Formula sheet for midterm exam

This page (both sides), and photocopies of the front and back covers of Griffiths, will be provided with the exam.

Electric field due to a source charge q at the origin:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \tag{1}$$

Force on test charge Q at $\vec{\mathbf{r}}$ due to source charge q at the origin:

$$\vec{\mathbf{F}} = Q\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\mathbf{r}} \tag{2}$$

Electric potential:

$$V(\vec{r}) = -\int_{\mathcal{O}}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}, \qquad \vec{\mathbf{E}} = -\vec{\nabla} V \tag{3}$$

Choosing $\mathcal{O}$ at infinity, potential of a source charge q at the origin:

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{4}$$

Gauss's law:

$$\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = \frac{Q_{\text{encl}}}{\epsilon_0}, \qquad \vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \tag{5}$$

Poisson's equation (reduces to Laplace's equation for $\rho = 0$):

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \tag{6}$$

Capacitance, definition of:

$$C = \frac{Q}{\Delta V}, \qquad C = \frac{A\epsilon_0}{d} \text{ for parallel plate} \tag{7}$$

Electrostatic energy for point charges and continuous charge distributions:

$$W = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}, \qquad W = \frac{1}{2}\int \rho\, V\, d\tau = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau \tag{8}$$

$$W = \frac{1}{2}C(\Delta V)^2 \text{ stored in a capacitor} \tag{9}$$

Discontinuities across a surface charge:

$$\vec{\mathbf{E}}_{\text{above}} - \vec{\mathbf{E}}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}, \tag{10}$$

$$V_{\text{above}} = V_{\text{below}}, \qquad \frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = -\frac{\sigma}{\epsilon_0} \tag{11}$$

Electrostatic pressure (on surface of conductor):

$$\vec{f} = \frac{\sigma^2}{2\epsilon_0}\hat{n} = \frac{\epsilon_0}{2}E^2\hat{n} \tag{12}$$

Method of Images for a grounded conducting sphere with radius R and a point charge q a distance a from the centre of the sphere:

$$q' = -\frac{R}{a}q, \qquad b = \frac{R^2}{a} \tag{13}$$

where q' is a distance b from the centre of the sphere. For a conducting sphere at potential V_0, add a second image charge q'' at the centre of the sphere such that

$$V_0 = \frac{1}{4\pi\epsilon_0}\frac{q''}{R} \tag{14}$$

Solutions of Laplace's equation:

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta) \tag{15}$$

$$V(s,\phi) = A_0 \ln s + B_0 + \sum_{m=1}^{\infty}(A_m \cos m\phi + B_m \sin m\phi)\left(s^m + C_m s^{-m}\right) \tag{16}$$

Legendre polynomials:

$P_0(x) = 1$	$P_3(x) = (5x^3 - 3x)/2$
$P_1(x) = x$	$P_4(x) = (35x^4 - 30x^2 + 3)/8$
$P_2(x) = (3x^2 - 1)/2$	$P_5(x) = (63x^5 - 70x^3 + 15x)/8$