Carleton University Physics Department

PHYS 4708 (Winter 2015, H. Logan)

Homework assignment #6

Handed out Wed. Mar. 4, 2015; due Mon. Mar. 16 at the start of class.
Problems are worth 5 points each unless noted otherwise.

1. (similar to Gasiorowicz 3rd ed. problem 16-1) Consider a particle of mass m in a three-dimensional harmonic oscillator with potential energy $m\omega^2 r^2/2$. Because the potential is radial, the energy eigenstates can be written in terms of spherical harmonics,

$$\psi(\vec{r}) = R_{n_r \ell}(r) Y_{\ell m_\ell}(\theta, \phi), \tag{1}$$

where $R_{n_r\ell}(r)$ is the radial wavefunction (not the same as the radial wavefunctions of the hydrogen atom). The energy eigenvalues are $E = \hbar\omega(2n_r + \ell + 3/2)$, where $n_r = 0, 1, 2, \ldots$ and $\ell = 0, 1, 2, \ldots$.

   (a) Work out the combinations of n_r and ℓ that contribute to the first five energy levels.

   (b) Now suppose the particle has charge q and the harmonic oscillator is placed in a *weak* constant uniform magnetic field $\vec{B}$. Write down the Schrödinger equation in the presence of this field and treat the new term(s) as a time-independent perturbing Hamiltonian H_1. Compute the first-order shifts $E^{(1)}$ in the energies due to this perturbation, and sketch the resulting spectrum (assume that the magnetic energy shifts are small compared to the harmonic oscillator level spacings).

   Hint: For a constant uniform magnetic field you can use $\vec{A} = -\vec{r} \times \vec{B}/2$. For a weak field, you can neglect terms of second order in $\vec{A}$. The vector identity $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$ may come in handy.

2. *[10 points]* Consider an electron confined to a ring of radius R in the x-y plane.

   (a) Solve the Schrödinger equation to get the energy eigenfunctions and quantized energy eigenvalues.

   (b) Now assume that the ring has been threaded by a solenoid (coaxial with the ring) of radius $R_s < R$ containing a total magnetic flux Φ ("pointing" in the $+z$ direction). $\vec{B} = 0$ outside the solenoid. Use Stokes' theorem to find the ϕ component of the vector potential A_ϕ at the position of the ring in terms of this flux. (An appropriate choice of gauge will set $A_\rho = A_z = 0$.)

   (c) Write down the Schrödinger equation for the electron on the ring in the presence of the solenoid. Show that your original energy eigenfunctions are still solutions of the new Hamiltonian, but that the corresponding energy eigenvalues are modified. Determine the values of the flux for which the energy spectrum is identical to the system without the solenoid.

continued....

3. *[10 points]* (Gasiorowicz 3rd ed. problem 16-4) Consider a charged particle in a magnetic field $\vec{B} = (0, 0, B)$ and a crossed electric field $\vec{E} = (E, 0, 0)$. Solve the eigenvalue problem. *(Hint: the choice of gauge is important. Start by reviewing section 16-4 on Landau levels and Example 11-1. You will not need to make any approximations, other than ignoring spin.)*