

**Carleton University Physics Department**  
**PHYS 4708 (Winter 2015, H. Logan)**  
**Homework assignment #8**

Handed out Wed. Mar. 25; due Mon. Apr. 6, 2015 at the start of class.  
*Problems are worth 5 points each unless noted otherwise.*

1. (Gasiorowicz 3rd ed. problem 19-1) Show that for a central potential  $V(\vec{r}) = V(r)$ , the matrix element  $M_{fi} = \langle \phi_f | V | \phi_i \rangle$  that appears in the Born approximation (see Gasiorowicz Eq. (19-77)) can be written in the form

$$M_{fi} = \frac{4\pi}{V} \int_0^\infty r V(r) \frac{\sin r \Delta}{\Delta} dr. \quad (1)$$

Note that this is a function of

$$\Delta \equiv |\vec{\Delta}| = \frac{1}{\hbar} |\vec{p}_f - \vec{p}_i|, \quad (2)$$

and hence is invariant under  $\vec{\Delta} \rightarrow -\vec{\Delta}$ .

2. Consider the scattering of two identical spin-1 particles. Following Gasiorowicz section 19-4, determine the appropriate form of the differential cross section for the states with total spin 0, 1, and 2. In each case, work out the differential cross section at  $\theta = \pi/2$  in terms of  $f(\pi/2)$ , including the interference term. What is the differential cross section for unpolarized scattering, including the interference?
3. (Gasiorowicz 3rd ed. problem 19-3) Consider the potential

$$V(r) = V_0 \frac{a}{r} e^{-r/a}. \quad (3)$$

If the range parameter is  $a = 1.2 \text{ fm} = 1.2 \times 10^{-15} \text{ m}$  and  $V_0 = 100 \text{ MeV}$  in magnitude, what is the total cross section for proton-proton scattering at 100 MeV centre-of-mass energy, calculated in the Born approximation? Pay attention to the fact that the two protons are identical fermions. (*This potential models scattering via the exchange of pions. It does not include the contribution from Coulomb scattering, which is a small effect anyway for the high centre-of-mass energy that we consider.*)

*When calculating the total cross section, it's convenient to write*

$$\hbar^2 \Delta^2 = |\vec{p}_f - \vec{p}_i|^2 = 2p^2(1 - \cos \theta) \quad (4)$$

*so that*

$$\int d\Omega = 2\pi \int d\cos \theta = \frac{\hbar^2 \pi}{p^2} \int d(\Delta^2). \quad (5)$$

*continued....*

4. (Gasiorowicz 3rd ed. problem 19-4) Suppose the scattering amplitude for neutron-proton scattering is given by the form

$$f(\theta) = \xi_f^\dagger (A + B \vec{\sigma}_p \cdot \vec{\sigma}_n) \xi_i, \quad (6)$$

where  $\xi_i$  and  $\xi_f$  are the initial and final spin states of the neutron-proton system. The possible states are

$$\xi_i = \begin{pmatrix} \chi_{\uparrow}^{(p)} \chi_{\uparrow}^{(n)} \\ \chi_{\uparrow}^{(p)} \chi_{\downarrow}^{(n)} \\ \chi_{\downarrow}^{(p)} \chi_{\uparrow}^{(n)} \\ \chi_{\downarrow}^{(p)} \chi_{\downarrow}^{(n)} \end{pmatrix} \quad \xi_f = \begin{pmatrix} \chi_{\uparrow}^{(p)} \chi_{\uparrow}^{(n)} \\ \chi_{\downarrow}^{(p)} \chi_{\downarrow}^{(n)} \\ \chi_{\uparrow}^{(p)} \chi_{\downarrow}^{(n)} \\ \chi_{\downarrow}^{(p)} \chi_{\uparrow}^{(n)} \end{pmatrix} \quad (7)$$

and the scalar product of the Pauli spin matrices can be written as

$$\vec{\sigma}_p \cdot \vec{\sigma}_n = \sigma_z^{(p)} \sigma_z^{(n)} + 2 \left( \sigma_+^{(p)} \sigma_-^{(n)} + \sigma_-^{(p)} \sigma_+^{(n)} \right). \quad (8)$$

Calculate the scattering amplitudes and corresponding total cross sections for all 16 initial and final spin combinations and make a table of your results.

*You can work in the representation where*

$$\sigma_+ = \frac{\sigma_x + i\sigma_y}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_- = \frac{\sigma_x - i\sigma_y}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (9)$$

and

$$\chi_{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (10)$$

5. (Gasiorowicz 3rd ed. problem 19-6) Neutron-proton scattering, continued.

- (a) Use the table of scattering amplitudes computed in the previous problem to calculate the amplitudes and total cross sections for total-spin triplet  $\rightarrow$  triplet and singlet  $\rightarrow$  singlet scattering. Show that the triplet  $\rightarrow$  singlet scattering vanishes. (*Note: for each total-spin state, add the amplitudes before squaring! The triplet comprises three states: for the total triplet  $\rightarrow$  triplet cross section, you'll need to average over the initial and sum over the final triplet states.*)
- (b) Check your results by recomputing the scattering amplitudes and cross sections in the  $|j, m_j\rangle$  basis (where  $\vec{J} = \vec{S}_p + \vec{S}_n$ ), by noting that

$$\vec{J} = \frac{\hbar}{2} \vec{\sigma}_p + \frac{\hbar}{2} \vec{\sigma}_n, \quad (11)$$

so that

$$\vec{\sigma}_p \cdot \vec{\sigma}_n = \frac{2J^2}{\hbar^2} - 3 = \begin{cases} 1 & \text{acting on triplet state,} \\ -3 & \text{acting on singlet state.} \end{cases} \quad (12)$$

*(Remember that there are three  $m_j$  states in the triplet, each contributing equally to the cross section.)*