Carleton University Physics Department

PHYS 3308 – Electromagnetism (Fall 2014)

Homework assignment #7

Handed out Thurs Nov 6; due Thurs Nov 13, 2014, at the start of class. H. Logan

Problems are worth 5 points each unless noted otherwise.

1. A parallel-plate capacitor with plates of area A and separation d has its bottom plate grounded ($V = 0$) and its top plate held at potential V_0.
   (a) Solve Laplace's equation inside the capacitor. *(Note: use Cartesian coordinates and ignore edge effects; then the potential depends on only one variable.)* Apply the boundary conditions and find V as a function of position, then take the gradient to get $\vec{E}$.
   (b) Use your result in part (a) to find the surface charge densities on the two plates. *(Note that this solution proves that σ is uniform in a parallel plate capacitor when you can neglect edge effects.)*
   (c) Now imagine that the space between the plates of the capacitor is filled with a linear dielectric with electric susceptibility χ_e. Do your results for V and/or $\vec{E}$ in part (a) change? Explain. Then find the polarization in the dielectric and all bound and free charge densities.
   (d) Find the capacitance of this capacitor using the definition $C = Q/\Delta V$, both without and with the dielectric. Check that the ratio of your results agrees with Griffiths example 4.6 (page 183).

2. A very thin spherical metal shell of radius a is centred inside a thick spherical metal shell with inner radius b and outer radius c, such that $a < b < c$ and both spheres have their centres at the origin. The outer sphere is left uncharged and a charge Q is put on the inner sphere. Then the space between the two spheres (i.e., between radii a and b) is filled with a linear dielectric (insulating) oil with electric susceptibility χ_e. Find all free and bound charge densities (surface and volume), the polarization in the dielectric, and the electric field *everywhere*.

3. A current I flows in the $+x$ direction through a rectangular bar of conducting material in the presence of a uniform magnetic field $\vec{B}$ pointing in the $+y$ direction.
   (a) If the moving charges are *positive*, in which direction are they deflected by the magnetic field?
   (b) This deflection results in an accumulation of charge on the upper and lower surfaces of the bar, which in turn produces an electric force to counteract the magnetic force. Equilibrium occurs when the two forces exactly cancel. (This phenomenon is known as the *Hall effect.*) Find the resulting potential difference (the *Hall voltage*) between the top and bottom of the bar, in terms of B, the speed v of the charges, and the relevant dimensions of the bar.
   (c) How would your analysis change if the moving charges were *negative*? Draw a schematic showing how you would measure the potential difference and thus determine the sign of the charge of the mobile charge carriers in a material.

(d) Copper has a conduction-electron density of 8.5×10^{28} m^{-3}. Find the Hall voltage for a 1 Ampere current flowing through a 1 cm $\times$ 1 cm copper bar in a 1 Tesla magnetic field.

4. An infinitely long cylindrical shell with radius R and carrying a uniform surface charge density σ is rotating about its axis with angular frequency ω.

   (a) Find the surface current density $\vec{K}$.

   (b) Use the Biot-Savart law to find the magnetic field at a point on the axis. *(Note: we will soon learn a much easier way to solve this problem using Ampère's law.)*

5. Two circular loops of wire of radius R are arranged facing each other with their axes along the z axis and their centres a distance d apart. Each loop carries a current I, in the same direction.

   (a) Find the magnetic field on the axis as a function of z, and show that $\partial B/\partial z$ is zero at the point midway between the loops.

   (b) Find the value of d such that $\partial^2 B/\partial z^2 = 0$ at the point midway between the loops, and show that the resulting magnetic field strength at that point is

$$B = \frac{8\mu_0 I}{5\sqrt{5}R}. \tag{1}$$

   (Note: This arrangement is known as a Helmholtz coil and is a convenient way of producing a relatively uniform magnetic field in the laboratory. You have probably encountered this setup while doing Thomson's experiment to measure the charge-to-mass ratio of the electron.)