## Formula sheet for final exam

This formula sheet, and copies of the front and back covers of Griffiths, will be provided with the exam.

Electric field due to a source charge $q$ at the origin:

$$
\begin{equation*}
\overrightarrow{\mathbf{E}}(\overrightarrow{\mathbf{r}})=\frac{1}{4 \pi \epsilon_{0}} \frac{q}{r^{2}} \hat{\mathbf{r}} \tag{1}
\end{equation*}
$$

Force on test charge $Q$ at $\overrightarrow{\mathbf{r}}$ due to source charge $q$ at the origin:

$$
\begin{equation*}
\overrightarrow{\mathbf{F}}=Q \overrightarrow{\mathbf{E}}(\overrightarrow{\mathbf{r}})=\frac{1}{4 \pi \epsilon_{0}} \frac{Q q}{r^{2}} \hat{\mathbf{r}} \tag{2}
\end{equation*}
$$

Electric potential:

$$
\begin{equation*}
V(\vec{r})=-\int_{\mathcal{O}}^{\overrightarrow{\mathbf{r}}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{l}}, \quad \overrightarrow{\mathbf{E}}=-\vec{\nabla} V \tag{3}
\end{equation*}
$$

Choosing $\mathcal{O}$ at infinity, potential of a source charge $q$ at the origin:

$$
\begin{equation*}
V(\overrightarrow{\mathbf{r}})=\frac{1}{4 \pi \epsilon_{0}} \frac{q}{r} \tag{4}
\end{equation*}
$$

Gauss's law:

$$
\begin{equation*}
\oint_{S} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{a}}=\frac{Q_{\mathrm{encl}}}{\epsilon_{0}}, \quad \vec{\nabla} \cdot \overrightarrow{\mathbf{E}}=\frac{\rho}{\epsilon_{0}} \tag{5}
\end{equation*}
$$

Poisson's equation (reduces to Laplace's equation for $\rho=0$ ):

$$
\begin{equation*}
\nabla^{2} V=-\frac{\rho}{\epsilon_{0}} \tag{6}
\end{equation*}
$$

Capacitance, definition of:

$$
\begin{equation*}
C=\frac{Q}{\Delta V}, \quad C=\frac{A \epsilon_{0}}{d} \text { for parallel plate } \tag{7}
\end{equation*}
$$

Electrostatic energy for point charges and continuous charge distributions:

$$
\begin{gather*}
W=\frac{1}{4 \pi \epsilon_{0}} \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{q_{i} q_{j}}{\left|\vec{r}_{i}-\vec{r}_{j}\right|}, \quad W=\frac{1}{2} \int \rho V d \tau=\frac{\epsilon_{0}}{2} \int_{\text {all space }} E^{2} d \tau  \tag{8}\\
W=\frac{1}{2} C(\Delta V)^{2} \text { stored in a capacitor } \tag{9}
\end{gather*}
$$

Discontinuities across a surface charge:

$$
\begin{gather*}
\overrightarrow{\mathbf{E}}_{\text {above }}-\overrightarrow{\mathbf{E}}_{\text {below }}=\frac{\sigma}{\epsilon_{0}} \hat{\mathbf{n}},  \tag{10}\\
V_{\text {above }}=V_{\text {below }}, \quad \frac{\partial V_{\text {above }}}{\partial n}-\frac{\partial V_{\text {below }}}{\partial n}=-\frac{\sigma}{\epsilon_{0}} \tag{11}
\end{gather*}
$$

Electrostatic pressure (on surface of conductor):

$$
\begin{equation*}
\vec{f}=\frac{\sigma^{2}}{2 \epsilon_{0}} \hat{n}=\frac{\epsilon_{0}}{2} E^{2} \hat{n} \tag{12}
\end{equation*}
$$

Method of Images for a grounded conducting sphere with radius $R$ and a point charge $q$ a distance $a$ from the centre of the sphere:

$$
\begin{equation*}
q^{\prime}=-\frac{R}{a} q, \quad b=\frac{R^{2}}{a} \tag{13}
\end{equation*}
$$

where $q^{\prime}$ is a distance $b$ from the centre of the sphere. For a conducting sphere at potential $V_{0}$, add a second image charge $q^{\prime \prime}$ at the centre of the sphere such that

$$
\begin{equation*}
V_{0}=\frac{1}{4 \pi \epsilon_{0}} \frac{q^{\prime \prime}}{R} \tag{14}
\end{equation*}
$$

Solutions of Laplace's equation:

$$
\begin{gather*}
V(r, \theta)=\sum_{l=0}^{\infty}\left(A_{l} r^{l}+\frac{B_{l}}{r^{l+1}}\right) P_{l}(\cos \theta)  \tag{15}\\
V(s, \phi)=A_{0} \ln s+B_{0}+\sum_{m=1}^{\infty}\left(A_{m} \cos m \phi+B_{m} \sin m \phi\right)\left(s^{m}+C_{m} s^{-m}\right) \tag{16}
\end{gather*}
$$

Legendre polynomials:

$$
\begin{array}{ll}
P_{0}(x)=1 & P_{3}(x)=\left(5 x^{3}-3 x\right) / 2 \\
P_{1}(x)=x & P_{4}(x)=\left(35 x^{4}-30 x^{2}+3\right) / 8 \\
P_{2}(x)=\left(3 x^{2}-1\right) / 2 & P_{5}(x)=\left(63 x^{5}-70 x^{3}+15 x\right) / 8
\end{array}
$$

Potential due to a dipole at the origin:

$$
\begin{equation*}
V(\overrightarrow{\mathbf{r}})=\frac{1}{4 \pi \epsilon_{0}} \frac{\hat{\mathbf{r}} \cdot \overrightarrow{\mathbf{p}}}{r^{2}} \tag{17}
\end{equation*}
$$

Bound charge in polarized materials:

$$
\begin{equation*}
\rho_{b}=-\vec{\nabla} \cdot \overrightarrow{\mathbf{P}} \quad \sigma_{b}=\overrightarrow{\mathbf{P}} \cdot \hat{\mathbf{n}} \tag{18}
\end{equation*}
$$

$\vec{D}$ field:

$$
\begin{equation*}
\overrightarrow{\mathbf{D}} \equiv \epsilon_{0} \overrightarrow{\mathbf{E}}+\overrightarrow{\mathbf{P}} \quad \vec{\nabla} \cdot \overrightarrow{\mathbf{D}}=\rho_{f} \quad \oint \overrightarrow{\mathbf{D}} \cdot d \overrightarrow{\mathbf{a}}=Q_{f, \text { encl }} \tag{19}
\end{equation*}
$$

Linear dielectrics: electric susceptibility $\chi_{e}$

$$
\begin{equation*}
\overrightarrow{\mathbf{P}}=\epsilon_{0} \chi_{e} \overrightarrow{\mathbf{E}} \quad \overrightarrow{\mathbf{D}}=\epsilon_{0}\left(1+\chi_{e}\right) \overrightarrow{\mathbf{E}} \equiv \epsilon_{0} \epsilon_{r} \overrightarrow{\mathbf{E}} \equiv \epsilon \overrightarrow{\mathbf{E}} \tag{20}
\end{equation*}
$$

Energy

$$
\begin{equation*}
W=\frac{1}{2} \int \overrightarrow{\mathbf{D}} \cdot \overrightarrow{\mathbf{E}} d \tau \quad \text { In vacuum, } W=\frac{\epsilon_{0}}{2} \int E^{2} d \tau \tag{21}
\end{equation*}
$$

Lorentz force on point charge or current

$$
\begin{equation*}
\overrightarrow{\mathbf{F}}=q \overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{B}} \quad \overrightarrow{\mathbf{F}}=\int(\overrightarrow{\mathbf{J}} \times \overrightarrow{\mathbf{B}}) d \tau \tag{22}
\end{equation*}
$$

Biot-Savart law

$$
\begin{equation*}
\overrightarrow{\mathbf{B}}(\overrightarrow{\mathbf{r}})=\frac{\mu_{0}}{4 \pi} \int \frac{\overrightarrow{\mathbf{J}}\left(\overrightarrow{\mathbf{r}^{\prime}}\right) \times\left(\overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{r}^{\prime}}\right)}{\left|\overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{r}^{\prime}}\right|^{3}} d \tau^{\prime} \quad \overrightarrow{\mathbf{B}}(\overrightarrow{\mathbf{r}})=\frac{\mu_{0} I}{4 \pi} \int \frac{d \overrightarrow{\mathbf{l}^{\prime}} \times\left(\overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{r}^{\prime}}\right)}{\left|\overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{r}^{\prime}}\right|^{3}} \tag{23}
\end{equation*}
$$

Ampère's law (magnetostatics)

$$
\begin{equation*}
\vec{\nabla} \times \overrightarrow{\mathbf{B}}=\mu_{0} \overrightarrow{\mathbf{J}} \quad \oint \overrightarrow{\mathbf{B}} \cdot d \overrightarrow{\mathbf{l}}=\mu_{0} I_{\mathrm{encl}} \tag{24}
\end{equation*}
$$

Vector potential

$$
\begin{equation*}
\overrightarrow{\mathbf{B}}=\vec{\nabla} \times \overrightarrow{\mathbf{A}} \quad \text { If } \vec{\nabla} \cdot \overrightarrow{\mathbf{A}}=0, \text { then } \nabla^{2} \overrightarrow{\mathbf{A}}=-\mu_{0} \overrightarrow{\mathbf{J}} \text { and } \overrightarrow{\mathbf{A}}(\overrightarrow{\mathbf{r}})=\frac{\mu_{0}}{4 \pi} \int \frac{\overrightarrow{\mathbf{J}}\left(\overrightarrow{\mathbf{r}^{\prime}}\right)}{\mid \overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{r}^{\prime} \mid}} d \tau^{\prime} \tag{25}
\end{equation*}
$$

Boundary conditions

$$
\begin{equation*}
\overrightarrow{\mathbf{B}}_{\text {above }}-\overrightarrow{\mathbf{B}}_{\text {below }}=\mu_{0}(\overrightarrow{\mathbf{K}} \times \hat{\mathbf{n}}) \tag{26}
\end{equation*}
$$

Torque and force on a magnetic dipole

$$
\begin{equation*}
\vec{\tau}=\overrightarrow{\mathbf{m}} \times \overrightarrow{\mathbf{B}} \quad \overrightarrow{\mathbf{F}}=\vec{\nabla}(\overrightarrow{\mathbf{m}} \cdot \overrightarrow{\mathbf{B}}) \tag{27}
\end{equation*}
$$

Vector potential due to a magnetic dipole at the origin

$$
\begin{equation*}
\overrightarrow{\mathbf{A}}(\overrightarrow{\mathbf{r}})=\frac{\mu_{0}}{4 \pi} \frac{\overrightarrow{\mathbf{m}} \times \hat{\mathbf{r}}}{r^{2}} \tag{28}
\end{equation*}
$$

Bound current in magnetized materials

$$
\begin{equation*}
\overrightarrow{\mathbf{J}}_{b}=\vec{\nabla} \times \overrightarrow{\mathbf{M}} \quad \overrightarrow{\mathbf{K}}_{b}=\overrightarrow{\mathbf{M}} \times \hat{\mathbf{n}} \tag{29}
\end{equation*}
$$

$\vec{H}$ field

$$
\begin{equation*}
\overrightarrow{\mathbf{H}} \equiv \frac{1}{\mu_{0}} \overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{M}} \quad \vec{\nabla} \times \overrightarrow{\mathbf{H}}=\overrightarrow{\mathbf{J}}_{f} \quad \oint \overrightarrow{\mathbf{H}} \cdot d \overrightarrow{\mathbf{l}}=I_{f, \text { encl }} \tag{30}
\end{equation*}
$$

Linear magnetic media: magnetic susceptibility $\chi_{m}$

$$
\begin{equation*}
\overrightarrow{\mathbf{M}}=\chi_{m} \overrightarrow{\mathbf{H}} \quad \overrightarrow{\mathbf{B}}=\mu_{0}\left(1+\chi_{m}\right) \overrightarrow{\mathbf{H}} \equiv \mu \overrightarrow{\mathbf{H}} \tag{31}
\end{equation*}
$$

Ohm's law and emf (here $\sigma \equiv$ conductivity)

$$
\begin{equation*}
\overrightarrow{\mathbf{J}}=\sigma \overrightarrow{\mathbf{f}}=\sigma(\overrightarrow{\mathbf{E}}+\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{B}}) \quad V=I R \quad \mathcal{E} \equiv \oint \overrightarrow{\mathbf{f}} \cdot d \overrightarrow{\mathbf{l}} \tag{32}
\end{equation*}
$$

Power

$$
\begin{equation*}
P=I \Delta V \tag{33}
\end{equation*}
$$

Motional emf, induction, and Faraday's law

$$
\begin{equation*}
\mathcal{E}=-\frac{d \Phi_{m}}{d t} \quad \vec{\nabla} \times \overrightarrow{\mathbf{E}}=-\frac{\partial \overrightarrow{\mathbf{B}}}{\partial t} \quad \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{l}}=-\int \frac{\partial \overrightarrow{\mathbf{B}}}{\partial t} \cdot d \overrightarrow{\mathbf{a}} \tag{34}
\end{equation*}
$$

Lenz's law: "Nature abhors a change in flux."
Mutual inductance and self-inductance

$$
\begin{equation*}
\Phi_{2}=M I_{1} \quad \Phi_{1}=M I_{2} \quad \Phi=L I \quad \mathcal{E}=-L \frac{d I}{d t} \tag{35}
\end{equation*}
$$

Energy

$$
\begin{equation*}
W=\frac{1}{2 \mu_{0}} \int B^{2} d \tau \tag{36}
\end{equation*}
$$

Continuity equation and Maxwell's fix to Ampère's law

$$
\begin{equation*}
\vec{\nabla} \cdot \overrightarrow{\mathbf{J}}=-\frac{\partial \rho}{\partial t} \quad \vec{\nabla} \times \overrightarrow{\mathbf{B}}=\mu_{0} \overrightarrow{\mathbf{J}}+\mu_{0} \epsilon_{0} \frac{\partial \overrightarrow{\mathbf{E}}}{\partial t} \tag{37}
\end{equation*}
$$

