Carleton University Physics Department

PHYS 4708 (Winter 2016, H. Logan)

Homework assignment #6

Handed out Wed. Mar. 16; due Mon. Mar. 28, 2016, at the start of class.
Problems are worth 5 points each unless noted otherwise.

1. Consider an electron confined to a ring of radius R in the x-y plane.
   (a) Solve the Schrödinger equation to get the energy eigenfunctions and quantized energy eigenvalues.
   (b) Now assume that the ring has been threaded by a solenoid (coaxial with the ring) of radius $R_s < R$ containing a total magnetic flux Φ ("pointing" in the $+z$ direction). $\vec{B} = 0$ outside the solenoid. Use Stokes' theorem to find the ϕ component of the vector potential A_ϕ at the position of the ring in terms of this flux. (An appropriate choice of gauge will set $A_\rho = A_z = 0$.)
   (c) Write down the Schrödinger equation for the electron on the ring in the presence of the solenoid. Show that your original energy eigenfunctions are still solutions of the new Hamiltonian, but that the corresponding energy eigenvalues are modified. Determine the values of the flux for which the energy spectrum is identical to the system without the solenoid.

2. *[10 points]* (Gasiorowicz 3rd ed. problem 16-4) Consider a charged particle in a magnetic field $\vec{B} = (0, 0, B)$ and a crossed electric field $\vec{E} = (E, 0, 0)$. Solve the eigenvalue problem. *(Hint: the choice of gauge is important. Start by reviewing section 16-4 on Landau levels and Example 11-1. You will not need to make any approximations, other than ignoring spin.)*

3. (Gasiorowicz 3rd edition problem 17-4) Calculate the "$2p \to 1s$" transition rate for a 3-dimensional harmonic oscillator. In this case the energy eigenvalues are given by $E = \hbar\omega(n + 3/2)$ where $n \equiv 2n_r + \ell$, where $n_r = 0, 1, 2, \ldots$ is the radial quantum number and $\ell = 0, 1, 2, \ldots$ is the usual total angular momentum quantum number. *(Hint: we are interested in the transition from the first excited state to the ground state, in the electric dipole approximation. You do not necessarily have to work in spherical coordinates.)*

continued....

4. Consider the following wavefunction for a spherical wave with angular part described by the ℓth Legendre polynomial:

$$\psi(\vec{r}) = C\frac{e^{\pm ikr}}{r}P_\ell(\cos\theta) \tag{1}$$

Compute the three-dimensional probability current density $\vec{j}$ (all three components) in spherical coordinates for $\ell = 0$, 1, and 2. In each case, integrate over a sphere of radius r to get the total inward- or outward-going flux.

For convenience, the three-dimensional probability current density is

$$\vec{j} = \frac{\hbar}{2im}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right), \tag{2}$$

the gradient in spherical coordinates is given by

$$\vec{\nabla}\psi = \hat{r}\frac{\partial\psi}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial\psi}{\partial\theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\varphi}, \tag{3}$$

and the first three Legendre polynomials are

$$P_0(\cos\theta) = 1, \qquad P_1(\cos\theta) = \cos\theta, \qquad P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}. \tag{4}$$