Carleton University Physics Department
PHYS 3308 – Electromagnetism (Fall 2014)
Homework assignment #8

Handed out Thurs Nov 13; due Thurs Nov 20, 2014, at the start of class. H. Logan
Problems are worth 5 points each unless noted otherwise.

1. A steady current I flows down a long cylindrical wire of radius R. Find the magnetic field, both inside and outside the wire, if
   (a) The current is uniformly distributed over the outside surface of the wire (i.e., there is a uniform surface current density).
   (b) The current is uniformly distributed over the cross section of the wire (i.e., there is a uniform volume current density).
   (c) The current is distributed in such a way that the magnitude of the volume current density is proportional to the distance from the axis, (i.e., $|\vec{J}| \propto s$).

2. When calculating the current enclosed by an Amperian loop, one must, in general, evaluate an integral of the form
$$I_{\text{encl}} = \int_{\mathcal{S}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}}. \tag{1}$$
But there are infinitely many surfaces that share the same boundary line! Which one are we supposed to use? Explain. *(If you argue that all surfaces give the same result, prove that it is true in magnetostatics.)*

3. Two very long, nested solenoids each has its axis along the z axis. They each carry current I, but with their windings in opposite directions. The inner solenoid has radius a and n_1 turns per unit length, while the outer solenoid has radius b and n_2 turns per unit length.
   (a) Using superposition, find $\vec{B}$ everywhere.
   (b) Show that the discontinuities in $\vec{B}$ across each of the solenoids are in agreement with the magnetostatic boundary conditions, $\vec{B}_{\text{above}} - \vec{B}_{\text{below}} = \mu_0 \vec{K} \times \hat{n}$.

4. A straight segment of wire of length L is oriented along the z axis from $-L/2$ to $+L/2$. It is carrying a current I in the $+z$ direction.
   (a) Find the vector potential everywhere.
   (b) Take the curl of the vector potential to get the magnetic field and check that your answer agrees with the direct calculation of $\vec{B}$ that we did in class. *(Recall that our result was $\vec{B} = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)\hat{\phi}$, where $\theta_{1,2}$ were the angles of the ends of the wires; see the picture on page 225 of Griffiths.)*

5. Detectors at particle colliders are normally constructed in layers. Surrounding the collision point is a volume of *tracker*, which is used to reconstruct the paths of particles produced in collisions. Around the tracker is a layer of *calorimeters*, solid detectors which stop almost all of the particles produced and measure the energy deposited. Usually only muons manage to pass through the calorimeter, so outside the calorimeter are *muon detectors* used to measure these particles.

   Most collider detectors use a solenoid to produce a uniform magnetic field in the tracker region, pointing parallel to the beam direction. This allows for measurements of the momentum of the particles produced in a collision, as well as of the sign of their electric charge.

   (a) Show that the radius of curvature of a particle's track in such a solenoidal magnetic field is given by $R = p_T/qB$, where q is the particle's charge, B is the strength of the magnetic field, and p_T is the transverse component of the particle's momentum, i.e., the component perpendicular to the beam direction.

   In addition to the usual solenoidal field in the inner part of the detector, the ATLAS detector also has a "toroidal" magnetic field[1], produced by large coils in the muon detector region outside the calorimeter. (Defining the beam direction to be the z axis, these toroids produce a magnetic field in the $\hat{\phi}$ direction, which varies with s—see our calculation in class, or Griffiths example 5.10. You can assume that the solenoidal field does not extend into the region of the toroids.)

   (b) Show that, inside the toroids, the radius of curvature of a particle's track is given by $R = p/qB$, where B is the local strength of the magnetic field (note R varies with position, but this can be dealt with in the track analysis) and p is the magnitude of the particle's *total* momentum.

   Such a toroidal magnetic field allows better measurements of the momentum of muons when the muons are produced at small angles from the beam direction, so that $p_T \ll p$.

[1] ATLAS stands for **A** **T**oroidal **L**HC **A**pparatu**S**. I am not joking.