Formula sheet for final exam *These pages will be provided with the exam.*

Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = H\Psi(\vec{r},t) \quad (1)$$

Energy eigenstates: $H = \vec{p}^2/2m + V(\vec{r})$

$$H\psi(\vec{r}) = E\psi(\vec{r}), \qquad \Psi(\vec{r},t) = \psi(\vec{r})\, e^{-iEt/\hbar} \quad (2)$$

Differential operators corresponding to momentum and energy:

$$\vec{p} \to -i\hbar\vec{\nabla} \qquad E \to i\hbar\frac{\partial}{\partial t} \quad (3)$$

Harmonic oscillator in 1 dimension:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2, \qquad H|n\rangle = E_n|n\rangle, \qquad E_n = \hbar\omega(n+1/2) \quad (4)$$

Raising and lowering operators:

$$a|n\rangle = \sqrt{n}|n-1\rangle, \qquad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \quad (5)$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a+a^\dagger), \qquad p_x = -i\sqrt{\frac{m\omega\hbar}{2}}(a-a^\dagger) \quad (6)$$

Harmonic oscillator in 3 dimensions (isotropic), $H = \vec{p}^2/2m + m\omega^2\vec{r}^2/2 = H_x + H_y + H_z$: spatial wavefunctions are just the products of the solutions of the 1-dim harmonic oscillator.

$$H|n_1n_2n_3\rangle = E_{n_1n_2n_3}|n_1n_2n_3\rangle, \qquad E_{n_1n_2n_3} = \hbar\omega(n_1+n_2+n_3+3/2); \quad (7)$$

the wavefunctions can also be written in terms of the spherical harmonics,

$$\langle\vec{r}|n_r\ell m\rangle = R_{n_r\ell}(r)Y_{\ell m}(\theta,\phi), \qquad E_{n_r\ell m} = \hbar\omega(2n_r+\ell+3/2). \quad (8)$$

Particle in a box (infinite square well), 1 dimensional:

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}, \qquad 0 \le x \le L, \qquad E_n = \frac{\hbar^2\pi^2}{2mL^2}n^2 \quad (9)$$

Particle in a box, 3 dimensional, $L \times L \times L$, one corner at the origin:

$$\psi(x,y,z) = \left(\frac{2}{L}\right)^{3/2}\sin\frac{n_1\pi x}{L}\sin\frac{n_2\pi y}{L}\sin\frac{n_3\pi z}{L}, \qquad E = \frac{\hbar^2\pi^2}{2mL^2}(n_1^2+n_2^2+n_3^2) \quad (10)$$

Angular momentum:

$$L^2|\ell,m\rangle = \hbar^2\ell(\ell+1)|\ell,m\rangle, \qquad L_z|\ell,m\rangle = \hbar m|\ell,m\rangle, \qquad m_{\max} = \ell \quad (11)$$

$$L_\pm = L_x \pm iL_y, \qquad [L^2, L_\pm] = 0, \qquad L_\pm|\ell,m\rangle = C_\pm(\ell,m)|\ell,m\pm1\rangle \quad (12)$$

$$C_+(\ell,m) = \hbar\sqrt{(\ell-m)(\ell+m+1)}, \qquad C_-(\ell,m) = \hbar\sqrt{(\ell+m)(\ell-m+1)} \quad (13)$$

$$\vec{L} = \vec{r}\times\vec{p} \qquad \text{so, e.g., } L_z = xp_y - yp_x. \quad (14)$$

Addition of angular momentum: when $\vec{J} = \vec{L} + \vec{S}$,

$$j = \ell+s, \ldots, |\ell-s|, \qquad J_z = L_z + S_z, \qquad J_\pm = L_\pm + S_\pm \quad (15)$$

Spherical harmonics, up to $\ell = 2$:

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \qquad\qquad Y_{\ell m}(-\hat{r}) = (-1)^\ell Y_{\ell m}(\hat{r}) \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi} \qquad Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_{1,-1} = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi} \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\phi} \qquad Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\phi} \qquad Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1) \\
& Y_{2,-1} = \sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{-i\phi} \qquad Y_{2,-2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{-2i\phi}
\end{aligned} \tag{16}$$

Pauli principle: under the exchange of any two identical particles, multi-particle wavefunctions are symmetric for bosons (integer spin), antisymmetric for fermions (half-odd-integer spin).

Time-independent perturbation theory:

$$H = H_0 + \lambda H_1, \qquad H_0|\phi_n\rangle = E_n^{(0)}|\phi_n\rangle, \qquad H|\psi_n\rangle = E_n|\psi_n\rangle \tag{17}$$

$$E_n = E_n^{(0)} + E_n^{(1)} + E_n^{(2)} + \cdots \tag{18}$$

$$E_n^{(1)} = \langle\phi_n|\lambda H_1|\phi_n\rangle \qquad E_n^{(2)} = \sum_{k\neq n}\frac{|\langle\phi_n|\lambda H_1|\phi_k\rangle|^2}{E_n^{(0)} - E_k^{(0)}} \tag{19}$$

$$|\psi_n\rangle = |\phi_n\rangle + \sum_{k\neq n}\frac{\langle\phi_k|\lambda H_1|\phi_n\rangle}{E_n^{(0)} - E_k^{(0)}}|\phi_k\rangle + \mathcal{O}(\lambda^2) \tag{20}$$

Degenerate perturbation theory: first find the combinations of states that diagonalize the matrix $\langle\phi_i|\lambda H_1|\phi_j\rangle$ made up of states degenerate at zeroth order. The eigenvalues of this matrix are $E_n^{(1)}$.

Time-dependent perturbation theory:

$$H_0|\phi_n\rangle = E_n^{(0)}|\phi_n\rangle, \qquad i\hbar\frac{d}{dt}|\psi(t)\rangle = (H_0 + \lambda V(t))|\psi(t)\rangle \tag{21}$$

$$|\psi(t)\rangle = \sum_n c_n(t)e^{-iE_n^{(0)}t/\hbar}|\phi_n\rangle \tag{22}$$

For $c_k(0) = 1$ and all other $c_n(0) = 0$, to first order in λ the coefficients are

$$c_m(t) = \frac{1}{i\hbar}\int_0^t dt' e^{i\omega_{mk}t'}\langle\phi_m|\lambda V(t')|\phi_k\rangle \tag{23}$$

where $\omega_{mk} = (E_m^{(0)} - E_k^{(0)})/\hbar$. The transition probability is $P_{k\to m}(t) = |c_m(t)|^2$.

Variational principle: for *any* wavefunction $|\Psi\rangle$, $\langle\Psi|H|\Psi\rangle \geq E_0$ = ground state energy of H.

Some math:

Taylor series about $x = 0$:

$$f(x) = \sum_{n=0}^{\infty}\frac{1}{n!}\left.\frac{d^n f}{dx^n}\right|_{x=0} x^n \tag{24}$$

Eigenvalues of a matrix M (I is the unit matrix; solve for λ):

$$\det(M - \lambda\cdot I) = 0 \tag{25}$$

Electron interacting with EM fields:

$$H = \frac{1}{2m_e}(\vec{p} + e\vec{A}(\vec{r},t))^2 - e\phi(\vec{r},t) \tag{26}$$

where $\vec{B} = \vec{\nabla} \times \vec{A}$ and $\vec{E} = -\vec{\nabla}\phi$.
Gauge transformation (ψ = electron wavefunction):

$$\vec{A}'(\vec{r},t) = \vec{A}(\vec{r},t) - \vec{\nabla} g(\vec{r},t), \qquad \phi'(\vec{r},t) = \phi(\vec{r},t) + \frac{\partial g(\vec{r},t)}{\partial t}, \qquad \psi'(\vec{r},t) = e^{ieg(\vec{r},t)/\hbar}\psi(\vec{r},t). \tag{27}$$

Transition rate for radiative decays:

$$\Gamma_{n\to m} = \frac{2\pi}{\hbar}|\langle\phi_m|V|\phi_n\rangle|^2\rho(E), \tag{28}$$

where $\rho(E)$ is the density of (final) states (the last expression uses $p = E/c$ for photons),

$$\rho(E) = \frac{V}{(2\pi\hbar)^3}p^2\frac{dp}{dE}d\Omega = \frac{V}{(2\pi\hbar)^3}\frac{\hbar^2\omega^2}{c^3}d\Omega. \tag{29}$$

Matrix element $\langle\phi_m|V|\phi_n\rangle$ involves

$$V = \frac{e}{m_e}\vec{A}\cdot\vec{p} = \frac{e}{m_e}\sqrt{\frac{\hbar}{2\epsilon_0\omega V}}e^{-i\vec{k}\cdot\vec{r}}\vec{\epsilon}^{(\lambda)}\cdot\vec{p}. \tag{30}$$

Electric dipole approximation: $e^{-i\vec{k}\cdot\vec{r}} \simeq 1$.
Photon is transverse: $\vec{\epsilon}^{(\lambda)}\cdot\vec{k} = 0$.
Handy trick for evaluating matrix elements of $\vec{p}$: use

$$\vec{p} = m_e\frac{d\vec{r}}{dt} = m_e\frac{-i}{\hbar}[H_0, \vec{r}]. \tag{31}$$

Scattering theory
Elastic scattering cross section $d\sigma = |f(\theta)|^2 d\Omega$ (k = wavenumber of beam):

$$f(\theta) = \sum_{\ell=0}^{\infty}(2\ell+1)\frac{S_\ell(k)-1}{2ik}P_\ell(\cos\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell(k)}\sin\delta_\ell(k)P_\ell(\cos\theta), \tag{32}$$

where $S_\ell(k) = e^{2i\delta_\ell(k)}$ is the phase shift. Total cross section is

$$\sigma_{\rm tot} = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell(k). \tag{33}$$

Inelastic scattering: $S_\ell(k) = e^{2i\delta_\ell(k)} \to \eta_\ell(k)e^{2i\delta_\ell(k)}$, $0 \le \eta_\ell(k) \le 1$.
Elastic, inelastic, and total cross sections:

$$\sigma_{\rm el} = \frac{4\pi}{k^2}\sum_\ell(2\ell+1)\frac{1+\eta_\ell^2 - 2\eta_\ell\cos 2\delta_\ell}{4} \tag{34}$$

$$\sigma_{\rm inel} = \frac{4\pi}{k^2}\sum_\ell(2\ell+1)\frac{1-\eta_\ell^2}{4} \tag{35}$$

$$\sigma_{\rm tot} = \sigma_{\rm el} + \sigma_{\rm inel} = \frac{4\pi}{k^2}\sum_\ell(2\ell+1)\frac{1-\eta_\ell\cos 2\delta_\ell}{2} \tag{36}$$

Born approximation: scattering rate:

$$R_{i\to f} = \frac{2\pi}{\hbar}\int \frac{V}{(2\pi\hbar)^3} d^3p_f |M_{fi}|^2 \delta(E_f - E_i) = \frac{1}{4\pi^2\hbar^4 V}\int d\Omega\, p_f m_f |\widetilde{V}(\vec{\Delta})|^2 \tag{37}$$

where $\vec{\Delta} = (\vec{p}_f - \vec{p}_i)/\hbar$ and

$$M_{fi} = \langle\psi_f|V(\vec{r})|\psi_i\rangle = \int d^3r \frac{1}{\sqrt{V}} e^{-i\vec{p}_f\cdot\vec{r}/\hbar} V(\vec{r}) \frac{1}{\sqrt{V}} e^{i\vec{p}_i\cdot\vec{r}/\hbar} = \frac{1}{V}\int d^3r\, e^{-i\vec{\Delta}\cdot\vec{r}} V(\vec{r}) \equiv \frac{1}{V}\widetilde{V}(\vec{\Delta}). \tag{38}$$

Cross section: divide scattering rate by incoming flux $= |v_{\rm rel}|/V$ (with $|v_{\rm rel}| = p_i/m_i$):

$$d\sigma = \frac{1}{4\pi^2\hbar^4}\frac{1}{|v_{\rm rel}|} p_f m_f |\widetilde{V}(\vec{\Delta})|^2 d\Omega. \tag{39}$$

Identical particles:

$$\frac{d\sigma}{d\Omega} = |f(\theta) \pm f(\pi - \theta)|^2 \tag{40}$$

with the $\pm$ sign chosen according to whether the spatial part of the wavefunction should be symmetric or antisymmetric under interchange of the two particles.
Unpolarized cross section: average over initial spins, sum over final spins.

40. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

J	J	...
M	M	...
m_1 m_2	Coefficients	
m_1 m_2		
⋮ ⋮		

1/2×1/2

m_1 m_2	J=1, M=+1
+1/2 +1/2	1

m_1 m_2	J=1, M=0	J=0, M=0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	J=1, M=−1
−1/2 −1/2	1

1×1/2

m_1 m_2	J=3/2, M=+3/2
+1 +1/2	1

m_1 m_2	3/2, +1/2	1/2, +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	3/2, −1/2	1/2, −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	3/2, −3/2
−1 −1/2	1

2×1

m_1 m_2	3, +3
+2 +1	1

m_1 m_2	3, +2	2, +2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	3, +1	2, +1	1, +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

m_1 m_2	3, 0	2, 0	1, 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	3, −1	2, −1	1, −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	3, −2	2, −2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	3, −3
−2 −1	1

1×1

m_1 m_2	2, +2
+1 +1	1

m_1 m_2	2, +1	1, +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	2, 0	1, 0	0, 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	2, −1	1, −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	2, −2
−1 −1	1

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2×1/2

m_1 m_2	5/2, +5/2
+2 +1/2	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	5/2, +1/2	3/2, +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	5/2, −1/2	3/2, −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	5/2, −3/2	3/2, −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	5/2, −5/2
−2 −1/2	1

3/2×1/2

m_1 m_2	2, +2
+3/2 +1/2	1

m_1 m_2	2, +1	1, +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	2, 0	1, 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	2, −1	1, −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	2, −2
−3/2 −1/2	1

3/2×1

m_1 m_2	5/2, +5/2
+3/2 +1	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	5/2, −3/2	3/2, −3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	5/2, −5/2
−3/2 −1	1

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^{\,j}_{m',m} = (-1)^{m-m'} d^{\,j}_{m,m'} = d^{\,j}_{-m,-m'}$$

2×3/2

m_1 m_2	7/2, +7/2
+2 +3/2	1

m_1 m_2	7/2, +5/2	5/2, +5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	−3/7

m_1 m_2	7/2, +3/2	5/2, +3/2	3/2, +3/2
+2 −1/2	1/7	16/35	2/5
+1 +1/2	4/7	1/35	−2/5
0 +3/2	2/7	−18/35	1/5

m_1 m_2	7/2, +1/2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+2 −3/2	1/35	6/35	2/5	2/5
+1 −1/2	12/35	5/14	0	−3/10
0 +1/2	18/35	−3/35	−1/5	1/5
−1 +3/2	4/35	−27/70	2/5	−1/10

m_1 m_2	7/2, −1/2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1 −3/2	4/35	27/70	2/5	1/10
0 −1/2	18/35	3/35	−1/5	−1/5
−1 +1/2	12/35	−5/14	0	3/10
−2 +3/2	1/35	−6/35	2/5	−2/5

m_1 m_2	7/2, −3/2	5/2, −3/2	3/2, −3/2
0 −3/2	2/7	18/35	1/5
−1 −1/2	4/7	−1/35	−2/5
−2 +1/2	1/7	−16/35	2/5

m_1 m_2	7/2, −5/2	5/2, −5/2
−1 −3/2	4/7	3/7
−2 −1/2	3/7	−4/7

m_1 m_2	7/2, −7/2
−2 −3/2	1

3/2×3/2

m_1 m_2	3, +3
+3/2 +3/2	1

m_1 m_2	3, +2	2, +2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	−1/2

m_1 m_2	3, +1	2, +1	1, +1
+3/2 −1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	−2/5
−1/2 +3/2	1/5	−1/2	3/10

m_1 m_2	3, 0	2, 0	1, 0	0, 0
+3/2 −3/2	1/20	1/4	9/20	1/4
+1/2 −1/2	9/20	1/4	−1/20	−1/4
−1/2 +1/2	9/20	−1/4	−1/20	1/4
−3/2 +3/2	1/20	−1/4	9/20	−1/4

m_1 m_2	3, −1	2, −1	1, −1
+1/2 −3/2	1/5	1/2	3/10
−1/2 −1/2	3/5	0	−2/5
−3/2 +1/2	1/5	−1/2	3/10

m_1 m_2	3, −2	2, −2
−1/2 −3/2	1/2	1/2
−3/2 −1/2	1/2	−1/2

m_1 m_2	3, −3
−3/2 −3/2	1

$$d^{\,1}_{0,0} = \cos\theta \qquad d^{\,1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{\,1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^{\,1}_{1,1} = \frac{1+\cos\theta}{2} \qquad d^{\,1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^{\,1}_{1,-1} = \frac{1-\cos\theta}{2}$$

2×2

m_1 m_2	4, +4
+2 +2	1

m_1 m_2	4, +3	3, +3
+2 +1	1/2	1/2
+1 +2	1/2	−1/2

m_1 m_2	4, +2	3, +2	2, +2
+2 0	3/14	1/2	2/7
+1 +1	4/7	0	−3/7
0 +2	3/14	−1/2	2/7

m_1 m_2	4, +1	3, +1	2, +1	1, +1
+2 −1	1/14	3/10	3/7	1/5
+1 0	3/7	1/5	−1/14	−3/10
0 +1	3/7	−1/5	−1/14	3/10
−1 +2	1/14	−3/10	3/7	−1/5

m_1 m_2	4, 0	3, 0	2, 0	1, 0	0, 0
+2 −2	1/70	1/10	2/7	2/5	1/5
+1 −1	8/35	2/5	1/14	−1/10	−1/5
0 0	18/35	0	−2/7	0	1/5
−1 +1	8/35	−2/5	1/14	1/10	−1/5
−2 +2	1/70	−1/10	2/7	−2/5	1/5

m_1 m_2	4, −1	3, −1	2, −1	1, −1
+1 −2	1/14	3/10	3/7	1/5
0 −1	3/7	1/5	−1/14	−3/10
−1 0	3/7	−1/5	−1/14	3/10
−2 +1	1/14	−3/10	3/7	−1/5

m_1 m_2	4, −2	3, −2	2, −2
0 −2	3/14	1/2	2/7
−1 −1	4/7	0	−3/7
−2 0	3/14	−1/2	2/7

m_1 m_2	4, −3	3, −3
−1 −2	1/2	1/2
−2 −1	1/2	−1/2

m_1 m_2	4, −4
−2 −2	1

$$d^{\,3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^{\,2}_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{\,2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^{\,2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^{\,2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{\,2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{\,2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^{\,2}_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 40.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).