Carleton University Physics Department

PHYS 3308 – Electromagnetism (Fall 2014)

Homework assignment #3

Handed out Tue Sept 23; due Tue Sept 30, 2014, at the start of class. H. Logan

Problems are worth 5 points each unless noted otherwise.

1. Starting from the definition of $\vec{\nabla}$ and working in Cartesian coordinates, prove that

   (a) $\vec{\nabla} \times (\vec{\nabla} T) = 0$,

   (b) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$, and

   (c) $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$.

   You may use the Levi-Civita tensor ϵ_{ijk} and/or the Kronecker delta δ_{ij} if you like. Recall also that $\epsilon_{kij}\epsilon_{kmn} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$.

2. *(10 points)* Find the potential inside and outside a uniformly charged solid sphere of radius R and total charge q in two ways:

   (a) Find $\vec{E}$ using Gauss's law and then integrate the $\vec{E}$ field inward from infinity to get the potential. *[Hint: you will need to do this in two steps, first outside and then inside the solid sphere.]*

   (b) Find V directly by integrating the volume charge density. *[Hint: find V along the z axis and then use symmetry to generalize to any position. Use the law of cosines to write $(\vec{r} - \vec{r}')$ in terms of z, r', and θ'. Be sure to take the positive square root: $\sqrt{r'^2 + z^2 - 2r'z} = (r' - z)$ if $r' > z$, but it's $(z - r')$ if $r' < z$.]*

   (c) Check that your result from parts (a) and (b) returns the correct $\vec{E}$ field when you take the gradient, both inside and outside the sphere.

   (d) Check that your result from parts (a) and (b) satisfies Poisson's equation both inside and outside the sphere.

3. Two infinitely long wires running parallel to the x axis and a distance $2a$ apart carry uniform line charge densities $+\lambda$ and $-\lambda$.

   (a) Find the electric field at any point (x, y, z).

   (b) Find the electric potential at any point (x, y, z), using a point midway between the two wires as your reference point.

   (c) Show that the equipotential surfaces are circular cylinders, and locate the axis and radius of the cylinder corresponding to a given potential V_0.

Problem 4 is on the next page...

4. Consider two point charges located on the z axis, q a distance d above the origin and $q' = -cq$ a distance d below the origin. Here c is a constant.

   (a) Compute the potential at any point (x, y, z), using infinity as your reference.

   (b) Show that the equipotential surface with $V = 0$ is a sphere, and determine its radius and the location of its centre as a function of d and c.

   Note: this problem forms the basis of the Method of Images involving a grounded spherical conductor.