Carleton University Physics Department
PHYS 4708 (Winter 2015, H. Logan)
Homework assignment #7

Handed out Mon. Mar. 16; due Wed. Mar. 25, 2015 at the start of class.
Problems are worth 5 points each unless noted otherwise.

1. *[10 points]* (Gasiorowicz 3rd edition problem 17-4) Calculate the "$2p \to 1s$" transition rate for a 3-dimensional harmonic oscillator. In this case the energy eigenvalues are given by $E = \hbar\omega(n + 3/2)$ where $n \equiv 2n_r + \ell$, where $n_r = 0, 1, 2, \ldots$ is the radial quantum number and $\ell = 0, 1, 2, \ldots$ is the usual total angular momentum quantum number. *(Hint: we are interested in the transition from the first excited state to the ground state, in the electric dipole approximation. You do not necessarily have to work in spherical coordinates.)*

2. *[10 points]* (Gasiorowicz 3rd edition problem 17-2) The matrix element for electromagnetic transitions can be expanded in powers of $\vec{k} \cdot \vec{r}$ to give

$$\langle \phi_f | e^{-i\vec{k}\cdot\vec{r}} \vec{\varepsilon} \cdot \vec{p} | \phi_i \rangle = \langle \phi_f | \vec{\varepsilon} \cdot \vec{p} | \phi_i \rangle + \langle \phi_f | -i\vec{k} \cdot \vec{r}\, \vec{\varepsilon} \cdot \vec{p} | \phi_i \rangle + \cdots \tag{1}$$

where

$$\vec{k} \cdot \vec{r}\, \vec{\varepsilon} \cdot \vec{p} = \frac{1}{2}(\vec{\varepsilon} \cdot \vec{p}\, \vec{k} \cdot \vec{r} + \vec{\varepsilon} \cdot \vec{r}\, \vec{p} \cdot \vec{k}) + \frac{1}{2}(\vec{k} \times \vec{\varepsilon}) \cdot (\vec{r} \times \vec{p}) \tag{2}$$

(see Eq. (17-35) of Gasiorowicz). Show that the first term in Eq. (2) above, called the electric quadrupole term, leads to $\Delta\ell = 2$ transitions by evaluating the term between a state $Y_{\ell m}(\theta, \phi)$ and the ground state Y_{00}.

3. Consider the following wavefunction for a spherical wave with angular part described by the ℓth Legendre polynomial:

$$\psi(\vec{r}) = C \frac{e^{\pm ikr}}{r} P_\ell(\cos\theta) \tag{3}$$

Compute the three-dimensional probability current density $\vec{j}$ (all three components) in spherical coordinates for $\ell = 0$, 1, and 2. In each case, integrate over a sphere of radius r to get the total inward- or outward-going flux.

For convenience, the three-dimensional probability current density is

$$\vec{j} = \frac{\hbar}{2im} \left(\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^* \right), \tag{4}$$

the gradient in spherical coordinates is given by

$$\vec{\nabla}\psi = \hat{r} \frac{\partial \psi}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial \psi}{\partial \theta} + \hat{\varphi} \frac{1}{r \sin\theta} \frac{\partial \psi}{\partial \varphi}, \tag{5}$$

and the first three Legendre polynomials are

$$P_0(\cos\theta) = 1, \qquad P_1(\cos\theta) = \cos\theta, \qquad P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}. \tag{6}$$