Carleton University Physics Department
PHYS 3308 – Electromagnetism (Fall 2014)
Homework assignment #5

Handed out Thurs Oct 9; due Thurs Oct 16, 2014, at the start of class. H. Logan
Problems are worth 5 points each unless noted otherwise.

1. Find the general solution to Laplace's equation in spherical coordinates, for the case where V depends only on r. Do the same for cylindrical coordinates, assuming V depends only on s. Then apply your results to find the potential in the specified volume for the following two setups:

   (a) Two nested conducting spherical shells, of radii a and b with $a < b$, centred at the origin. The inner shell is grounded and the outer shell is held at a potential V_0. The volume of interest is in between the two shells.

   (b) An infinitely long coaxial cable running along the z axis, with its inner wire of radius a and its outer conducting cylindrical shell of radius b. The inner wire is grounded and the outer shell is held at a potential V_0. The volume of interest is in the space between the inner wire and the outer cylindrical shell.

2. An infinitely long line of charge with linear charge density λ is held parallel to, and a distance d away from, an infinite grounded conducting plane.

   (a) Find the electric field and the electric potential everywhere.

   (b) Find the force per unit length on the line charge.

3. *(10 points)* In deriving the method of images for a conducting spere (see Griffiths Example 3.2 on pages 124–125), we assumed that the conducting sphere was grounded ($V = 0$). But with the addition of a second image charge, the same basic model will handle the case of a sphere at *any* potential V_0 (relative to infinity).

   (a) Find the charge and location of the second image charge required.

   (b) Find the force of attraction between a point charge q and a *neutral* conducting sphere.

   (c) Now consider a conducting sphere of radius R carrying total charge Q located near a point charge q. Find the electric field outside the conductor and the surface charge density as a function of position on the conductor. Verify that your expression for the surface charge density integrates to the correct total charge. *(Note: this problem is the full solution of part (e) of problem 3 of homework #4.)*

4. The potential at the surface of an isolated, hollow sphere of radius R is given by $V(R, \theta) = V_0 \cos 2\theta$, where V_0 is a constant. Use the solutions of Laplace's equation to find the potential inside and outside the sphere, and find the surface charge density $\sigma(\theta)$ on the sphere.