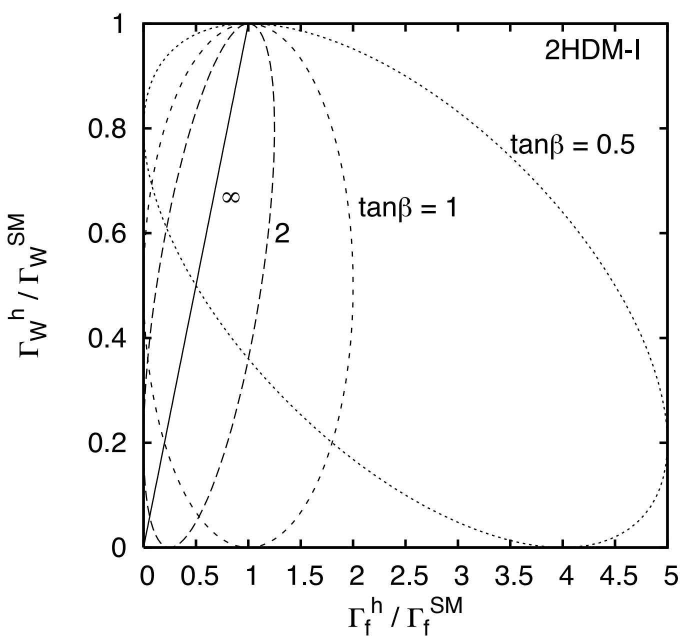
$$b'_0 \equiv \langle \phi'_0 | \phi_v \rangle \quad \rightarrow \quad b_f^2 + b_0'^2 = \omega^2 \leq 1$$

Some vev can be carried by the combination of ϕ_{0i} orthogonal to h ("vev sharing"). 5 params, 4 eqns → no unique solution.

$$\bar{g}_W = \omega \sqrt{1 - \delta^2}$$
$$\bar{g}_f = (1/\omega) \left[\sqrt{1 - \delta^2} + \cot\beta \; \delta \right]$$

Compare 2HDM-I:

- Footprints are the same.
- Can't tell the models apart based on h couplings.
- Inverse relations will give a solution but it will be wrong.



Fermion masses from two doublets

3 ways to couple fermions:

1. 2HDM-II

2. Flipped 2HDM

3. Lepton-specific 2HDM

Extensions:
- singlet(s)
- extra doublet(s)

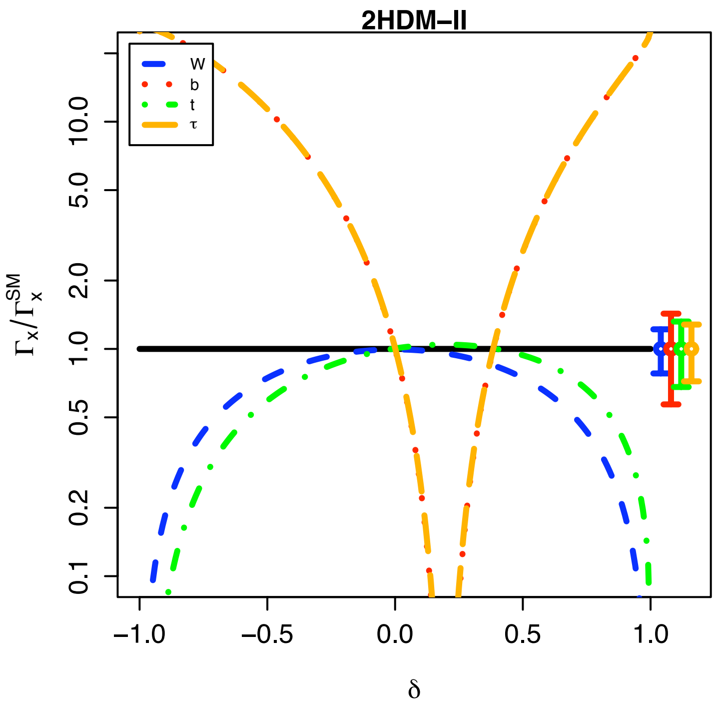
MSSM (violation of natural flavour conservation assumption)

2HDM-II

Field content: 1 doublet Φ_u gives mass to up-type quarks; 2nd doublet Φ_d gives mass to down-type quarks and charged leptons.

Constraints: $a_u^2 + a_d^2 = 1$; $b_u^2 + b_d^2 = 1$ $h = a_u \phi_u + a_d \phi_d$

Couplings: $\bar{g}_W = a_u b_u + a_d b_d$; $\bar{g}_u = a_u / b_u$; $\bar{g}_d = \bar{g}_\ell = a_d / b_d$



Key signature: $\bar{g}_d = \bar{g}_\ell \neq \bar{g}_u$

Notation: $\tan\beta \equiv v_u / v_d = b_u / b_d$,
$\delta \equiv \cos(\beta - \alpha) = a_u b_d - a_d b_u$.

$$\bar{g}_W = \sqrt{1 - \delta^2}$$
$$\bar{g}_u = \sqrt{1 - \delta^2} + \cot\beta\ \delta$$
$$\bar{g}_d = \bar{g}_\ell = \sqrt{1 - \delta^2} - \tan\beta\ \delta$$

Plot: $\tan\beta = 5$

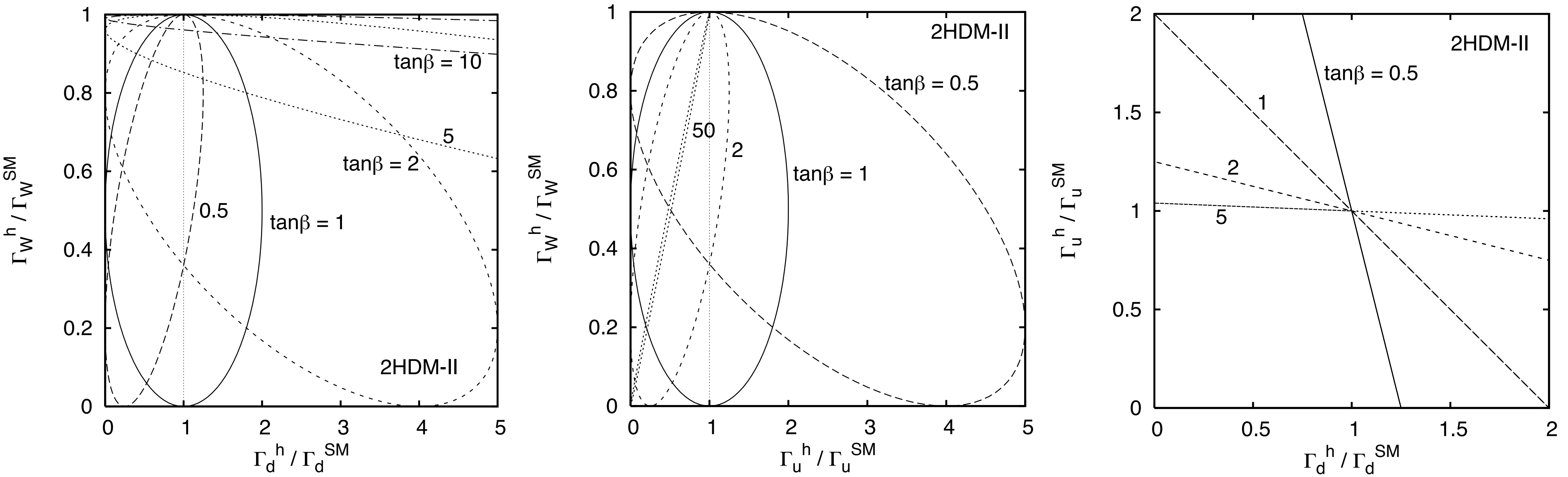
2HDM-II

3 different couplings $(\bar{g}_W, \bar{g}_u, \bar{g}_d)$ controlled by only 2 parameters $(\tan\beta, \delta)$: model occupies a 2-dim subspace of 3-dim coupling space.

Key signature: "pattern relation" [Ginzburg, Krawczyk & Osland 2001]

$$P_{ud} \equiv \bar{g}_W(\bar{g}_u + \bar{g}_d) - \bar{g}_u\bar{g}_d = 1$$

equiv patt reln $P_{u\ell} = 1$



2HDM-II

Inverse relations:

$$b_u = \left[\frac{\bar{g}_W - \bar{g}_d}{\bar{g}_u - \bar{g}_d}\right]^{1/2} = \left[\frac{1 - \bar{g}_d^2}{\bar{g}_u^2 - \bar{g}_d^2}\right]^{1/2} \qquad a_u = b_u \bar{g}_u$$

$$b_d = \left[\frac{\bar{g}_W - \bar{g}_u}{\bar{g}_d - \bar{g}_u}\right]^{1/2} = \left[\frac{1 - \bar{g}_u^2}{\bar{g}_d^2 - \bar{g}_u^2}\right]^{1/2} \qquad a_d = b_d \bar{g}_d$$

Unique solution for b_u, b_d even if relative signs of couplings are not known (used pattern relation).

2HDM-II + singlet(s)

Constraints: $a_f^2 + a_0^2 + a_s^2 = 1; \;\; b_f^2 + b_0^2 = 1$

Multiple singlets: $a_0^2 \to \sum a_{0i}^2$

Couplings: $\bar{g}_W = a_f b_f + a_0 b_0; \;\; \bar{g}_f = a_f / b_f$

Parameterize singlet mixing: $\xi \equiv 1 - a_s^2 = a_f^2 + a_0^2$

Couplings:

$$\bar{g}_W = \sqrt{\xi}\sqrt{1-\delta^2}$$

$$\bar{g}_u = \sqrt{\xi}\left[\sqrt{1-\delta^2} + \cot\beta\ \delta\right]$$

$$\bar{g}_d = \bar{g}_\ell = \sqrt{\xi}\left[\sqrt{1-\delta^2} - \tan\beta\ \delta\right]$$

Distinguishable from 2HDM-II using pattern relation!

$$P_{ud} \equiv \bar{g}_W(\bar{g}_u + \bar{g}_d) - \bar{g}_u \bar{g}_d = \xi \leq 1$$

"Footprint": model fills volume in 3-dim coupling space between 2HDM-II surface ($P_{ud} = 1$) and origin ($\bar{g}_W = \bar{g}_u = \bar{g}_d = 0$).

2HDM-II + singlet(s)

Inverse relations:

$$b_u = \left[\frac{\bar{g}_W - \bar{g}_d}{\bar{g}_u - \bar{g}_d}\right]^{1/2} = \left[\frac{\xi - \bar{g}_d^2}{\bar{g}_u^2 - \bar{g}_d^2}\right]^{1/2} \qquad a_u = b_u \bar{g}_u$$

$$b_d = \left[\frac{\bar{g}_W - \bar{g}_u}{\bar{g}_d - \bar{g}_u}\right]^{1/2} = \left[\frac{\xi - \bar{g}_u^2}{\bar{g}_d^2 - \bar{g}_u^2}\right]^{1/2} \qquad a_d = b_d \bar{g}_d$$

$$a_s = \sqrt{1 - \xi}$$

Unique solutions for all parameters if relative signs of couplings are known (use pattern relation to get ξ).

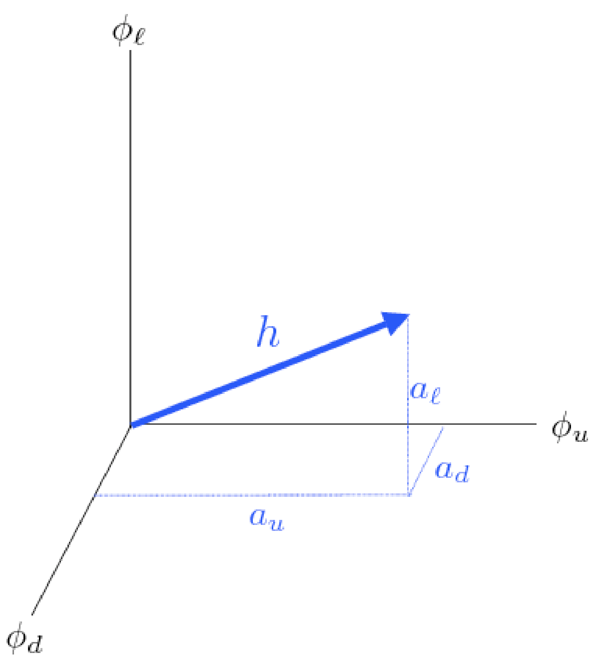
If signs are not known, get discrete ambiguities.

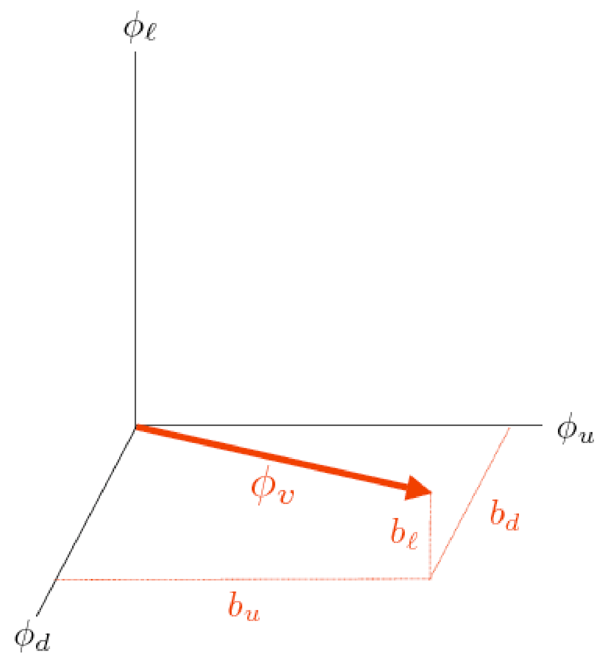
2HDM-II + extra doublet(s)

Constraints: $a_u^2 + a_d^2 + a_0^2 = 1$; $b_u^2 + b_d^2 + b_0^2 = 1$

Couplings: $\bar{g}_W = a_u b_u + a_d b_d + a_0 b_0$; $\bar{g}_u = a_u / b_u$; $\bar{g}_d = \bar{g}_\ell = a_d / b_d$

Physical picture:





$$\bar{g}_W = \langle h | \phi_v \rangle$$

$$\bar{g}_u = \langle h | \phi_u \rangle / \langle \phi_v | \phi_u \rangle$$

$$\bar{g}_d = \bar{g}_\ell = \langle h | \phi_d \rangle / \langle \phi_v | \phi_d \rangle$$

2HDM-II + extra doublet(s)

Limiting cases:

1) When $b_0 \to 0$, 3rd doublet "acts like a singlet": it can mix into h, but does not couple to fermions or gauge bosons. Duplicates 2HDM-II + singlet(s) ($P_{ud} \leq 1$):

$$\bar{g}_W = \sqrt{\xi}\sqrt{1-\delta^2} \qquad \bar{g}_u = \sqrt{\xi}\left[\sqrt{1-\delta^2} + \cot\beta\ \delta\right]$$

$$\bar{g}_d = \bar{g}_\ell = \sqrt{\xi}\left[\sqrt{1-\delta^2} - \tan\beta\ \delta\right]$$

2) When $a_0 \to 0$, 3rd doublet serves to reduce the vev carried by the doublets that constitute h. Similar to 2HDM-I + extra doublet(s):

$$\bar{g}_W = \omega\sqrt{1-\delta^2} \qquad \bar{g}_u = (1/\omega)\left[\sqrt{1-\delta^2} + \cot\beta\ \delta\right]$$

$$\bar{g}_d = \bar{g}_\ell = (1/\omega)\left[\sqrt{1-\delta^2} - \tan\beta\ \delta\right]$$

P_{ud} can be > 1 or < 0.

Footprint is larger than 2HDM-II + singlet(s).

Other fermion coupling structures

[Barnett et al; Grossman]

2HDM-II: $\Phi_u \leftrightarrow u, \quad \Phi_d \leftrightarrow d, \ell$

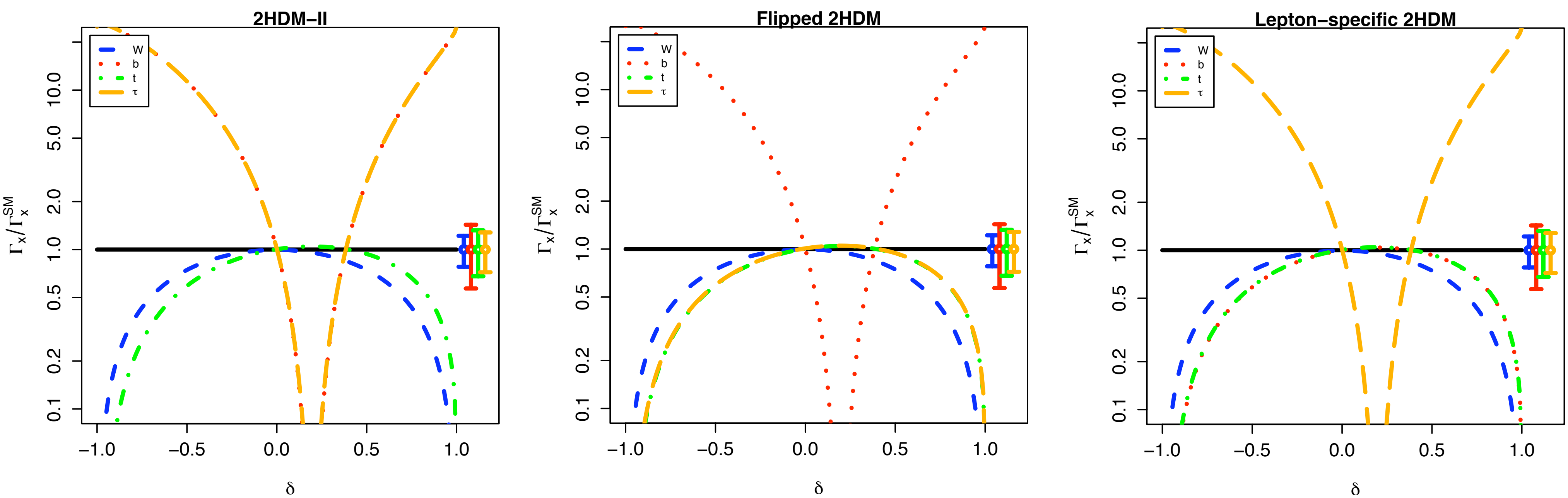
Pattern reln: $P_{ud} \equiv \bar{g}_W(\bar{g}_u + \bar{g}_d) - \bar{g}_u \bar{g}_d = 1 = P_{u\ell}$

Flipped 2HDM: $\Phi_u \leftrightarrow u, \ell, \quad \Phi_d \leftrightarrow d$

Pattern reln: $P_{ud} \equiv \bar{g}_W(\bar{g}_u + \bar{g}_d) - \bar{g}_u \bar{g}_d = 1 = P_{\ell d}$

Lepton-specific 2HDM: $\Phi_q \leftrightarrow u, d, \quad \Phi_\ell \leftrightarrow \ell$

Pattern reln: $P_{u\ell} \equiv \bar{g}_W(\bar{g}_u + \bar{g}_\ell) - \bar{g}_u \bar{g}_\ell = 1 = P_{d\ell}$



MSSM

At tree level, MSSM Higgs sector = 2HDM-II.

Beyond tree level, sbottom-gluino and stop-chargino loops can induce a coupling of ϕ_u to $b\bar{b}$. Violates natural flavour conservation.

Correction to b quark mass parameterized as
$m_b = (y_b v_{SM}/\sqrt{2}) \cos\beta \, (1 + \Delta_b)$

$hb\bar{b}$ coupling is modified compared to 2HDM-II:

$$\bar{g}_b \;=\; \sqrt{1-\delta^2} - \tan\beta\,\delta \left[\frac{1-\cot^2\beta\,\Delta_b}{1+\Delta_b}\right]$$

SUSY corrections to other couplings are small, neglect them:

$$\bar{g}_W = \sqrt{1-\delta^2}, \qquad \bar{g}_u = \sqrt{1-\delta^2} + \cot\beta\,\delta,$$

$$\bar{g}_\ell = \sqrt{1-\delta^2} - \tan\beta\,\delta$$

MSSM

Key features:

1) $\bar{g}_b \neq \bar{g}_\ell$

2) But, 2HDM-II pattern relation still holds among W, u, and ℓ couplings: $P_{u\ell} = \bar{g}_W(\bar{g}_u + \bar{g}_\ell) - \bar{g}_u\bar{g}_\ell = 1$.

Inverse relations:

- Solve for 2HDM-II parameters using $\bar{g}_W$, $\bar{g}_u$, and $\bar{g}_\ell$.

- Get Δ_b from $\Delta_b = (\bar{g}_b - \bar{g}_\ell)/(\bar{g}_u - \bar{g}_b)$.

Fermion masses from three doublets

1. Democratic 3HDM

2. 3HDM-D + singlet(s)

3. 3HDM-D + extra doublet(s)

Democratic 3HDM

Field content:

1 doublet Φ_u gives mass to up-type quarks;

2nd doublet Φ_d gives mass to down-type quarks;

3rd doublet Φ_ℓ gives mass to charged leptons.

Constraints: $a_u^2 + a_d^2 + a_\ell^2 = 1, \qquad b_u^2 + b_d^2 + b_\ell^2 = 1$

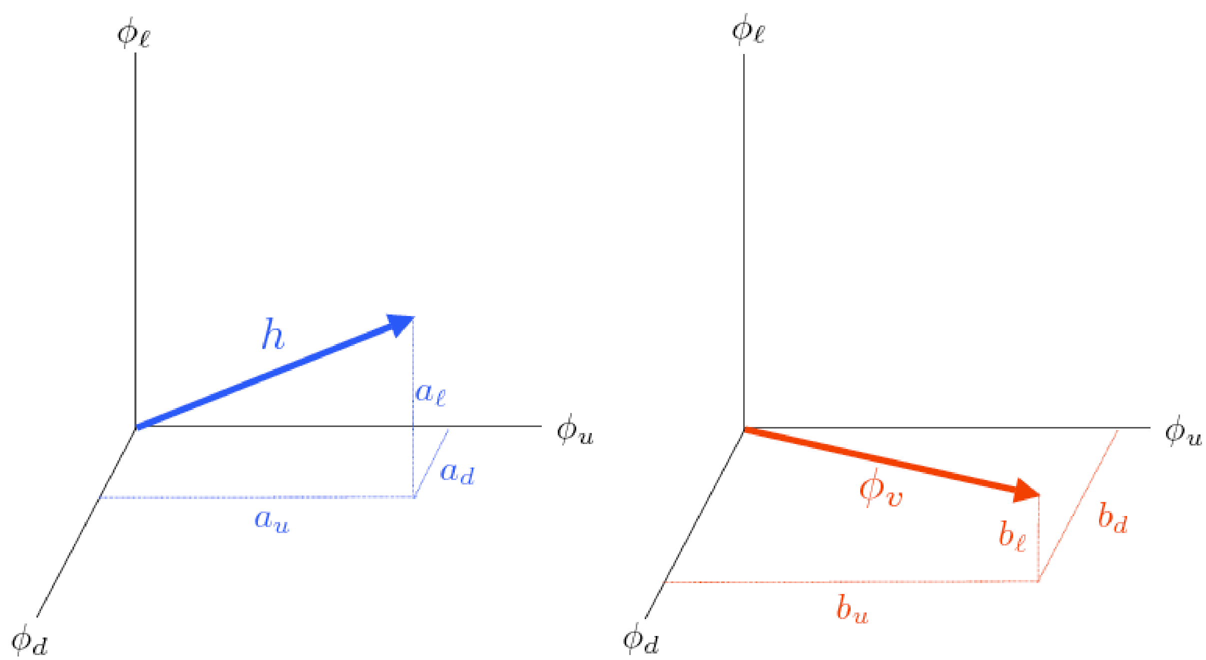
Couplings: $\bar{g}_W = a_u b_u + a_d b_d + a_\ell b_\ell$

$$\bar{g}_u = a_u / b_u, \quad \bar{g}_d = a_d / b_d, \quad \bar{g}_\ell = a_\ell / b_\ell$$

One key feature: $\bar{g}_u \neq \bar{g}_d \neq \bar{g}_\ell$ and MSSM pattern relation is not satisfied.

Democratic 3HDM

Analysis quite similar to 2HDM-II + extra doublet:



$$\bar{g}_W = \langle h|\phi_v\rangle$$

$$\bar{g}_u = \langle h|\phi_u\rangle / \langle\phi_v|\phi_u\rangle$$

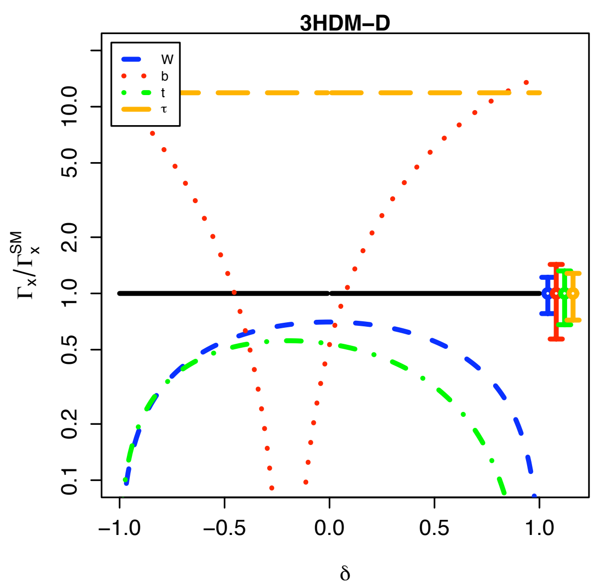
$$\bar{g}_d = \langle h|\phi_d\rangle / \langle\phi_v|\phi_d\rangle$$

$$\bar{g}_\ell = \langle h|\phi_\ell\rangle / \langle\phi_v|\phi_\ell\rangle$$

Democratic 3HDM

Couplings: Can write 4 couplings in terms of 4 parameters

$$\begin{aligned}
\bar{g}_W &= \sqrt{1-\delta^2} \\
\bar{g}_u &= \sqrt{1-\delta^2} + \delta \left[\sin\gamma \frac{\cos\beta}{\cos\Omega} - \cos\gamma \tan\Omega \right] \\
\bar{g}_d &= \sqrt{1-\delta^2} + \delta \left[-\sin\gamma \frac{\tan\beta}{\cos\Omega} - \cos\gamma \tan\Omega \right] \\
\bar{g}_\ell &= \sqrt{1-\delta^2} + \delta \left[\cos\gamma \cot\Omega \right]
\end{aligned}$$



Notation:

$\tan\beta = v_u/v_d = b_u/b_d$

$\sin\Omega = b_\ell$

$\delta = \sin(\text{angle between } h \text{ and } \phi_v)$

$\gamma = \text{azimuthal angle of } h \text{ about } \phi_v \text{ axis}$

Plot: $\tan\beta = 5$, $b_\ell = 0.2$, $a_\ell = 1/\sqrt{2}$

Democratic 3HDM

Inverse relations:

$$b_u = \left[\frac{1 - \bar{g}_W(\bar{g}_d + \bar{g}_\ell) + \bar{g}_d\bar{g}_\ell}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)}\right]^{1/2}$$

$$b_d = \left[\frac{1 - \bar{g}_W(\bar{g}_u + \bar{g}_\ell) + \bar{g}_u\bar{g}_\ell}{(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)}\right]^{1/2}$$

$$b_\ell = \left[\frac{1 - \bar{g}_W(\bar{g}_u + \bar{g}_d) + \bar{g}_u\bar{g}_d}{(\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)}\right]^{1/2}$$

$$a_u = b_u\bar{g}_u, \qquad a_d = b_d\bar{g}_d, \qquad a_\ell = b_\ell\bar{g}_\ell$$

If relative signs of couplings are known then the solution is unique; otherwise there are discrete ambiguities.

Democratic 3HDM + singlet(s) or extra doublet(s)

Key to this analysis is the inverse relations for b_i in terms of couplings in democratic 3HDM.

Consider the combinations of couplings:

$$\begin{aligned} X_u &= \left[\frac{1 - \bar{g}_W(\bar{g}_d + \bar{g}_\ell) + \bar{g}_d \bar{g}_\ell}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)}\right] \\ X_d &= \left[\frac{1 - \bar{g}_W(\bar{g}_u + \bar{g}_\ell) + \bar{g}_u \bar{g}_\ell}{(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)}\right] \\ X_\ell &= \left[\frac{1 - \bar{g}_W(\bar{g}_u + \bar{g}_d) + \bar{g}_u \bar{g}_d}{(\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)}\right] \end{aligned}$$

By construction, $X_u + X_d + X_\ell = 1$.

In democratic 3HDM, $X_i = b_i^2$, so $0 \leq X_i \leq 1$.

Democratic 3HDM + singlet(s) or extra doublet(s)

In democratic 3HDM + singlet,

$$\begin{aligned} X_u &= b_u^2 + \frac{a_s^2}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)} \\ X_d &= b_d^2 + \frac{a_s^2}{(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)} \\ X_\ell &= b_\ell^2 + \frac{a_s^2}{(\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)} \end{aligned}$$

Exactly one of the three denominators must be negative.
$\longrightarrow$ In part of the parameter space one of the X_i can be negative (impossible for b_i^2).

This means the footprint of this model is larger than that of the democratic 3HDM: the models are distinguishable (in part of the parameter space).

(Adding additional singlets: $a_s^2 \to \sum a_{si}^2$, footprint stays the same.)

Democratic 3HDM + singlet(s) or extra doublet(s)

If one of the X_i is negative, we can also get a lower bound on a_s (the singlet content of h).

Define

$$
\begin{aligned}
Y &= \begin{cases} (\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)X_u & \text{if } X_u < 0, \\ (\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)X_d & \text{if } X_d < 0, \\ (\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)X_\ell & \text{if } X_\ell < 0. \end{cases} \\
&= a_s^2 + (\text{a negative number}) \times b_i^2 \\
&\leq a_s^2.
\end{aligned}
$$

Note $0 < Y \leq 1$ by construction.

Democratic 3HDM + singlet(s) or extra doublet(s)

In democratic 3HDM + extra doublet,

$$X_u = b_u^2 + \frac{a_0^2 + b_0^2 \bar{g}_d \bar{g}_\ell - a_0 b_0 (\bar{g}_d + \bar{g}_\ell)}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)}$$

$$X_d = b_d^2 + \frac{a_0^2 + b_0^2 \bar{g}_u \bar{g}_\ell - a_0 b_0 (\bar{g}_u + \bar{g}_\ell)}{(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)}$$

$$X_\ell = b_\ell^2 + \frac{a_0^2 + b_0^2 \bar{g}_u \bar{g}_\ell - a_0 b_0 (\bar{g}_u + \bar{g}_d)}{(\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)}$$

- If $b_0 \to 0$, this reduces to same form as 3HDM + singlet.
- If $b_0 \neq 0$, numerator of 2nd term can be < 0 or > 1.

Define Y as before. In part of parameter space can get $Y < 0$; in other parts can get $Y > 1$. Impossible in 3HDM + singlet.

Thus footprint of 3HDM + extra doublet is larger than the other models.

(Adding even more doublets or singlets: footprint stays the same.)

Future directions

1) Experimental prospects.
We studied the theoretical "footprints": which models can be distinguished *in principle*.
Obvious next step: how well will experiment do?

2) Going beyond restrictive assumptions.
- SU(2) multiplets larger than doublets – must be careful with ρ parameter. Triplet models, ...
- Models without natural flavour conservation – must be careful with FCNCs. Type-III 2HDM, "Private Higgs," ...
- Impact of electroweak radiative corrections. QCD corr's universal

3) Adding observables from other Higgs states.
- Additional neutral CP-even states (coupling sum rules!)
- CP-odd states; CP mixtures
- Charged Higgses