Custodial symmetry violation in the Georgi-Machacek model

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based on work with Ben Keeshan and Terry Pilkington

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Motivation

Georgi & Machacek 1985; Chanowitz & Golden 1985

The Georgi-Machacek model is a nice benchmark for studying contributions to electroweak symmetry breaking from scalars in SU(2)$_L$ representations larger than the doublet.

- Extends SM by one real and one complex triplet.

- Avoids stringent constraints on triplet vevs from ρ parameter by imposing global SU(2)$_L\times$SU(2)$_R$ symmetry on Higgs potential.

$\Rightarrow$ SM-like Higgs boson can have $\kappa_V > 1$. favoured at 1.4σ by ATLAS...
1909.02845

$\Rightarrow$ Spectrum of new scalars preserves custodial symmetry.

$\Rightarrow$ Interesting phenomenology of custodial fiveplet states $(H_5^{++}, H_5^+, H_5^0, H_5^-, H_5^{--})$: fermiophobic, couple to VV at tree level $\propto v_\chi$; nice VBF signatures.

Motivation

The GM model has a long-known problem when going beyond tree level:

Global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is broken by gauging hypercharge!

Gunion, Vega & Wudka 1991

True also in the SM, but not a problem because global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is an accidental symmetry of the Higgs potential: 1-loop corrections to ρ parameter (i.e., T) are finite and calculable.

In the GM model, the global-$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$-preserving relations among params of *full* gauge-invariant scalar potential can only hold at one energy scale: violated by running due to hypercharge. Garcia-Pepin, Gori, Quiros, Vega, Vega-Morales, Yu 2014

Need the UV completion to be nearby.

What are the implications?

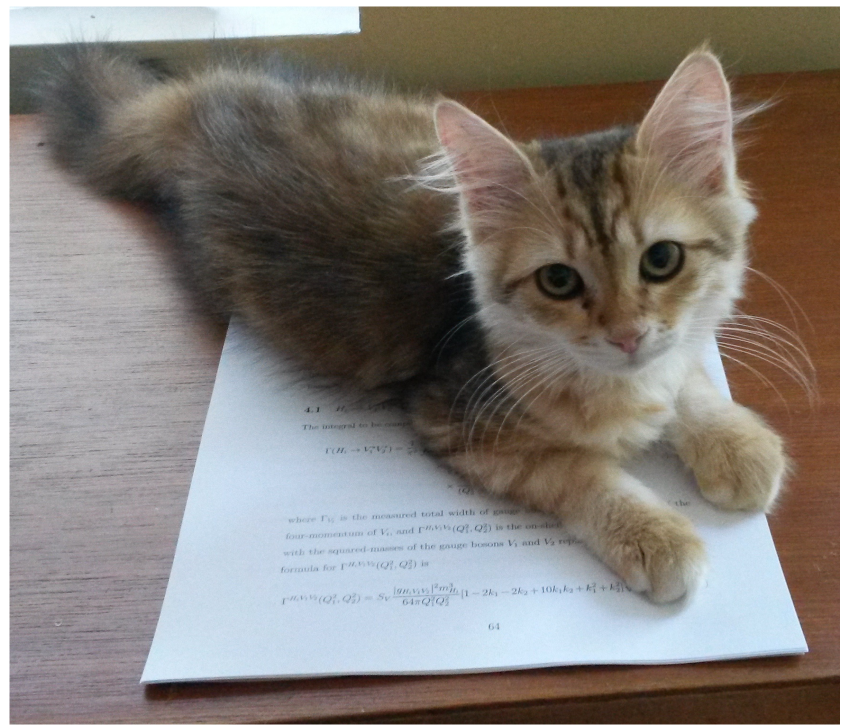
Outline

Georgi-Machacek model and its phenomenology

Loop-induced custodial symmetry violation

Our implementation and results

Conclusions and outlook



Georgi-Machacek model

Georgi & Machacek 1985; Chanowitz & Golden 1985

SM Higgs bidoublet + the two triplets in a bitriplet:

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \qquad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

Impose a global SU(2)$_L\times$SU(2)$_R$ and write down the scalar potential (this is not the most general gauge invariant potential):

$$\begin{aligned} V(\Phi, X) \;=\; & \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 \\ & +\lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) + \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) \\ & +\lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \\ & -M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab} \end{aligned}$$

9 parameters, 2 fixed by G_F and m_h $\rightarrow$ 7 free parameters. Aoki & Kanemura, 0712.4053

Chiang & Yagyu, 1211.2658; Chiang, Kuo & Yagyu, 1307.7526

Hartling, Kumar & HEL, 1404.2640

Spontaneous symmetry breaking can be achieved preserving the custodial SU(2) $\rightarrow \langle X\rangle = v_\chi \times I_{3\times 3}$, so $v_\xi = v_\chi$ naturally!

Georgi-Machacek model

Georgi & Machacek 1985; Chanowitz & Golden 1985

SM Higgs bidoublet + the two triplets in a bitriplet:

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \qquad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

Physical spectrum controlled by transformation under SU(2)$_L\times$SU(2)$_R \to$ SU(2)$_{\text{custodial}}$:

Bidoublet: $2 \otimes 2 \to 1 \oplus 3$ Bitriplet: $3 \otimes 3 \to 1 \oplus 3 \oplus 5$

- Two custodial singlets mix $\to h^0$, H^0 m_h, m_H, angle α
  Usually identify $h^0 = h(125)$ $\lambda_{WZ} = 1$

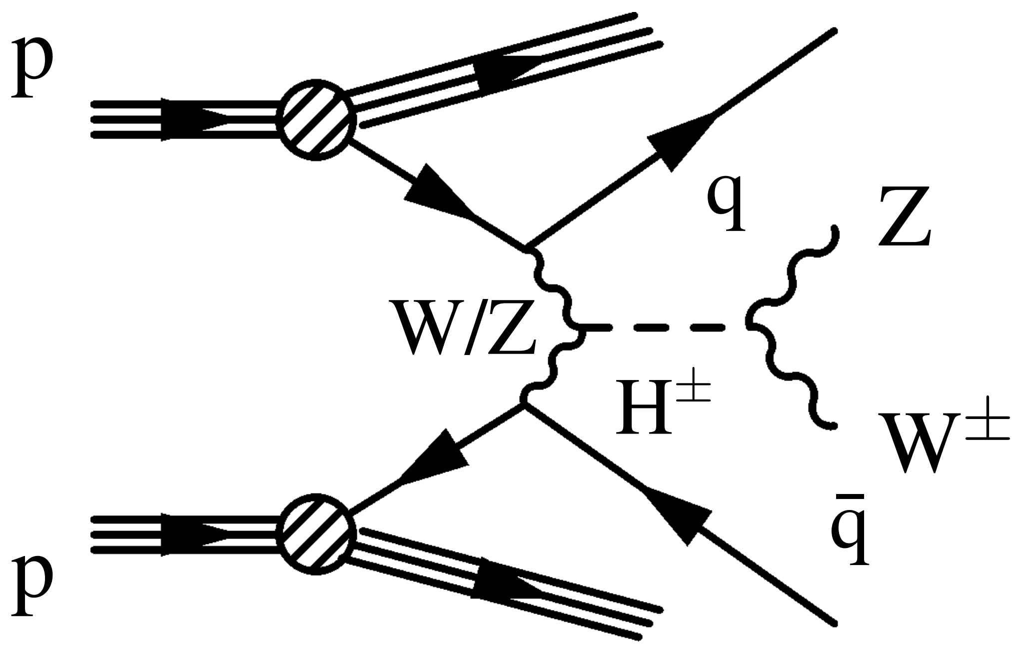
- Two custodial triplets mix $\to (H_3^+, H_3^0, H_3^-)$ m_3 + Goldstones
  Phenomenology very similar to $H^\pm, A^0$ in 2HDM Type I, $\tan\beta \to \cot\theta_H$

- Custodial fiveplet $(H_5^{++}, H_5^+, H_5^0, H_5^-, H_5^{--})$ m_5
  Fermiophobic; H_5VV couplings $\propto s_H \equiv \sqrt{8}v_\chi/v_{\text{SM}}$
  $\lambda_{WZ} = -1/2$ for H_5^0 $s_H^2 \equiv$ exotic fraction of M_W^2, M_Z^2

Smoking-gun processes involve $(H_5^{++}, H_5^{+}, H_5^{0}, H_5^{-}, H_5^{--})$:

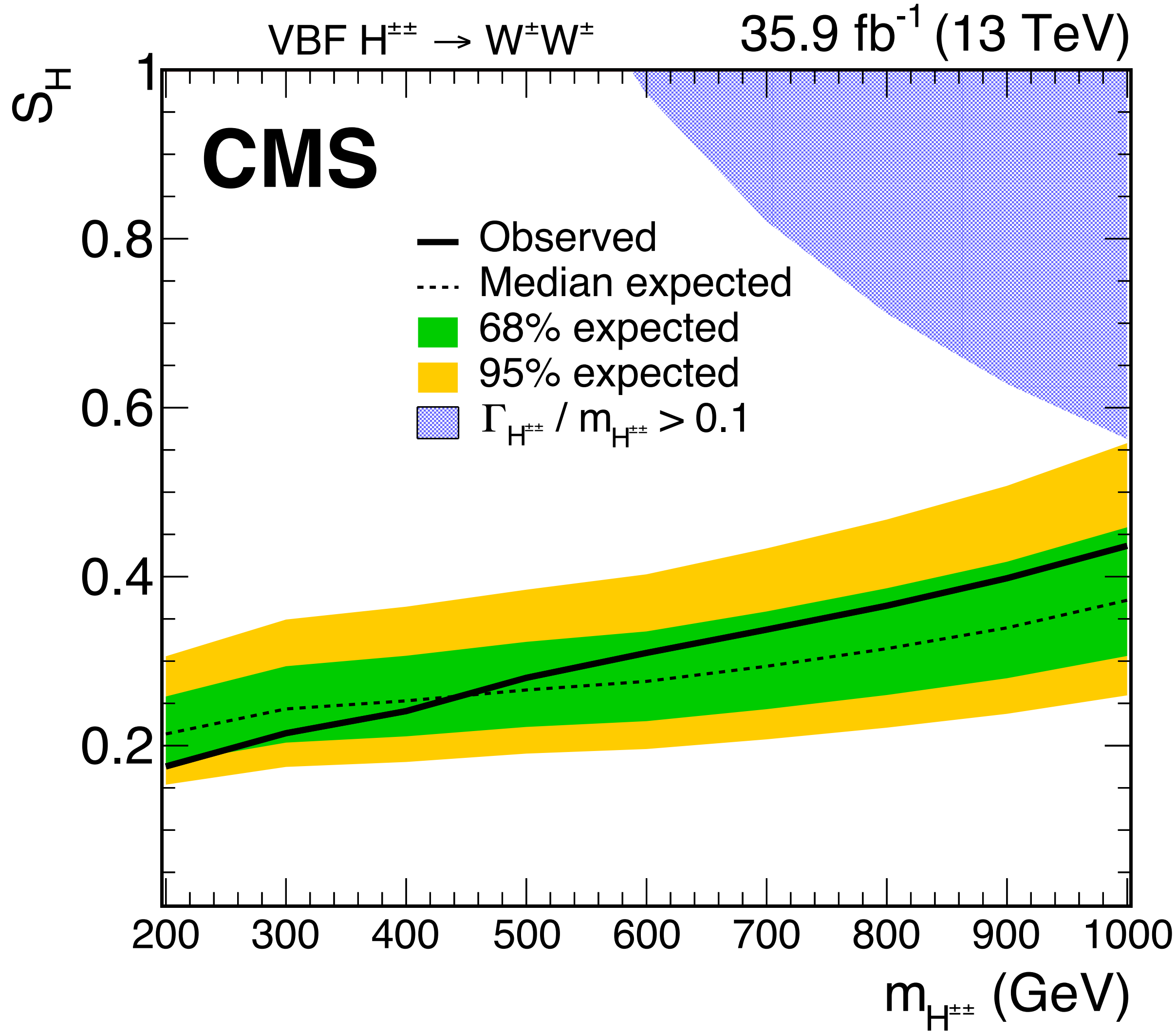
VBF $\to H_5^{\pm\pm} \to W^\pm W^\pm$ VBF + like-sign dileptons + MET

VBF $\to H_5^{\pm} \to W^\pm Z$ VBF + $qq\ell\ell$; VBF + 3ℓ + MET



Cross section $\propto s_H^2 \equiv$ fraction of M_W^2, M_Z^2 due to exotic scalars.

Most stringent constraint: VBF $\to H_5^{\pm\pm} \to W^\pm W^\pm \to \ell^\pm\ell^\pm$ MET

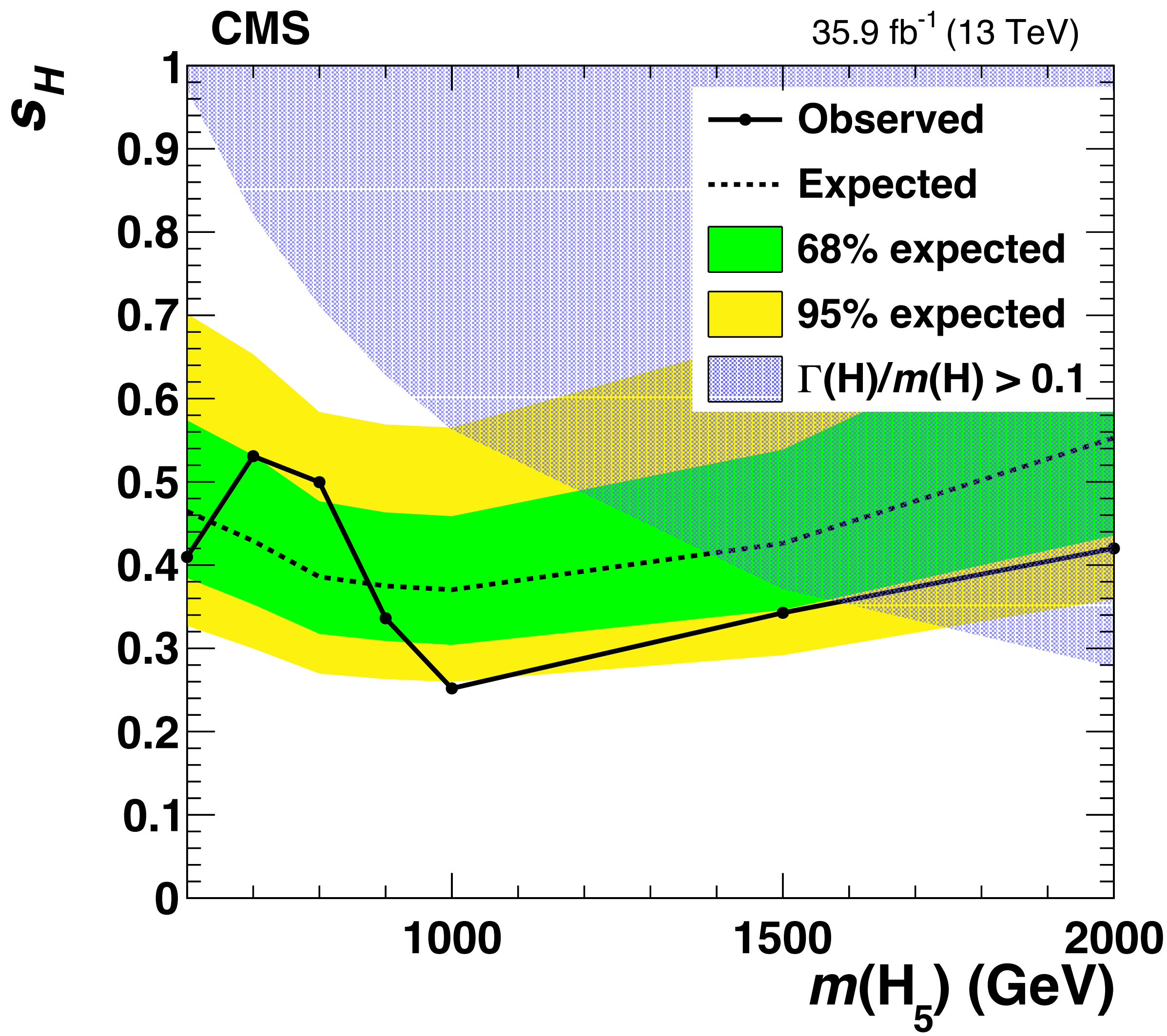


CMS, arXiv:1709.05822

Cross section $\propto s_H^2 \equiv$ fraction of M_W^2, M_Z^2 due to exotic scalars

Probed by direct searches in GM model: $\sim 4\% - 20\%$

Also (new): VBF $\to H_5^\pm/H_5^{\pm\pm} \to W^\pm Z/W^\pm W^\pm \to qq\ell\ell/qq\ell^\pm\nu$



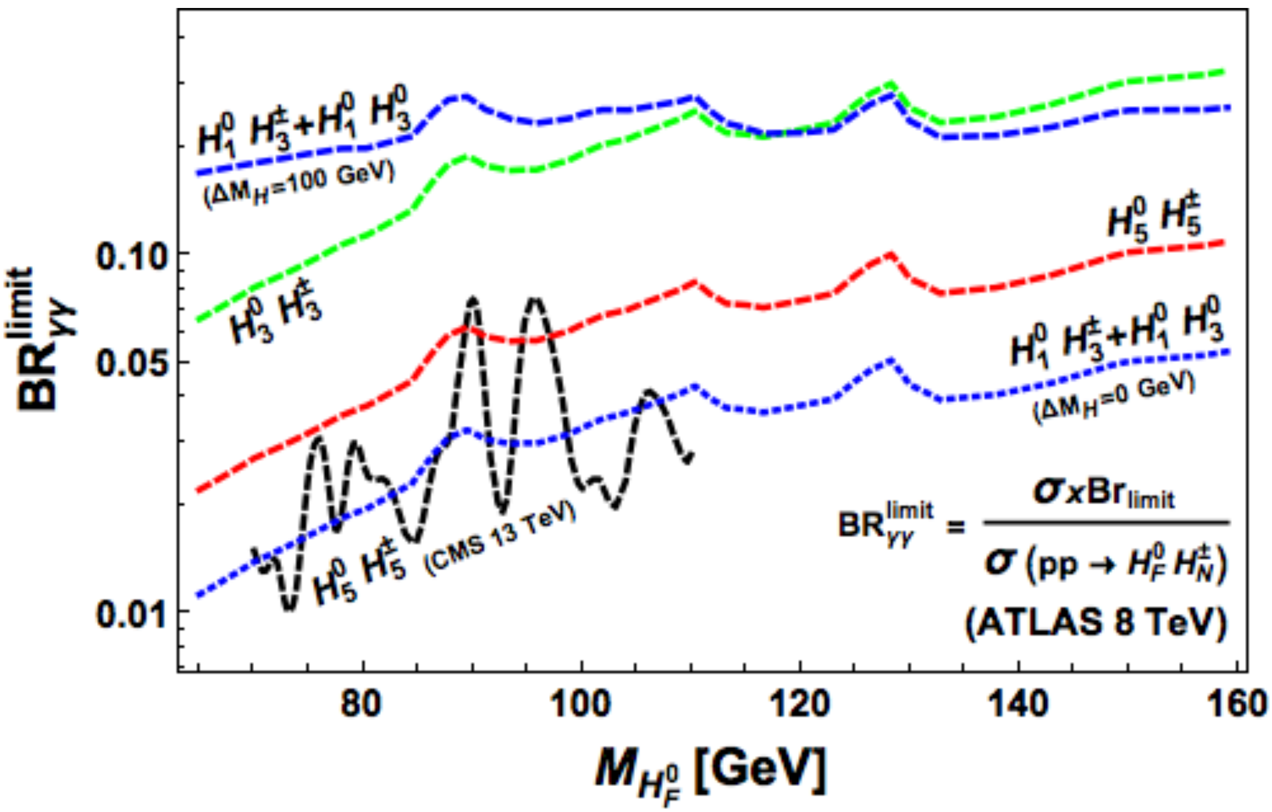
CMS, arXiv:1905.07445

Extends expt reach beyond 1000 GeV!

(Expected limit comparable; downward fluctuation in data.)

At tree level, H_5^0 is fermiophobic due to custodial symmetry:
$H_5^0 \to \gamma\gamma$ gives a powerful search channel at low mass!

Drell-Yan $pp \to H_5^0 H_5^\pm$ depends only on m_5 and gauge couplings:

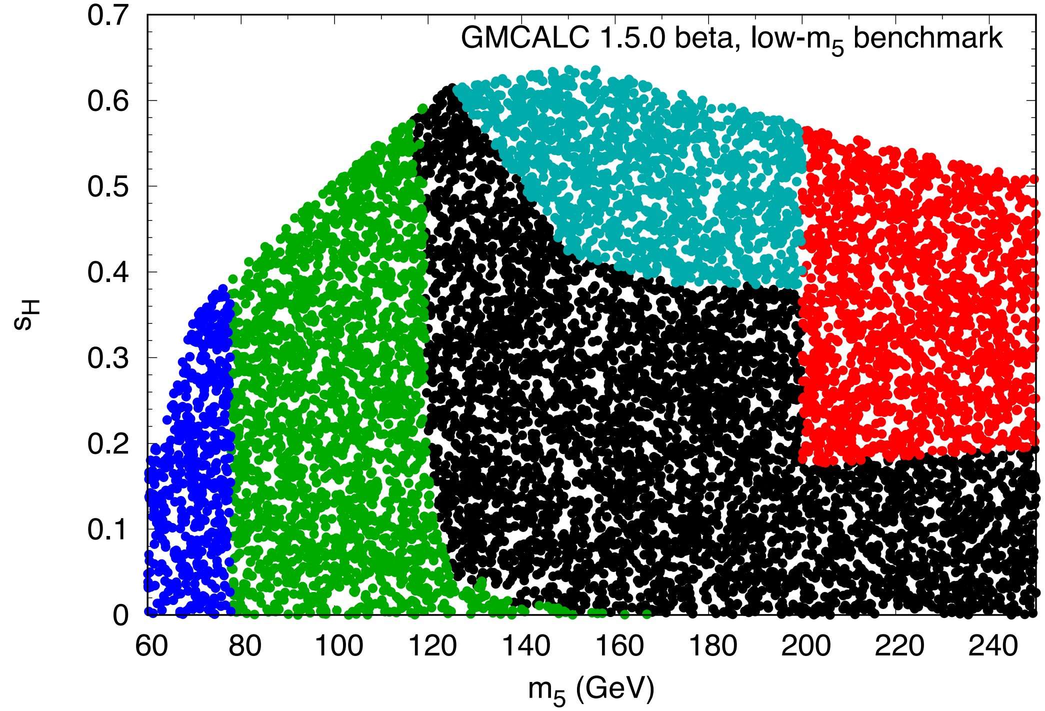


Vega, Vega-Morales & Xie, 1805.01970

$H_5^0 \to \gamma\gamma$ constrains H_5 between ~ 80 and ~ 120 GeV!

Recast ATLAS Run1 VBF $\to W^\pm W^\pm$, 1407.5053

Recast ATLAS Run1 dimuons, 1412.7603, 1502.01275



Recast ATLAS Run1 $\gamma\gamma$ resonance, in prep

(Preliminary)

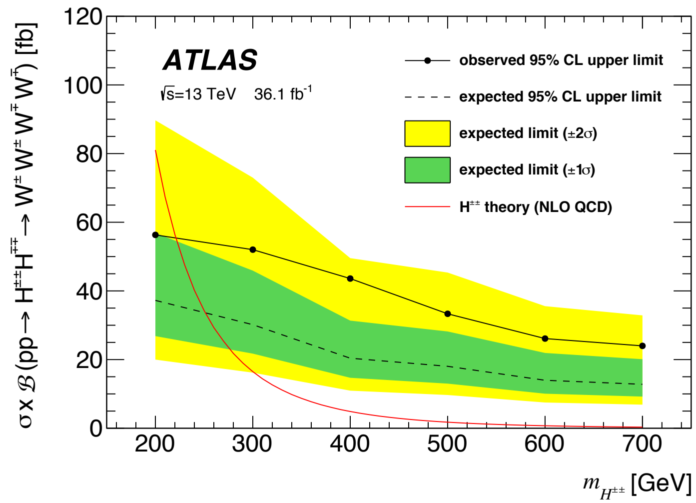
[with B. Keeshan & Y. Wu]

CMS VBF $\to H^{\pm\pm} \to W^\pm W^\pm$

1709.05822

Allowed

Drell-Yan $H^{++}H^{--} \to 4W$ should also have sensitivity over this whole region – important to extend search below 200 GeV!



Custodial symmetry violation in the GM model: a long history

Gunion, Vega & Wudka 1991 showed that computing the T parameter in the GM model yields infinity due to an uncancelled UV divergence caused by hypercharge violating the custodial symmetry at 1-loop. Full gauge-invariant but SU(2)$_L\times$SU(2)$_R$-violating scalar potential yields the needed counterterm.

Englert, Re & Spannowsky 1302.6505 applied S, T parameter constraints by subtracting a counterterm for T. (just divergence?)

Chiang, Kuo & Yagyu 1804.02633 calculated 1-loop renormalized predictions for h couplings in GM model and used measured T parameter as input to fix the relevant custodial-symmetry-violating counterterm.

Blasi, De Curtis & Yagyu 1704.08512 computed the RGEs and studied custodial violation from running up from custodial-symmetric theory at the weak scale. (RGEs independently calculated by us.)

Full gauge-invariant potential: notation follows GVW'91 + dimensionful terms

$$\begin{aligned} V(\phi,\chi,\xi) &= \tilde{\mu}_2^2\phi^\dagger\phi + \tilde{\mu}_3'^2\chi^\dagger\chi + \frac{\tilde{\mu}_3^2}{2}\xi^\dagger\xi \\ &+\tilde{\lambda}_1(\phi^\dagger\phi)^2 + \tilde{\lambda}_2|\tilde{\chi}^\dagger\chi|^2 + \tilde{\lambda}_3(\phi^\dagger\tau^a\phi)(\chi^\dagger t^a\chi) \\ &+\left[\tilde{\lambda}_4(\tilde{\phi}^\dagger\tau^a\phi)(\chi^\dagger t^a\xi) + \text{h.c.}\right] + \tilde{\lambda}_5(\phi^\dagger\phi)(\chi^\dagger\chi) \\ &+\tilde{\lambda}_6(\phi^\dagger\phi)(\xi^\dagger\xi) + \tilde{\lambda}_7(\chi^\dagger\chi)^2 + \tilde{\lambda}_8(\xi^\dagger\xi)^2 \\ &+\tilde{\lambda}_9|\chi^\dagger\xi|^2 + \tilde{\lambda}_{10}(\chi^\dagger\chi)(\xi^\dagger\xi) \\ &+\frac{\tilde{M}_1}{\sqrt{2}}\phi^\dagger\Delta_0\phi - \frac{1}{2}\left[\tilde{M}_1'\phi^\dagger\Delta_2\tilde{\phi} + \text{h.c.}\right] - 6\tilde{M}_2\chi^\dagger\overline{\Delta}_0\chi \end{aligned}$$

where

$$\begin{aligned} \Delta_0 &\equiv \sqrt{2}\tau^a U_{ai}\xi_i = \begin{pmatrix} \xi^0/\sqrt{2} & -\xi^+ \\ -\xi^{+*} & -\xi^0/\sqrt{2} \end{pmatrix}, \\ \Delta_2 &\equiv \sqrt{2}\tau^a U_{ai}\chi_i = \begin{pmatrix} \chi^+/\sqrt{2} & -\chi^{++} \\ \chi^0 & -\chi^+/\sqrt{2} \end{pmatrix}, \\ \overline{\Delta}_0 &\equiv -t^a U_{ai}\xi_i = \begin{pmatrix} -\xi^0 & \xi^+ & 0 \\ \xi^{+*} & 0 & \xi^+ \\ 0 & \xi^{+*} & \xi^0 \end{pmatrix}. \end{aligned}$$

Minimize potential, compute mass matrices, etc.

16 Lagrangian parameters compared to 9 in original GM model: Matching gauge-invariant potential to original GM model yields

$$
\begin{aligned}
\tilde{\mu}_2^2 &= \mu_2^2 \\
\tilde{\mu}_3'^2 &= \mu_3^2 \\
\tilde{\mu}_3^2 &= \mu_3^2 \\
\tilde{\lambda}_1 &= 4\lambda_1 \\
\tilde{\lambda}_2 &= 2\lambda_3 \\
\tilde{\lambda}_3 &= -2\lambda_5 \\
\tilde{\lambda}_4 &= -\sqrt{2}\lambda_5 \\
\tilde{\lambda}_5 &= 4\lambda_2
\end{aligned}
\qquad
\begin{aligned}
\tilde{\lambda}_6 &= 2\lambda_2 \\
\tilde{\lambda}_7 &= 2\lambda_3 + 4\lambda_4 \\
\tilde{\lambda}_8 &= \lambda_3 + \lambda_4 \\
\tilde{\lambda}_9 &= 4\lambda_3 \\
\tilde{\lambda}_{10} &= 4\lambda_4 \\
\tilde{M}_1' &= M_1 \\
\tilde{M}_1 &= M_1 \\
\tilde{M}_2 &= M_2
\end{aligned}
$$

RGEs with $g' = 0$ preserve these relations.

Allowing $g' \neq 0$ violates these relations and introduces custodial symmetry violation through the RGE running.

Our implementation

B. Keeshan, HEL & T. Pilkington 1807.11511

- Assume custodial symmetry at some high scale Λ.

(accidental SU(2)$_L\times$SU(2)$_R$ coming from UV completion e.g. composite Higgs)

- Run down to weak scale $\Rightarrow$ custodial violation generated.

(1-loop RGEs, tree-level matching $\equiv$ leading log approximation)

(Have to do some iteration to get correct low-scale G_F, m_h, m_t.)

- Use measured value of ρ_0 to put an upper bound on scale Λ.

(Also require perturbative unitarity constraint on quartic couplings.)

- Subject to ρ_0 constraint (and perturbativity at Λ), quantify maximum allowed custodial symmetry violation and its phenomenological consequences.

(Use a combination of benchmark plane and general parameter scans to study effects over the GM model parameter space.)

Some observations:

- $g' \neq 0$ is the ONLY source of custodial symmetry violation at one loop.

$y_t \neq y_b$ does not cause custodial-breaking running at one loop (because the fermions couple directly only to the doublet, whose part of the scalar potential preserves custodial symmetry accidentally).

Custodial symmetry preserving RGEs

($g' = 0$; $t \equiv \log \mu$)

$$16\pi^2 \frac{d\left(\mu_2^2\right)}{dt} = \frac{9}{2}M_1^2 + \mu_2^2\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{9}{2}g_2^2 + 48\lambda_1\right) + 36\mu_3^2\lambda_2,$$

$$16\pi^2 \frac{d\left(\mu_3^2\right)}{dt} = M_1^2 + 144M_2^2 + 16\mu_2^2\lambda_2 + \mu_3^2\left(-12g_2^2 + 56\lambda_3 + 88\lambda_4\right),$$

$$16\pi^2 \frac{d\lambda_1}{dt} = -\frac{3}{2}y_b^4 - \frac{3}{2}y_t^4 - \frac{1}{2}y_\tau^4 + \lambda_1\left(12y_b^2 + 12y_t^2 + 4y_\tau^2 - 9g_2^2 + 96\lambda_1\right) + \frac{9}{32}g_2^4 + 18\lambda_2^2 + \frac{3}{2}\lambda_5^2,$$

$$16\pi^2 \frac{d\lambda_2}{dt} = \lambda_2\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{33}{2}g_2^2 + 48\lambda_1 + 16\lambda_2 + 56\lambda_3 + 88\lambda_4\right) + \frac{3}{2}g_2^4 + 4\lambda_5^2,$$

$$16\pi^2 \frac{d\lambda_3}{dt} = \frac{3}{2}g_2^4 + \lambda_3\left(-24g_2^2 + 80\lambda_3 + 96\lambda_4\right) - \lambda_5^2,$$

$$16\pi^2 \frac{d\lambda_4}{dt} = \frac{3}{2}g_2^4 + \lambda_4\left(-24g_2^2 + 136\lambda_4 + 112\lambda_3\right) + 8\lambda_2^2 + 24\lambda_3^2 + \lambda_5^2,$$

$$16\pi^2 \frac{d\lambda_5}{dt} = \lambda_5\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{33}{2}g_2^2 + 16\lambda_1 + 32\lambda_2 - 8\lambda_3 + 16\lambda_4 - 4\lambda_5\right),$$

$$16\pi^2 \frac{dM_1}{dt} = M_1\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{21}{2}g_2^2 + 16\lambda_1 + 16\lambda_2 - 16\lambda_5\right) - 48M_2\lambda_5,$$

$$16\pi^2 \frac{dM_2}{dt} = -M_1\lambda_5 + M_2\left(-18g_2^2 - 24\lambda_3 + 48\lambda_4\right).$$

Some observations:

- The full gauge-invariant scalar potential allows for explicit CP violation through the complex coupling parameters $\tilde{\lambda}_4$ and $\tilde{M}'_1$.

But the custodial-symmetric scalar potential is CP-invariant; if CP invariance is imposed at one scale, it is preserved by the 1-loop running.

$$
\begin{aligned}
V(\phi,\chi,\xi) \;=\;& \tilde{\mu}_2^2 \phi^\dagger \phi + \tilde{\mu}_3'^2 \chi^\dagger \chi + \frac{\tilde{\mu}_3^2}{2} \xi^\dagger \xi \\
&+ \tilde{\lambda}_1 (\phi^\dagger \phi)^2 + \tilde{\lambda}_2 |\tilde{\chi}^\dagger \chi|^2 + \tilde{\lambda}_3 (\phi^\dagger \tau^a \phi)(\chi^\dagger t^a \chi) \\
&+ \left[\tilde{\lambda}_4 (\tilde{\phi}^\dagger \tau^a \phi)(\chi^\dagger t^a \xi) + \text{h.c.} \right] + \tilde{\lambda}_5 (\phi^\dagger \phi)(\chi^\dagger \chi) \\
&+ \tilde{\lambda}_6 (\phi^\dagger \phi)(\xi^\dagger \xi) + \tilde{\lambda}_7 (\chi^\dagger \chi)^2 + \tilde{\lambda}_8 (\xi^\dagger \xi)^2 \\
&+ \tilde{\lambda}_9 |\chi^\dagger \xi|^2 + \tilde{\lambda}_{10} (\chi^\dagger \chi)(\xi^\dagger \xi) \\
&+ \frac{\tilde{M}_1}{\sqrt{2}} \phi^\dagger \Delta_0 \phi - \frac{1}{2} \left[\tilde{M}'_1 \phi^\dagger \Delta_2 \tilde{\phi} + \text{h.c.} \right] - 6 \tilde{M}_2 \chi^\dagger \overline{\Delta}_0 \chi
\end{aligned}
$$

Some observations:

There are a few other symmetries which are preserved by the 1-loop running:

- Setting $\tilde{M}_1 = \tilde{M}_1' = \tilde{M}_2 = 0$ preserves $(\chi, \xi) \to (-\chi, -\xi)$

- Setting $\tilde{\lambda}_4 = \tilde{M}_1' = 0$ preserves $\chi \to -\chi$

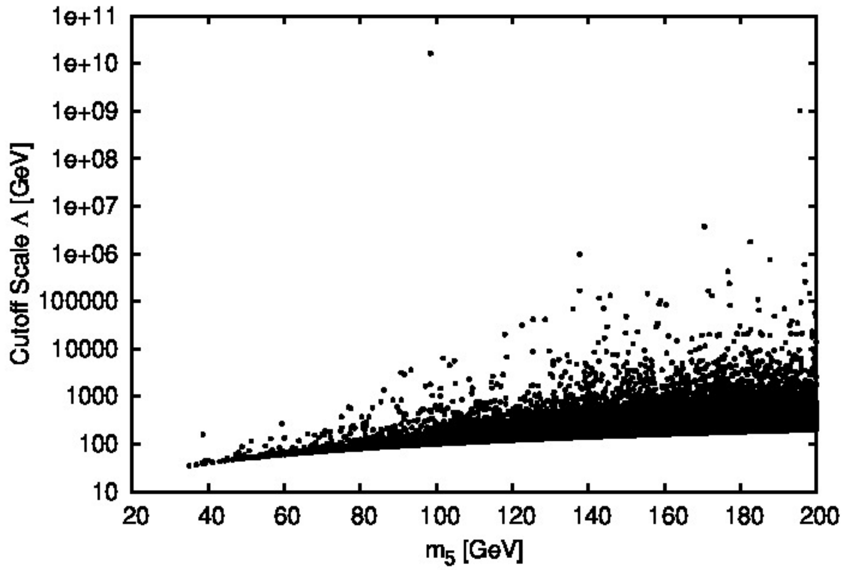
- Setting $\tilde{\lambda}_4 = \tilde{M}_1 = \tilde{M}_2 = 0$ preserves $\xi \to -\xi$

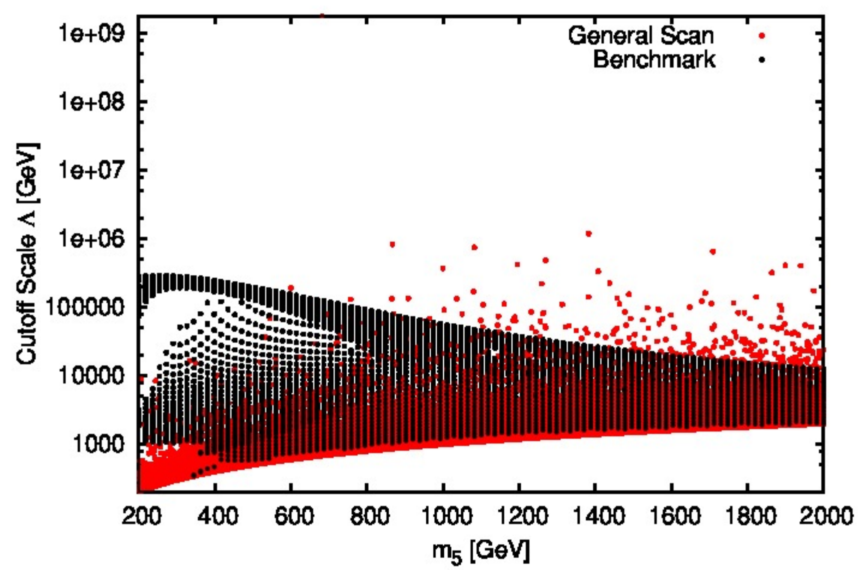
- Setting all the dimensionful parameters to zero preserves scale invariance.

As usual, the RGEs for a coupling of given mass dimension depend only on couplings of equal or lesser mass dimension. (Dimensionful couplings do not enter the RGEs of dimensionless Lagrangian parameters.)

Numerical results: maximum cutoff scale Λ

B. Keeshan, HEL & T. Pilkington 1807.11511 + revisions in preparation





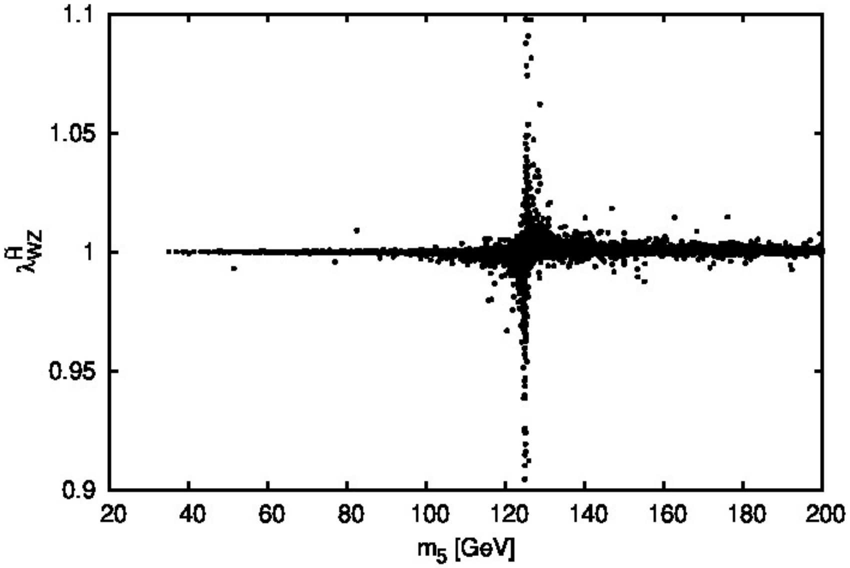
UV completion generally must appear below 10s to 100s of TeV.

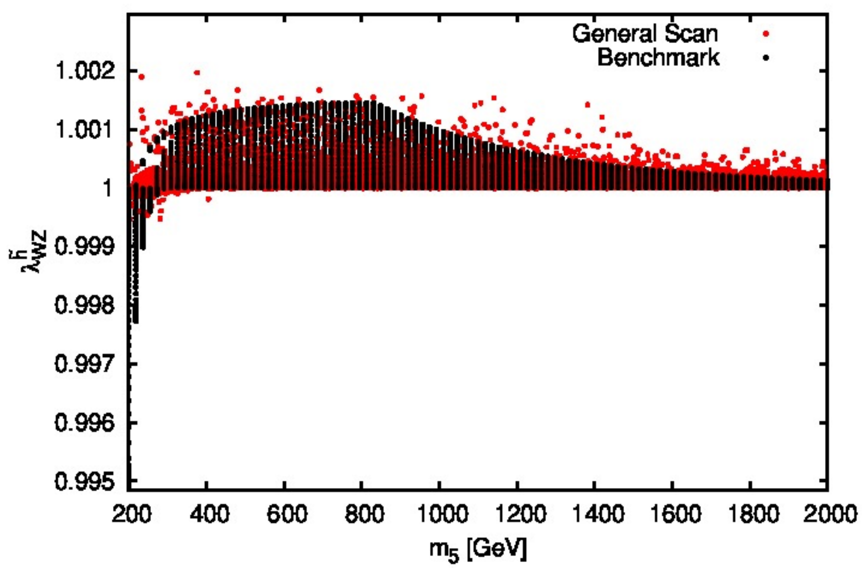
Not too far away! Hierarchy problem is only "little".

But also not right on top of our heads: generally high enough to be able to ignore loop effects or dimension-6 operators induced by the UV completion.

Numerical results: $\lambda_{WZ} \equiv hWW/hZZ$ normalized to SM

B. Keeshan, HEL & T. Pilkington 1807.11511 + revisions in preparation





Deviation from SM prediction ($\lambda^h_{WZ} = 1$) below percent-level except for resonant mixing between h and H_5^0 at $m_5 \sim 125$ GeV.

Current LHC precision: $\lambda^h_{WZ} = 0.95 \pm 0.08$ ATLAS Run 2, 1909.02845

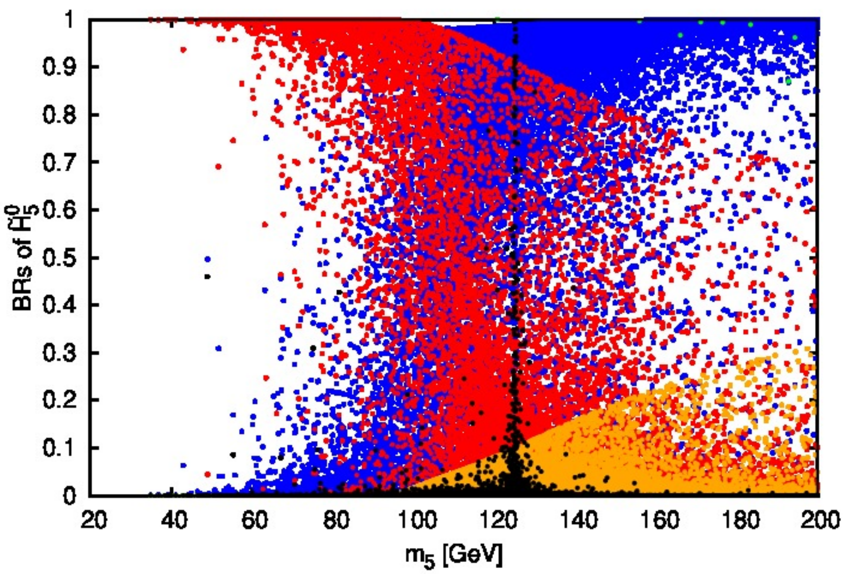
Future: HL-LHC few % / ILC $\sim 0.5\%$ / FCC-ee $\sim 0.2\%$

Numerical results: custodial-violating mixing of Higgs states

B. Keeshan, HEL & T. Pilkington 1807.11511 + revisions in preparation

Custodial symmetry violation mixes doublet into H_5^0, can induce fermionic decays that might compete with $\gamma\gamma$.

The effect is generally very small unless H_5^0–h mixing is resonant.



$H_5^0 \to \gamma\gamma$

$H_5^0 \to WW + ZZ$

$H_5^0 \to Z\gamma$

$H_5^0 \to f\bar{f}$ (points around $m_5 \sim 125$ GeV)

$H_5^0 \to H_3 H_3$ (few points above 180 GeV)

BR($H_5^0 \to f\bar{f}$) remains small except for resonant mixing region $\Rightarrow$ $H_5^0 \to \gamma\gamma$ still strongly constraining for masses below ~ 110 GeV.

Conclusions and outlook

Custodial symmetry is accidental in the Standard Model: though violated by hypercharge and $y_t \neq y_b$, effects are finite and calculable (no tree-level counterterms).

Imposing custodial symmetry "by hand" in BSM models works at tree level, but is violated at one loop.

One way to think about this: Custodial symmetry accidental at a cutoff scale Λ, violated by RG running down to weak scale; effects quantifiable!

We quantified this explicitly in the Georgi-Machacek model, a prototype for LHC searches for "exotic" extended Higgs sectors:

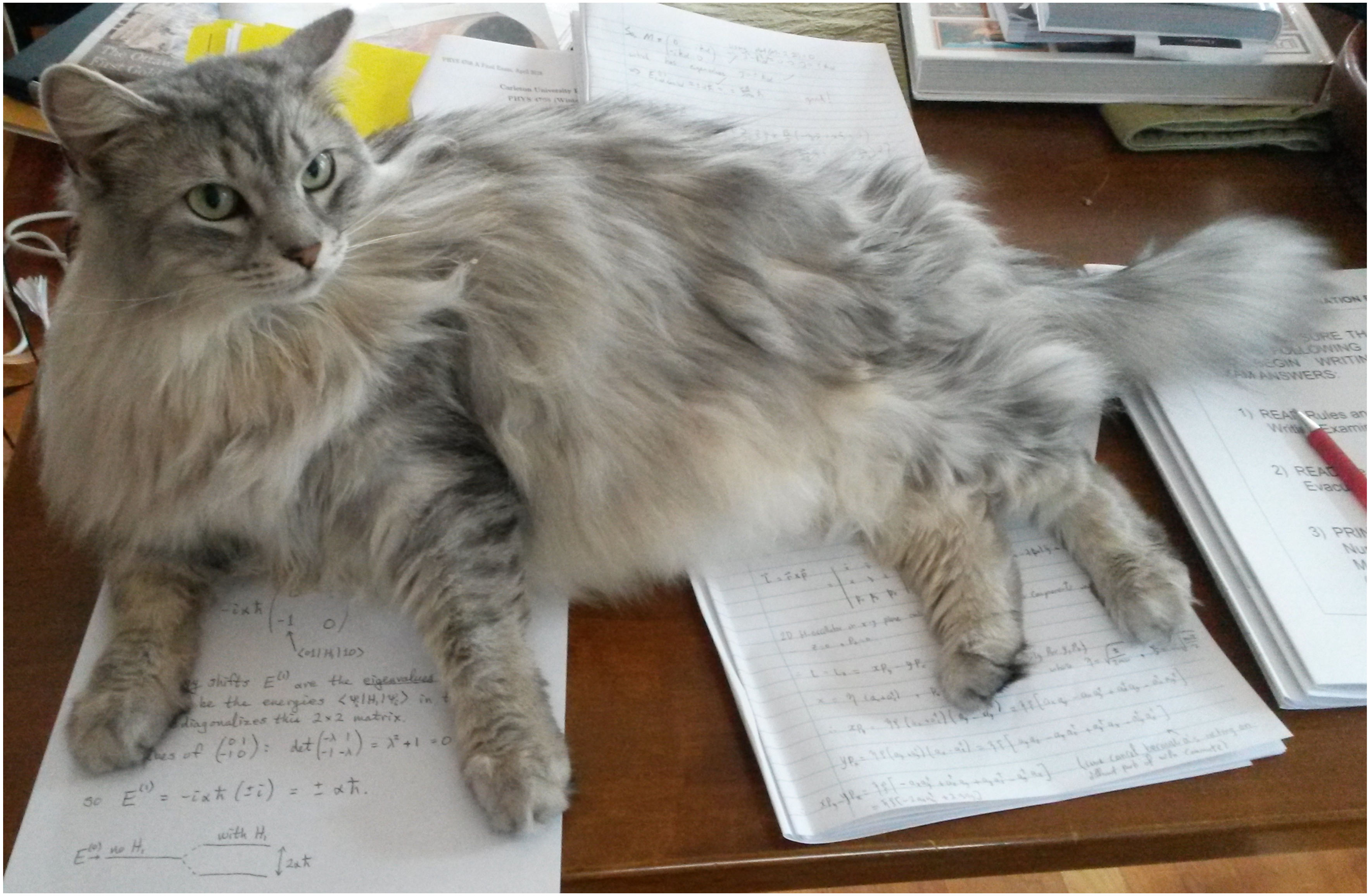
- UV completion generally lies below 10s to 100s of TeV

  forced by perturbative unitarity + measured ρ parameter
- Custodial-violating effects are generally small

  assumption of custodial-symmetric GM is sufficient for LHC searches

Next step: compute T parameter constraints in a sensible way.

~ THE END ~



BACKUP SLIDES

Custodial symmetry preserving RGEs ($g' = 0$; $t \equiv \log\mu$)

$$16\pi^2 \frac{d\left(\mu_2^2\right)}{dt} = \frac{9}{2}M_1^2 + \mu_2^2\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{9}{2}g_2^2 + 48\lambda_1\right) + 36\mu_3^2\lambda_2,$$

$$16\pi^2 \frac{d\left(\mu_3^2\right)}{dt} = M_1^2 + 144M_2^2 + 16\mu_2^2\lambda_2 + \mu_3^2\left(-12g_2^2 + 56\lambda_3 + 88\lambda_4\right),$$

$$16\pi^2 \frac{d\lambda_1}{dt} = -\frac{3}{2}y_b^4 - \frac{3}{2}y_t^4 - \frac{1}{2}y_\tau^4 + \lambda_1\left(12y_b^2 + 12y_t^2 + 4y_\tau^2 - 9g_2^2 + 96\lambda_1\right) + \frac{9}{32}g_2^4 + 18\lambda_2^2 + \frac{3}{2}\lambda_5^2,$$

$$16\pi^2 \frac{d\lambda_2}{dt} = \lambda_2\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{33}{2}g_2^2 + 48\lambda_1 + 16\lambda_2 + 56\lambda_3 + 88\lambda_4\right) + \frac{3}{2}g_2^4 + 4\lambda_5^2,$$

$$16\pi^2 \frac{d\lambda_3}{dt} = \frac{3}{2}g_2^4 + \lambda_3\left(-24g_2^2 + 80\lambda_3 + 96\lambda_4\right) - \lambda_5^2,$$

$$16\pi^2 \frac{d\lambda_4}{dt} = \frac{3}{2}g_2^4 + \lambda_4\left(-24g_2^2 + 136\lambda_4 + 112\lambda_3\right) + 8\lambda_2^2 + 24\lambda_3^2 + \lambda_5^2,$$

$$16\pi^2 \frac{d\lambda_5}{dt} = \lambda_5\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{33}{2}g_2^2 + 16\lambda_1 + 32\lambda_2 - 8\lambda_3 + 16\lambda_4 - 4\lambda_5\right),$$

$$16\pi^2 \frac{dM_1}{dt} = M_1\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{21}{2}g_2^2 + 16\lambda_1 + 16\lambda_2 - 16\lambda_5\right) - 48M_2\lambda_5,$$

$$16\pi^2 \frac{dM_2}{dt} = -M_1\lambda_5 + M_2\left(-18g_2^2 - 24\lambda_3 + 48\lambda_4\right).$$

Complete RGEs ($g_1 \equiv \sqrt{5/3}g'$)

$$16\pi^2 \frac{d\left(\tilde{\mu}_2^2\right)}{dt} = \frac{3}{2}\tilde{M}_1^2 + 3|\tilde{M}_1'|^2 + \tilde{\mu}_2^2\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 + 12\tilde{\lambda}_1\right) + 6\tilde{\mu}_3^2\tilde{\lambda}_6 + 6\tilde{\mu}_3'^2\tilde{\lambda}_5,$$

$$16\pi^2 \frac{d\left(\tilde{\mu}_3'^2\right)}{dt} = |\tilde{M}_1'|^2 + 144\tilde{M}_2^2 + \tilde{\mu}_3'^2\left(8\tilde{\lambda}_2 + 16\tilde{\lambda}_7 - \frac{18}{5}g_1^2 - 12g_2^2\right) + 4\tilde{\mu}_2^2\tilde{\lambda}_5 + 2\tilde{\mu}_3^2\left(\tilde{\lambda}_9 + 3\tilde{\lambda}_{10}\right),$$

$$16\pi^2 \frac{d\left(\tilde{\mu}_3^2\right)}{dt} = \tilde{M}_1^2 + 144\tilde{M}_2^2 + 4\tilde{\mu}_3^2\left(10\tilde{\lambda}_8 - 3g_2^2\right) + 8\tilde{\mu}_2^2\tilde{\lambda}_6 + 4\tilde{\mu}_3'^2\left(\tilde{\lambda}_9 + 3\tilde{\lambda}_{10}\right),$$

$$16\pi^2 \frac{d\tilde{\lambda}_1}{dt} = -6y_b^4 - 6y_t^4 - 2y_\tau^4 + \tilde{\lambda}_1\left(12y_b^2 + 12y_t^2 + 4y_\tau^2 - \frac{9}{5}g_1^2 - 9g_2^2 + 24\tilde{\lambda}_1\right) + \frac{27}{200}g_1^4 + \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2g_2^2 + \frac{1}{2}\tilde{\lambda}_3^2 + 2|\tilde{\lambda}_4|^2 + 3\tilde{\lambda}_5^2 + 6\tilde{\lambda}_6^2,$$

$$16\pi^2 \frac{d\tilde{\lambda}_2}{dt} = 3g_2^4 - \frac{36}{5}g_1^2g_2^2 + 12\tilde{\lambda}_2\left(\tilde{\lambda}_2 + 2\tilde{\lambda}_7 - \frac{3}{5}g_1^2 - 2g_2^2\right) - \frac{1}{2}\tilde{\lambda}_3^2 + \tilde{\lambda}_9^2,$$

$$16\pi^2 \frac{d\tilde{\lambda}_3}{dt} = \tilde{\lambda}_3\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 + 4\tilde{\lambda}_1 - 8\tilde{\lambda}_2 + 8\tilde{\lambda}_5 + 4\tilde{\lambda}_7 - \frac{9}{2}g_1^2 - \frac{33}{2}g_2^2\right) + \frac{36}{5}g_2^2g_1^2 + 4|\tilde{\lambda}_4|^2,$$

$$16\pi^2 \frac{d\tilde{\lambda}_4}{dt} = \tilde{\lambda}_4\left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{27}{10}g_1^2 - \frac{33}{2}g_2^2 + 4\tilde{\lambda}_1 + 2\tilde{\lambda}_3 + 4\tilde{\lambda}_5 + 8\tilde{\lambda}_6 - 2\tilde{\lambda}_9 + 4\tilde{\lambda}_{10}\right),$$

Complete RGEs ($g_1 \equiv \sqrt{5/3}g'$)

$$16\pi^2\frac{d\tilde\lambda_5}{dt} = \tilde\lambda_5\left(6y_b^2+6y_t^2+2y_\tau^2+4\tilde\lambda_5+12\tilde\lambda_1+8\tilde\lambda_2+16\tilde\lambda_7-\frac{9}{2}g_1^2-\frac{33}{2}g_2^2\right) + \frac{27}{25}g_1^4+6g_2^4+2\tilde\lambda_3^2+4|\tilde\lambda_4|^2+4\tilde\lambda_6\tilde\lambda_9+12\tilde\lambda_6\tilde\lambda_{10},$$

$$16\pi^2\frac{d\tilde\lambda_6}{dt} = \tilde\lambda_6\left(6y_b^2+6y_t^2+2y_\tau^2+8\tilde\lambda_6+12\tilde\lambda_1+40\tilde\lambda_8-\frac{9}{10}g_1^2-\frac{33}{2}g_2^2\right)+3g_2^4+4|\tilde\lambda_4|^2+2\tilde\lambda_5\tilde\lambda_9+6\tilde\lambda_5\tilde\lambda_{10},$$

$$16\pi^2\frac{d\tilde\lambda_7}{dt} = \frac{54}{25}g_1^4+9g_2^4+\frac{36}{5}g_2^2g_1^2+\left(-\frac{36}{5}g_1^2-24g_2^2+16\tilde\lambda_2+28\tilde\lambda_7\right)+16\tilde\lambda_2^2+\frac{1}{2}\tilde\lambda_3^2+2\tilde\lambda_5^2+\tilde\lambda_9^2+2\tilde\lambda_{10}\left(3\tilde\lambda_{10}+2\tilde\lambda_9\right),$$

$$16\pi^2\frac{d\tilde\lambda_8}{dt} = 3g_2^4+8\tilde\lambda_8\left(-3g_2^2+11\tilde\lambda_8\right)+2\tilde\lambda_6^2+\tilde\lambda_9\left(\tilde\lambda_9+2\tilde\lambda_{10}\right)+3\tilde\lambda_{10}^2,$$

$$16\pi^2\frac{d\tilde\lambda_9}{dt} = 6g_2^4+2\tilde\lambda_9\left(-12g_2^2-\frac{9}{5}g_1^2+5\tilde\lambda_9+4\tilde\lambda_2+2\tilde\lambda_7+8\tilde\lambda_8+8\tilde\lambda_{10}\right)-2|\tilde\lambda_4|^2,$$

$$16\pi^2\frac{d\tilde\lambda_{10}}{dt} = 6g_2^4+2\tilde\lambda_{10}\left(-\frac{9}{5}g_1^2-12g_2^2+4\tilde\lambda_2+8\tilde\lambda_7+20\tilde\lambda_8+4\tilde\lambda_{10}\right)+2|\tilde\lambda_4|^2+2\tilde\lambda_9^2+4\tilde\lambda_5\tilde\lambda_6+4\tilde\lambda_9\left(\tilde\lambda_7+2\tilde\lambda_8\right),$$

Complete RGEs ($g_1 \equiv \sqrt{5/3}g'$)

$$16\pi^2 \frac{d\tilde{M}_1'}{dt} = \tilde{M}_1' \left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{27}{10}g_1^2 - \frac{21}{2}g_2^2 + 4\tilde{\lambda}_1 + 4\tilde{\lambda}_3 + 4\tilde{\lambda}_5 \right) + 4\sqrt{2}\tilde{\lambda}_4^* \left(\tilde{M}_1 + 6\tilde{M}_2 \right),$$

$$16\pi^2 \frac{d\tilde{M}_1}{dt} = \tilde{M}_1 \left(6y_b^2 + 6y_t^2 + 2y_\tau^2 - \frac{9}{10}g_1^2 - \frac{21}{2}g_2^2 + 4\tilde{\lambda}_1 + 8\tilde{\lambda}_6 \right) + 24\tilde{M}_2\tilde{\lambda}_3 + 8\sqrt{2}\mathrm{Re}\left[\tilde{M}_1'\tilde{\lambda}_4 \right],$$

$$16\pi^2 \frac{d\tilde{M}_2}{dt} = \tilde{M}_2 \left(-\frac{18}{5}g_1^2 - 18g_2^2 - 8\tilde{\lambda}_2 + 4\tilde{\lambda}_7 - 4\tilde{\lambda}_9 + 8\tilde{\lambda}_{10} \right) + \frac{1}{6}\tilde{M}_1\tilde{\lambda}_3 + \frac{1}{3}\sqrt{2}\mathrm{Re}\left[\tilde{M}_1'\tilde{\lambda}_4 \right],$$

The running of g_1 and g_2 is modified by the extra triplets:

$$16\pi^2 \frac{dg_1}{dt} = \frac{47}{10}g_1^3 \qquad \text{or equivalently} \qquad 16\pi^2 \frac{dg'}{dt} = \frac{47}{6}g'^3,$$

$$16\pi^2 \frac{dg_2}{dt} = -\frac{13}{6}g_2^3,$$

Complete RGEs ($g_1 \equiv \sqrt{5/3}g'$)

The running of g_3 and the Yukawa couplings is the same as in the SM:

$$16\pi^2 \frac{dg_3}{dt} = -7g_3^3.$$

$$16\pi^2 \frac{dy_t}{dt} = \left(-\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \frac{3}{2}y_b^2 + \frac{9}{2}y_t^2 + y_\tau^2\right) y_t,$$

$$16\pi^2 \frac{dy_b}{dt} = \left(-\frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \frac{9}{2}y_b^2 + \frac{3}{2}y_t^2 + y_\tau^2\right) y_b,$$

$$16\pi^2 \frac{dy_\tau}{dt} = \left(-\frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + 3y_b^2 + 3y_t^2 + \frac{5}{2}y_\tau^2\right) y_\tau.$$

where $y_f = \sqrt{2}m_f/\tilde{v}_\phi$.

(In our numerical work we neglect y_b and y_τ.)

MORE BACKUP SLIDES

There are only two known approaches:

1) Use the septet $(T, Y) = (3, 2)$: $\rho = 1$ by accident!
Doublet $\left(\frac{1}{2}, \frac{1}{2}\right)$ + septet $(3, 2)$: Scalar septet model

Hisano & Tsumura, 1301.6455; Kanemura, Kikuchi & Yagyu, 1301.7303

2) Use global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ imposed on the scalar potential
Global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \rightarrow$ custodial SU(2) ensures tree-level $\rho = 1$
Doublet + triplets $(1, 0) + (1, 1)$: Georgi-Machacek model

Georgi & Machacek 1985; Chanowitz & Golden 1985

Doublet + quartets $\left(\frac{3}{2}, \frac{1}{2}\right) + \left(\frac{3}{2}, \frac{3}{2}\right)$:
Doublet + quintets $(2, 0) + (2, 1) + (2, 2)$:
Doublet + sextets $\left(\frac{5}{2}, \frac{1}{2}\right) + \left(\frac{5}{2}, \frac{3}{2}\right) + \left(\frac{5}{2}, \frac{5}{2}\right)$:
Generalized Georgi-Machacek models

Galison 1984; Robinett 1985; HEL 1999; Chang et al 2012; HEL & Rentala 2015

Larger than sextets $\rightarrow$ too many large multiplets, violates perturbativity

Can also have duplications, combinations $\rightarrow$ ignore that here.

Both approaches have theoretical "issues":

1) Can't give the septet a vev through spontaneous breaking without generating a physical massless Goldstone boson.

Have to couple it to the SM doublet through a dimension-7 $X\Phi^*\Phi^5$ term Hisano & Tsumura 2013

Need the UV completion to be nearby!

2) Global SU(2)$_L\times$SU(2)$_R$ is broken by gauging hypercharge.

Gunion, Vega & Wudka 1991

Special relations among params of *full* gauge-invariant scalar potential can only hold at one energy scale: violated by running due to hypercharge. Garcia-Pepin, Gori, Quiros, Vega, Vega-Morales, Yu 2014

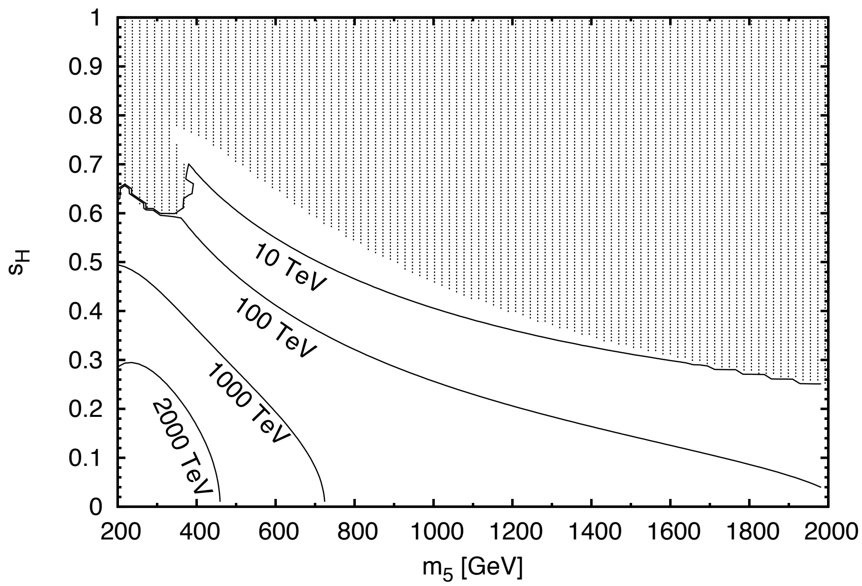
Need the UV completion to be nearby!

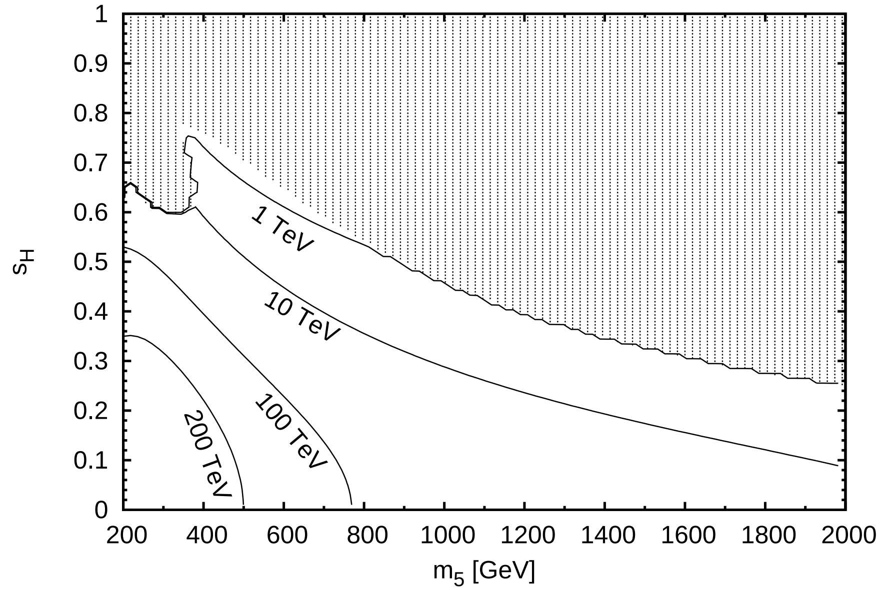
This talk: quantify (2) in the Georgi-Machacek model.

H5plane benchmark (introduced by HXSWG for H_5 LHC searches)

Fixed Parameters	Variable Parameters	Dependent Parameters
$G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}	$m_5 \in [200, 3000]$ GeV	$\lambda_2 = 0.4 m_5/(1000$ GeV$)$
$m_h = 125$ GeV	$s_H \in (0, 1)$	$M_1 = \sqrt{2} s_H (m_5^2 + v^2)/v$
$\lambda_3 = -0.1$		$M_2 = M_1/6$
$\lambda_4 = 0.2$		

Results (within H5plane benchmark): cutoff scale



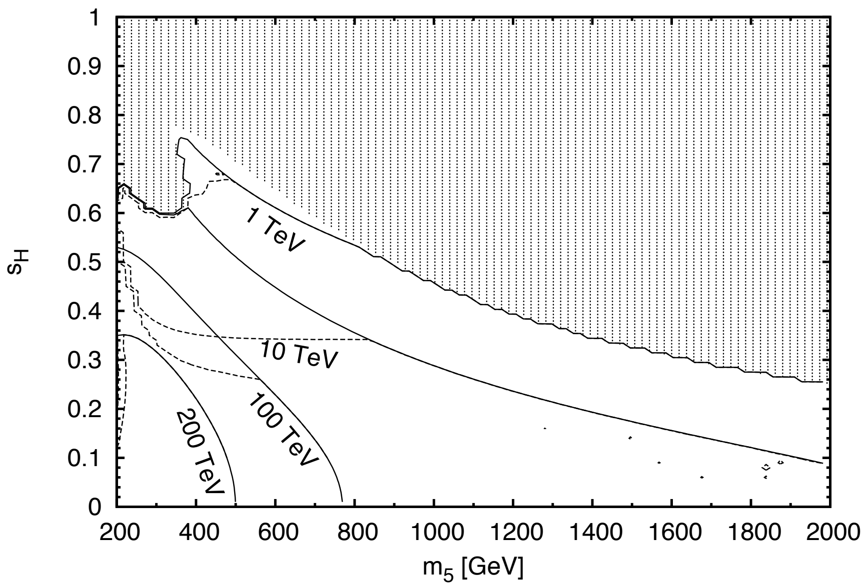


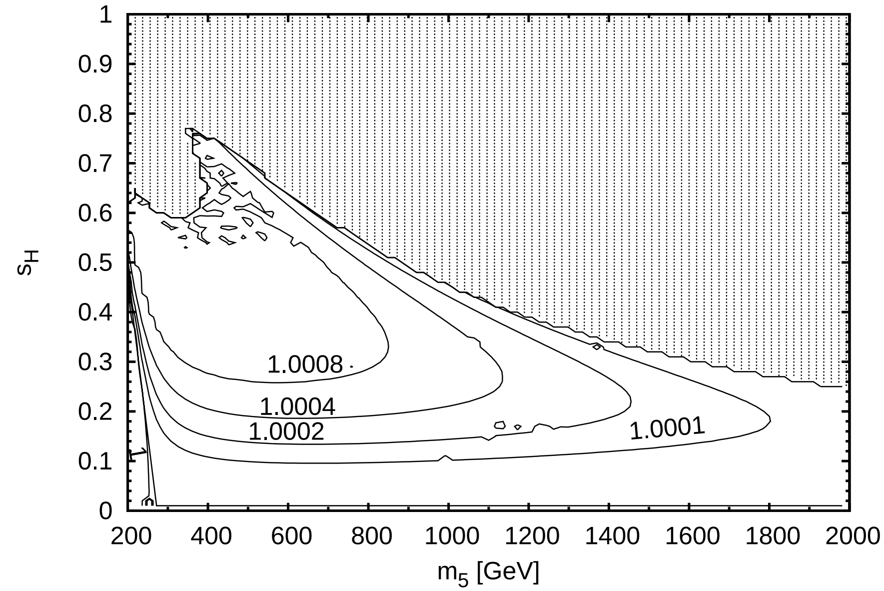
Left: Scale of Landau pole

Right: Highest scale at which perturbative unitarity constraints on custodial-symmetric λ_i remain satisfied

UV completion must appear below 10s to 100s of TeV

Results (within H5plane benchmark): ρ parameter





Left: Maximum cutoff scale including ρ parameter constraint (dashed) + perturbative unitarity (solid)

Right: Weak-scale value of ρ, for Λ as large as possible

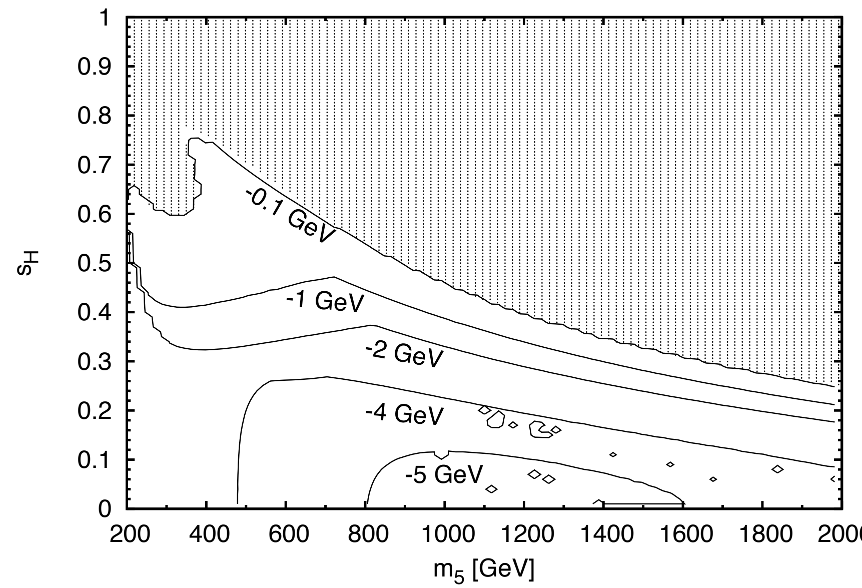
ρ_0 samples full 2σ allowed range

$\Delta\rho_0$ is positive in most of H5plane benchmark parameter space

Results (within H5plane benchmark): mass splittings

Plot: $m_{H_3^\pm} - m_{H_3^0}$
for Λ as large as possible

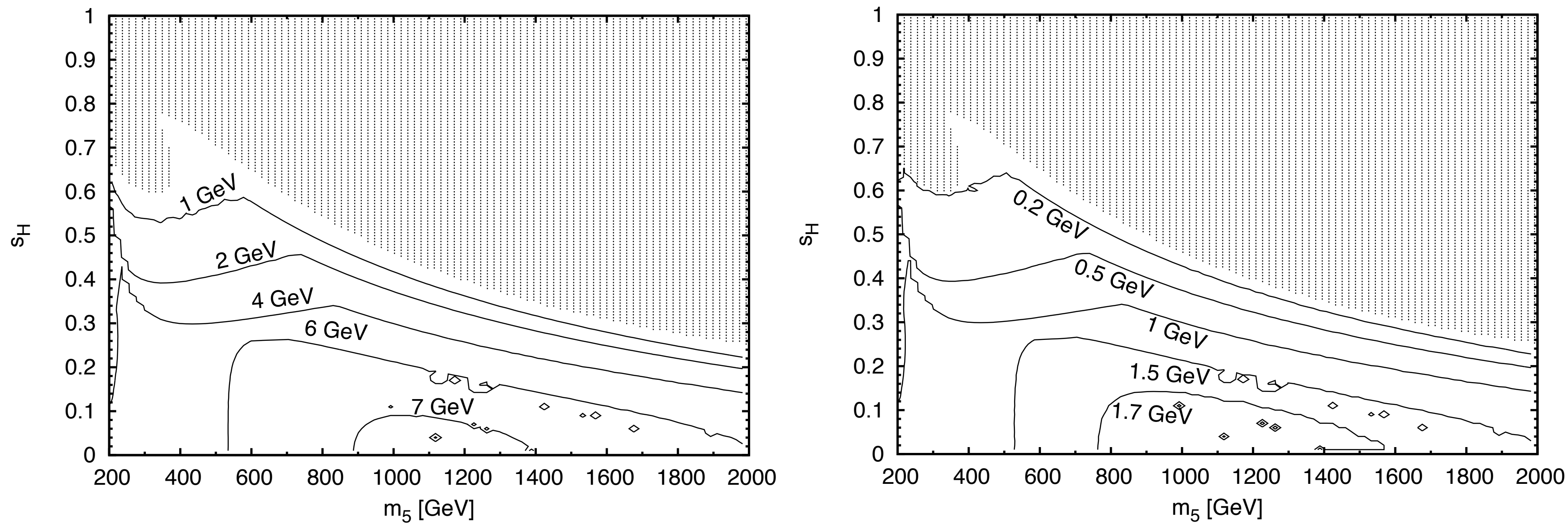
(negative values: $H_3^\pm$ is lighter)



Custodial-violating mass splitting of $H_3^{0,\pm}$ is at most 5.3 GeV.
$m_{H_3^0} > m_{H_3^\pm}$ everywhere in H5plane benchmark.

Measurement prospects: $H_3^0 \to b\bar{b}, t\bar{t}$; $H_3^+ \to t\bar{b}$
Couplings as in Type-I 2HDM: down-type decays not enhanced
Mass splitting too small to detect at LHC

Results (within H5plane benchmark): mass splittings



Left: $m_{H_5^{\pm\pm}} - m_{H_5^0}$

for Λ as large as possible

Right: $m_{H_5^{\pm}} - m_{H_5^0}$

Custodial-violating mass splitting of $H_5^{0,\pm,\pm\pm}$ is at most 7.2 GeV.

$m_{H_5^{\pm\pm}} > m_{H_5^{\pm}} > m_{H_5^0}$ everywhere in H5plane benchmark.

Decays are to VV – similar challenges to detect small mass splittings at LHC.

Minimal nontrivial representation of the Higgs field (Lorentz scalar) is a complex SU(2)$_L$ doublet with hypercharge $Y = 1/2$:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

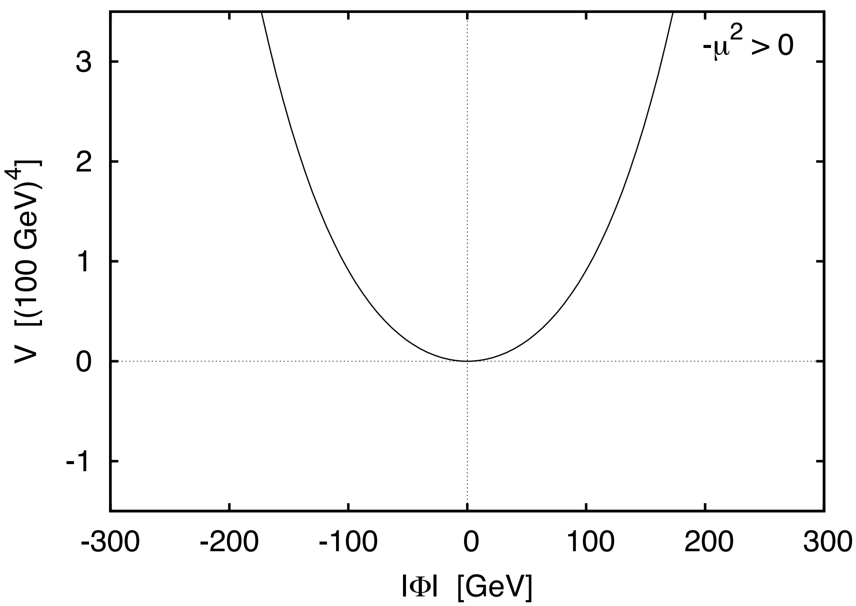
The most general gauge-invariant potential for this field (the so-called Higgs potential) is

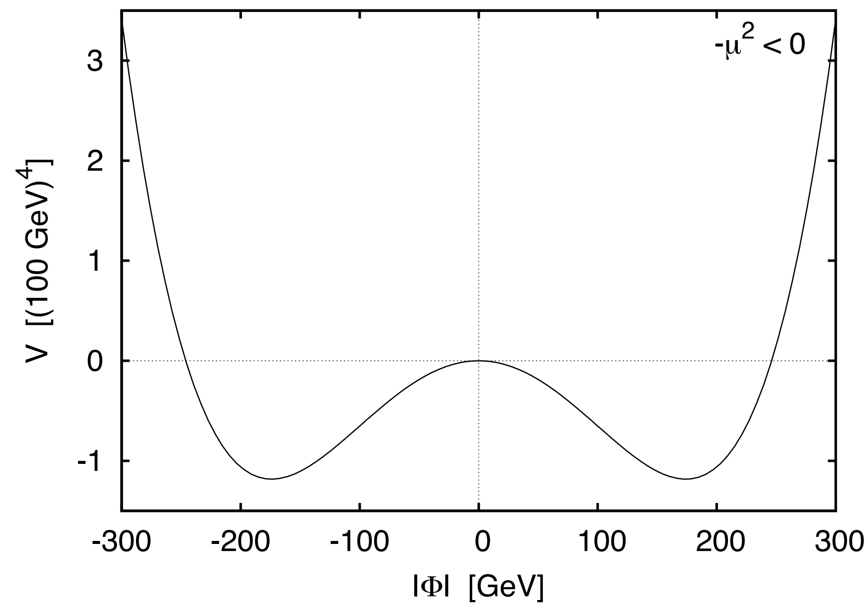
$$\begin{aligned} V &= -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \\ &= -\frac{\mu^2}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) + \frac{\lambda}{4}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)^2 \end{aligned}$$

Clearly this potential is invariant under more than just SU(2)$_L \times$U(1)$_Y$: there is a global SO(4) symmetry (homomorphic to SU(2)$\times$SU(2)) under which $(\phi_1, \phi_2, \phi_3, \phi_4)$ transforms as a vector.

 Weinberg 1967

Spontaneous symmetry breaking: coefficient of $\Phi^\dagger\Phi$ is negative





Vacuum: $(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) \equiv v^2 = \mu^2/\lambda$

Vacuum value of $(\phi_1, \phi_2, \phi_3, \phi_4)$ must choose a direction:
Breaks three SO(4) rotations, preserves the remaining three.
$\cong$ Breaks SU(2)$\times$SU(2) down to diagonal SU(2) subgroup.
This is the custodial SU(2).

Another way to see this: rewrite Φ as a "bidoublet":

$$\overline{\Phi} = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}$$

- Second column is the original Φ.
- First column is the conjugate doublet $\tilde{\Phi} \equiv i\sigma^2 \Phi^*$ (also transforms as a doublet because SU(2) is pseudo-real).

$$V = -\frac{\mu^2}{2} \mathrm{Tr}(\overline{\Phi}^\dagger \overline{\Phi}) + \frac{\lambda}{4} [\mathrm{Tr}(\overline{\Phi}^\dagger \overline{\Phi})]^2$$

V is invariant under SU(2)$_L \times$SU(2)$_R$ transformations:

$$\overline{\Phi} \to \exp(i\theta_L^a \tau^a) \overline{\Phi} \exp(-i\theta_R^b \tau^b)$$

Vacuum preserves diagonal subgroup $\theta_L^a = \theta_R^a$: (custodial SU(2))

$$\langle \overline{\Phi} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \propto I_{2\times 2}$$

These are global symmetries. Match them back to the gauge symmetries? SU(2)$_L\times$SU(2)$_R \leftarrow$? $\rightarrow$ SU(2)$_L\times$U(1)$_Y$

- Global SU(2)$_L$ is the gauged SU(2)$_L$.

- The T^3 generator of global SU(2)$_R$ is the hypercharge U(1)$_Y$ generator.

- The T^3 generator of the custodial SU(2) is the electric charge operator (unbroken).

- Gauging only the one (hypercharge) generator of SU(2)$_R$ breaks the global symmetry without promoting it to a full SU(2) gauge symmetry. $\rightarrow$ hypercharge is going to cause some trouble down the line....

Gauge boson masses in the SM come from the gauge-covariant derivative terms in the Lagrangian acting upon the Higgs field's vacuum expectation value. ($Y = 1/2$, $\tau^a = \sigma^a/2$)

$$\mathcal{L} \supset (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi), \qquad \mathcal{D}_\mu = \partial_\mu - ig' Y B_\mu - ig \tau^a W^a_\mu$$

Gauge boson mass terms generated:
write in matrix form in basis (W^1, W^2, W^3, B):

$$M^2 = \frac{v^2}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}$$

- $W^\pm_\mu = (W^1_\mu \mp i W^2_\mu)/\sqrt{2}$ have the same mass $M_W = gv/2$ and do not mix with anything else (charge is conserved).
- W^3_μ and B_μ mix by $\theta_W = \tan^{-1}(g'/g)$ to produce the massive Z with $M_Z = \sqrt{g^2 + g'^2}\, v/2$ and the massless photon.

Gauge boson masses in the SM come from the gauge-covariant derivative terms in the Lagrangian acting upon the Higgs field's vacuum expectation value. ($Y = 1/2$, $\tau^a = \sigma^a/2$)

$$\mathcal{L} \supset (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi), \qquad \mathcal{D}_\mu = \partial_\mu - i g' Y B_\mu - i g \tau^a W^a_\mu$$

Gauge boson mass terms generated:
write in matrix form in basis (W^1, W^2, W^3, B):

$$M^2 = \frac{v^2}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}$$

The custodial symmetry manifests here in the limit $g' \to 0$ as an invariance under SU(2) rotations among (W^1, W^2, W^3).

Consequence with $g' \neq 0$ is that $\rho_0 \equiv M_W^2 / M_Z^2 \cos^2\theta_W = 1$.
Experiment: $\rho_0 = 1.00039 \pm 0.00019$ (PDG 2018).

 Weinberg 1967

Higgs bidoublet is $2 \otimes 2$ under SU(2)$_L\times$SU(2)$_R$:

$$\overline{\Phi} = \begin{pmatrix} \phi^{0*} & \phi^{+} \\ -\phi^{+*} & \phi^{0} \end{pmatrix}$$

Breaking SU(2)$_L\times$SU(2)$_R \to$ SU(2)$_{\text{custodial}} \Rightarrow 2 \otimes 2 \to 3 \oplus 1$.
- Custodial triplet $(\phi^{+}, \sqrt{2}\text{Im}\phi^{0}, \phi^{+*})$ are the (eaten) Goldstone bosons.
- Custodial singlet $\sqrt{2}\text{Re}\phi^{0} = h$ is the (physical) Higgs boson.

Higgs couplings to $W^{+}W^{-}$ and ZZ have a characteristic pattern:

$$hW^{+}_{\mu}W^{-}_{\nu}: \quad 2i\frac{M_W^2}{v}g_{\mu\nu}$$

$$hZ_{\mu}Z_{\nu}: \quad 2i\frac{M_Z^2}{v}g_{\mu\nu}$$

Experiment: $\lambda_{WZ} \equiv (g_{hWW}/M_W^2)/(g_{hZZ}/M_Z^2) = 0.88^{+0.10}_{-0.09}$
(ATLAS + CMS 2016).

Models without custodial symmetry?

To get an appreciation of the importance of custodial symmetry, let's look at some ways of breaking the SM gauge symmetry that do not preserve it.

Example 1: Real triplet with $Y = 0$.

$$\Xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ -\xi^{+*} \end{pmatrix}, \qquad \langle \Xi \rangle = \begin{pmatrix} 0 \\ v_\xi \\ 0 \end{pmatrix}$$

Gauge boson mass matrix generated:

$$M^2_{\Xi} = v_\xi^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Real triplet generates a mass for W, but no mass for Z!

see also Georgi & Glashow 1972

Combine with a doublet: θ_W stays the same, but now M_W gets an extra contribution. $\rho_0 \equiv M_W^2/M_Z^2 \cos^2\theta_W > 1$.

Models without custodial symmetry?

To get an appreciation of the importance of custodial symmetry, let's look at some ways of breaking the SM gauge symmetry that do not preserve it.

Example 2: Complex triplet with $Y = 1$.

$$X = \begin{pmatrix} \chi^{++} \\ \chi^{+} \\ \chi^{0} \end{pmatrix}, \qquad \langle X \rangle = \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}$$

Gauge boson mass matrix generated:

$$M_X^2 = v_\chi^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & 2g^2 & -2gg' \\ 0 & 0 & -2gg' & 2g'^2 \end{pmatrix}$$

Complex triplet generates $\sqrt{2}$ more mass for Z than for W!

Combine with a doublet: θ_W stays the same, but now M_Z gets more contribution. $\rho_0 \equiv M_W^2/M_Z^2 \cos^2\theta_W < 1$.

Models without custodial symmetry?

What if we combine the real triplet and the complex triplet?
(At least one doublet is needed to generate the fermion masses.)

$$M_W^2 = \frac{g^2}{4}(v_\phi^2 + 4v_\xi^2 + 4v_\chi^2), \qquad M_Z^2 = \frac{g^2 + g'^2}{4}(v_\phi^2 + 8v_\chi^2)$$

so (using $g^2 + g'^2 = g^2/\cos^2\theta_W$),

$$\rho_0 = \frac{v_\phi^2 + 4v_\xi^2 + 4v_\chi^2}{v_\phi^2 + 8v_\chi^2}$$

If we just fine-tune $v_\xi = v_\chi$ then we are in good shape!

But that is ugly, since the fine-tuning has to be pretty extreme.
Experiment: $\rho_0 = 1.00039 \pm 0.00019$ (PDG 2018).

Instead, let's construct a model including both of the triplets with custodial symmetry re-imposed! Georgi & Machacek 1985