Neutralino annihilation
at next-to-leading order

Heather Logan
University of Wisconsin, Madison

Argonne Theory Institute
May 9-13, 2005

with V. Barger, G. Shaughnessy, & A. Tregre, hep-ph/05xxxxx

Outline

- Introduction: the relic density codes
- Leading order processes
- QCD corrections
- Outlook

Introduction

Our standard picture of dark matter:

- Thermal production in the early universe

- Freeze-out when expansion of the universe competes with pair annihilation

- Relic density is controlled by the annihilation cross section

$\longrightarrow$ I'm going to assume SUSY $\longleftarrow$

Program for understanding dark matter:

1. Measure the parameters (couplings and masses) that determine the annihilation cross section. LHC + ILC program.

2. Compute the resulting relic density, assuming thermal production.

3. Compare to cosmological dark matter density.
   - If they agree, then we understand the microphysics of the dark matter!
   - If they disagree, then there is more going on in the early universe than is dreamt of in our philosophy, e.g. nonthermal production, superWIMPs, etc.

[Similar to BBN.]

Cosmological dark matter density:

- Already measured to impressive precision - WMAP: 10%
- Future will be even better - PLANCK, etc: few %

Computing the relic density:

There are a number of sophisticated relic density codes on the market:
- DarkSUSY, MicrOMEGAs, IsaRED
- plus private codes by Ellis et al and Rozskowski et al

... that calculate to high precision:

- Inclusion of coannihilations, resonance annihilation, thresholds

- Careful treatment of thermal distributions and velocity dependence:
1% accuracy claimed by DarkSUSY

High precision motivates us to consider QCD corrections to annihilation rates.

Neutralino annihilation processes

The most important process for neutralino annihilation depends on the region of parameter space.

- t-channel sfermion exchange
  - squarks and/or sleptons
  - the old “bulk region”

- coannihilation
  - important when lightest neutralino is nearly degenerate with another SUSY particle (e.g., stau or stop or chargino). They freeze out together.

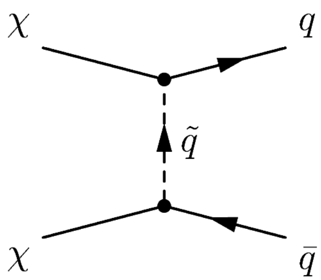
- resonant annihilation
  - through the pseudoscalar Higgs pole (usually at large tan β)
  - through the Z or h poles (in odd corners of parameter space)
  - QCD corrections already included - usual $A \to b\bar{b}$ calculation

- mixed Higgsino-gaugino region
  - annihilation through t-channel chargino exchange, to W pairs
  - happens in Focus Point / Hyperbolic Branch region

We are interested in QCD corrections to t-channel squark exchange.

Will be important only when this process dominates the annihilation cross section.

$\chi\chi \to q\bar{q}$



Neutralinos freeze out at low relative velocity.

$\to$ Can expand the annihilation cross section in powers of v_{rel}:

$$\sigma v_{\mathrm{rel}} = a + b v_{\mathrm{rel}}^2 + \cdots$$

Ellis, Hagelin, Nanopoulos, Olive & Srednicki (1984)

DM codes contain exact $\chi\chi \to q\bar{q}$ cross section

χ is a Majorana fermion: must antisymmetrize initial $\chi\chi$ state.
In the zero-velocity limit, the initial-state spinors simplify:

$$u(p_1)\bar{v}(p_2) - u(p_2)\bar{v}(p_1) = \frac{1}{\sqrt{2}}(2M_\chi + \not{p}_1 + \not{p}_2)\,\gamma^5 = \sqrt{2}M_\chi\left(1+\gamma^0\right)\gamma^5$$

$\longrightarrow$ $\chi\chi$ system behaves like a pseudoscalar.

$\longrightarrow$ s-wave cross section is helicity-suppressed by the final-state quark mass:

$$a \propto m_q^2/M_\chi^2$$

If $M_\chi < m_t$ (sufficiently), then $\chi\chi \to t\bar{t}$ does not contribute.
Then $m_q \ll M_\chi$ for all final state quark pairs.

What about the p-wave part?

b is not suppressed by small mass factors $\longrightarrow$ "p-wave annihilator".

But b is multiplied by $v_{\rm rel}^2$, which is fairly small at freeze-out:

$$b v_{\rm rel}^2 \text{ suppressed by small } v_{\rm rel}^2$$

Can we make the neutralino an "s-wave annihilator"?

Radiating an extra gluon lifts the m_q suppression of the s-wave cross section.

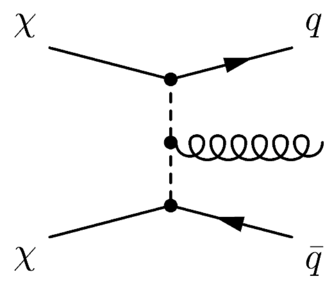
Analogy from B physics:

- B meson is a pseudoscalar, like $\chi\chi$ system at $v_{\text{rel}} = 0$.

- $B_s \to \mu^+\mu^-$ decay suppressed by m_μ^2/m_B^2: need helicity flip to conserve angular momentum. SM cross section is tiny, $\sim 3 \times 10^{-9}$

- $B \to K^{(*)}\mu^+\mu^-$ is much larger, $\sim 10^{-6}$; already observed by Belle and BaBar! Emitting the extra $K^{(*)}$ lifts the helicity suppression.

$\chi\chi \to q\bar{q}g$



$+$ additional diagrams with g attached to external quark legs.

Calculated for $\tilde{\gamma}\tilde{\gamma} \to f\bar{f}\gamma$ with degenerate squarks

Bergstrom (1989); Flores, Olive & Rudaz (1989)

We generalize this to:
- general neutralino composition
- non-degenerate $\tilde{q}_L$ and $\tilde{q}_R$

Our approximations:

- neglect m_q − reasonable for $M_\chi < m_t$.
- neglect v_{rel}^2.

$$\sigma v_{\text{rel}} = a_0 + b_0 v_{\text{rel}}^2 + a_1 + \cdots$$

DM codes do not contain $\chi\chi \to q\bar{q}g$

General neutralino composition:

$$\chi = N_{11}\tilde{B} + N_{12}\tilde{W}^3 + N_{13}\tilde{H}_1^0 + N_{14}\tilde{H}_2^0$$

Neglecting m_q:
- Neglect Higgsino couplings: $\propto$ Yukawas.
- Neglect $\tilde{q}_L - \tilde{q}_R$ mixing.

Leading term in $1/M_{\tilde{q}}$ expansion (for one quark species):

$$a_1 = \frac{\alpha_s}{240\pi^2} M_\chi^6 \left[g_L^4 \frac{1}{M_{\tilde{q}_L}^8} + g_R^4 \frac{1}{M_{\tilde{q}_R}^8} \right]$$

with L and R couplings $(\chi - \tilde{q} - q)$

$$g_L = -\sqrt{2} g'(T^3 - Q_q) N_{11} + \sqrt{2} g T^3 N_{12}, \qquad g_R = -\sqrt{2} g' Q_q N_{11}$$

Note $M_{\tilde{q}}^8$ suppression in the cross section:
from TWO squark propagators.

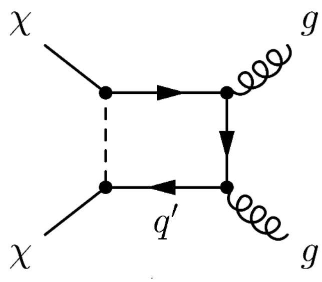
Contribution falls quickly with increasing squark mass.

Compare $M_{\tilde{q}}^4$ suppression in the cross section from the tree-level diagram.

Can the squark mass suppression be reduced, while still avoiding m_q^2 suppression?

Yes, but only by going to one-loop.

$\chi\chi \to gg$



$\tilde{\gamma}\tilde{\gamma} \to \gamma\gamma$: Rudaz (1989); Bouquet, Salati & Silk (1989)
$\tilde{\gamma}\tilde{\gamma} \to gg$: Flores, Olive & Rudaz (1989)

$$a_1 = \frac{16}{3}\frac{\alpha_s^2}{64\pi} M_\chi^2 \mathcal{F}^2$$

$\mathcal{F}$ is a formfactor containing the neutralino couplings and the loop function.

DM codes include $\chi\chi \to gg$ already

For massless quarks q' in the loop, the loop function simplifies:

$$\mathcal{F}(m_q = 0) = \sum_{q'} \left[g_L^2 \frac{1}{M^2_{\tilde{q}'_L}} + g_R^2 \frac{1}{M^2_{\tilde{q}'_R}} \right]$$

Good approximation for 5 light quarks.

Note squark mass dependence:

$$a_1 \propto \mathcal{F}^2 \sim \frac{1}{M^4_{\tilde{q}'}}$$

For a massive quark in the loop, $\mathcal{F}$ contains a 3-point function.
- Consistency: keep $\tilde{q}_L - \tilde{q}_R$ mixing, Higgsino couplings.
- Complete formula for $\mathcal{F}$ is ugly but straightforward.

Contribution of massive quark decouples fairly quickly for $M_\chi < m_t$.

Anything else? ...Yes, another one-loop contribution:

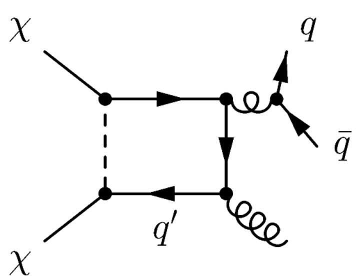
$\chi\chi \to q\bar{q}g$ at one-loop

In general there are MANY diagrams, including IR divergences that must be canceled by tree-level $\chi\chi \to q\bar{q}gg$, $\chi\chi \to q\bar{q}q\bar{q}$.

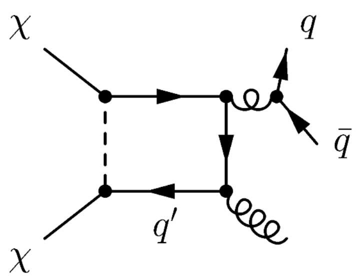
We can get rid of almost all the diagrams by taking the most important (and IR-finite) piece:

- Set $m_q = 0$ for the external $q\bar{q}$
- Set $v_{\text{rel}} = 0$
- Expand in $1/M_{\tilde{q}'}^2$; keep only leading term in amplitude

We are left with one diagram (and its crossings):



DM codes do not contain $\chi\chi \to q\bar{q}g$ at one loop (no surprise...)



Calculated for $\tilde{\gamma}\tilde{\gamma} \to q\bar{q}g$ with degenerate squarks, massless internal quarks

Flores, Olive & Rudaz (1989)

We generalize this to:

- general neutralino composition
- non-degenerate $\tilde{q}_L$ and $\tilde{q}_R$
- massive internal quark (top), including $\tilde{t}_L - \tilde{t}_R$ mixing and Higgsino couplings

Our approximations:

- neglect external m_q: 5 light quarks
- neglect v_{rel}
- leading term in $1/M_{\tilde{q}}$ expansion

Full expression: Must integrate loop function over phase space - no analytic form.

For massless internal quarks q', the loop integral simplifies and the cross section can be integrated analytically:

$$a_1 = \frac{\alpha_s^3}{36\pi^2} M_\chi^2 \left[\sum_{q'} (g_L^2 + g_R^2) \frac{1}{M_{\tilde{q}'}^2} \right]^2 \left[\log \frac{M_\chi^2}{m_q^2} - \frac{11}{6} \right]$$

(given here for degenerate $\tilde{q}_L$ and $\tilde{q}_R$)

Note the $\log(M_\chi^2/m_q^2)$:

- comes from the integration of the off-shell gluon propagator:

$$\int_{p^2=4m_q^2}^{p^2=M_\chi^2} \frac{1}{p^2}$$

- log divergence cut off by the quark mass

- gives a large enhancement of the cross section!

Flores, Olive & Rudaz (1989)

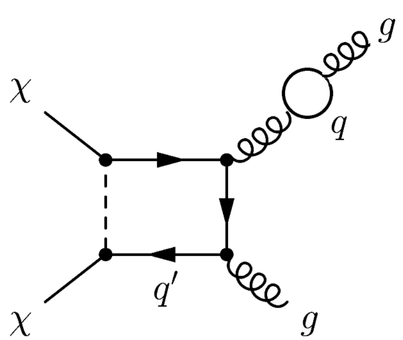
"Everyone needs a log"

That log divergence looks suspicious...

It looks like an IR divergence, cut off by the finite quark mass!

Our one-loop $\chi\chi \to q\bar{q}g$ diagram is the real radiation part of $\chi\chi \to gg$ at next-to-leading (2-loop) order!

We must include the quark bubbles that renormalize the gluon legs.



Gluon leg renormalization:

$$g_s^{\text{bare}} = \frac{1}{\sqrt{Z_3}} g_s^{\text{ren}} = g_s^{\text{ren}} - \frac{1}{2}\hat{\Pi}_2(0) g_s^{\text{ren}}$$

Can read $\hat{\Pi}(0)$ off from the quark loop part of the strong coupling RGE:

$$\hat{\Pi}_2(p^2 \to 0) = -\frac{\alpha_s}{6\pi} \sum_{q_i} \log(m_{q_i}^2/\mu^2)$$

The bubble diagram amounts to a correction to g_s in the original $\chi\chi \to gg$ matrix element.

Original $\chi\chi \to gg$ matrix element $\propto g_s^2$.

LO-NLO interference term:

$$2\,\mathrm{Re}\left[\mathcal{M}^{\mathrm{LO}}_{\chi\chi\to gg} \times \mathcal{M}^{\mathrm{NLO}}_{\chi\chi\to gg}\right] = -2\,\mathrm{Re}\left[\hat{\Pi}_2(0)\right]\left|\mathcal{M}^{\mathrm{LO}}_{\chi\chi\to gg}\right|^2$$

$$a_2 = -2\,\mathrm{Re}\left[\hat{\Pi}_2(0)\right] a_1 = -2\left[-\frac{\alpha_s}{6\pi}\sum_q \log(m_q^2/\mu^2)\right]\left[\frac{16}{3}\frac{\alpha_s^2}{64\pi}M_\chi^2\mathcal{F}^2\right]$$

Plugging in $\mathcal{F}$ for massless internal quarks, this exactly cancels the log part of the one-loop $\chi\chi \to q\bar{q}g$, leaving $\log(M_\chi^2/\mu^2)$ (where μ is the renormalization scale).

This result is brand new, and certainly not in the DM codes

Outlook

The main point:
When t-channel squark exchange dominates the neutralino annihilation cross section, QCD corrections can be important.

- The dark matter relic density is / will be known to high accuracy.
  - WMAP: 10%
  - future: few %

- LHC + ILC will measure the SUSY mass spectrum and couplings.
  - first-principles calculation of neutralino relic density will be possible.

- If squarks are relatively light, QCD corrections to neutralino annihilation will be important.

We've tried to give simple compact formulae that can be implemented into existing DM codes
- including general neutralino composition, nondegenerate squarks, and top squark mixing in 1-loop diagrams.
- making approximations for NLO parts: $m_q = 0$, $v_{\text{rel}} = 0$, leading $1/M_{\tilde{q}}$ pieces.

Numerical work is in progress
- K-factor, impact on relic density for sample SUSY point(s).