Phenomenology of Non-Commutative Field Theories

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1. Motivation for NCQFT Phenomenology
2. 60-second overview of NCQFT
3. NC QED
4. NCQED Phenomenology in e^+e^- colliders
5. Summary

Motivation for NCQFT Phenomenology

For recent review see: Hinchliffe and Kersting, hep-ph/0205040

- NCQFT arises in string/M theory by describing low-energy excitations of D-branes in background of “EM-like” fields
- Numerous ideas from string theory have stimulated particle physics phenomenology
- Yet another example of String/M theory stimulating phenomenology
- Why Not?
  - It’s interesting
  - Not explored nor ruled out

Testable differences between QFT and NCQFT

Non-commutative Field Theory

[For recent review see: M.R. Douglas and N.A. Nekrasov Rev. Mod. Phys73, 977 (2002)]

In NCQFT the conventional commuting coordinates are replaced with non-commuting space-time operators:

$$\left[\hat{X}_m, \hat{X}_n\right] = i q_{mn} \equiv \frac{i}{\Lambda_{NC}^2} C_{mn}$$

Λ_{NC} is the scale where NC effects become relevant

What is the most likely value for Λ_{NC}?

$m_{Pl} \sim 10^{19}$ GeV

$m_\lambda \sim$ TeV; large extra dimensions where gravity becomes strong at scales O(1 TeV)

$C_{\mu\nu}$ is a real antisymmetric matrix **NOT a tensor**

C can be parametrized as (with elements 0(1)):

$$C_{mn} = \begin{pmatrix} 0 & C_{01} & C_{02} & C_{03} \\ -C_{01} & 0 & C_{12} & -C_{13} \\ -C_{02} & -C_{12} & 0 & C_{23} \\ -C_{03} & C_{13} & -C_{23} & 0 \end{pmatrix}$$

The components are identical in all frames

violates Lorentz invariance at Λ_{NC}

Can rewrite in terms of 2 fixed, frame independent vectors:

"E" field $C_{01} = \sin a \cos b \quad C_{02} = \sin a \sin b \quad C_{03} = \cos a$

"B" field $C_{12} = \cos g \quad C_{13} = \sin g \sin b \quad C_{23} = -\sin g \cos b$

- For origin of ϕ axis choose $\beta=\pi/2$
- Can parametrize with 2 angles α and γ
- Since experiments are sensitive to direction of C-vectors must employ astronomical coordinate system and time stamp data

NCQFT General Properties

Can cast NCQFT in form of conventional QFT using two approaches: Seiberg-Witten and Weyl-Moyal

1. In Moyal-Weyl

Sheikh-Jabbari, JHEP 06, 015 (1999); Hayakawa, PL B478, 394 (2000); Armani, NP B593, 229 (2001)

   only U(n) Lie algebras are closed under Moyal brackets

   $\Rightarrow$ NC gauge theories only based on U(n)

   Covariant derivatives only for fields with Q=0, ± 1

2. NCSM developed using Sieberg-Witten approach but non-renormalizable order by order

Seiberg, Witten, JHEP 9909, 032 (1999); Wulkenhaar, hep-th/0112248; Jurco *et al*, hep-th/0104153

Difficult to construct NCSM but NCQED well defined in in WM approach, simplest extension of SM and no need to define EW gauge interactions

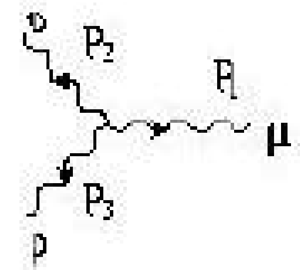
I will restrict myself to NCQED

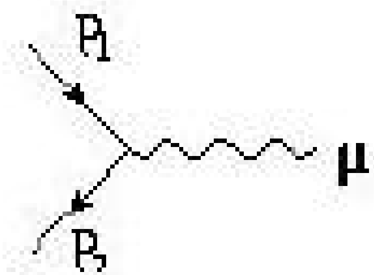
Note that others have studied NCSM

Provides testing ground for the ideas of NCQFT

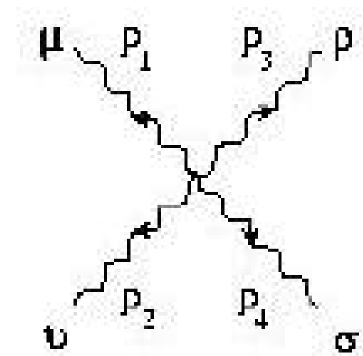
Non-commutative QED

- Introduce non-commuting coordinates
- Can be cast in form of conventional commuting QFT
- Gives rise to
  - Momentum dependent phase factors
  - New 3- and 4-point photon vertices
  - Violates Lorentz invariance





$$= ig\gamma^{\mu}\exp(ip_1\theta p_2/2)$$



J. Hewett F.J. Petriello, T. G. Rizzo PR D64, 075012 (2001)
Armoni, NP B593, 229 (2001)
Sheikh-Jabbari, J. HEP 06, 015 (1999)
Krajewski, Wulkenharr Int J. Mod. Phys. A15, 1011 (2000)
Ardalan, Sadooghi, Int. J. Mod. Phys. A16, 3151 (2001)

The hallmark signal is azimuthal dependence in cross section
In $2 \to 2$ processes resulting from existence of preferred direction

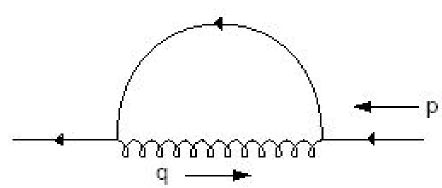
Another Approach:

Carlson, Carone, Lebed, PL B518, 201 (2001)
Mocioiu, Pospelov, Roibon, PL B489, 390 (2000)

Consider Lorentz violating operators:

$$\text{eg.}\quad q^{mn}\bar{q}s_{mn}q$$

And place limits on scale of noncommutativity



Leads to set of Lorentz violating operators

$$mq^{mn}\bar{q}s_{mn}q, \quad q^{mn}\bar{q}s_{mn}D_h g^h q, \quad q^{mn}D_m\bar{q}s_{nr}D^r q$$

$mq^{mn}\bar{q}s_{mn}q$ acts like $\vec{s}\cdot\vec{B}$ interaction with a fixed B

•Cs and Hg atomic clocks have different sensitivities to external $\vec{s}\cdot\vec{B}$ interaction leads to estimate

$$\frac{\alpha_s}{12\pi}\Lambda^2 \theta m_q < \Delta E \;\Rightarrow\; \theta\Lambda^2 \leq 10^{-29} \Rightarrow \Lambda_{NC} > 10^{17}\ \text{GeV}$$

•Also $\theta^{\mu\nu}\,\overline{N}\sigma_{\mu\nu}N$ where N are nucleon wavefunctions

•Leads to shift in nuclear magnetic moments and observed Sidereal variation in hyperfine splittings in atoms

$$\Lambda_{NC} > 10^{15}\ \text{GeV}$$

•If one accepts these conclusions it is unlikely that colliders can probe NC phenomenology

•Ways out include NC restricted to extra spatial directions but introduces KK excitations of photons

Carlson, Carone, hep-ph/0112143
Mocioiu, Pospelov, Roiban, PL B489, 390 (2000) [hep-ph/0005191]
Anisimov, Banks, Dine, Graesser, PR D65, 085032 (2002) [hep-ph/0106356]

Collider Tests of NCQED

$e\gamma \to e\gamma$	Compton scattering	Doncheski Godfrey PR D65 015005 (2001) Mathews PR D63, 075012 (2001) Arfaei, Yavartanoo hep-th/0010244
$\gamma\gamma \to e^+e^-$	Pair production	Doncheski Godfrey PR D65 015005 (2001) Baek, Ghosh, He, Hwang PR D64, 056001 (2001)
$e^+e^- \to \gamma\gamma$	Pair annihilation	Hewett Petriello Rizzo PR D64 075012 (2001)
$e^-e^- \to e^-e^-$	Moller scattering	Hewett Petriello Rizzo PR D64 075012 (2001)
$e^+e^- \to e^+e^-$	Bhabha scattering	Hewett Petriello Rizzo PR D64 075012 (2001) Arfaei, Yavartanoo hep-th/0010244
$\gamma\gamma \to \gamma\gamma$		Hewett Petriello Rizzo PR D64 075012 (2001)
$\gamma\gamma \to H^+H^-$	Higgs production	Grosse, Liao, PR D64, 115007 (2001)

Non-Collider Tests of NCSM

$b \to s\gamma$, sg	Iltan, hep-ph/0204129, hep-ph/0202011
Lamb shift	Chaichian, Sheikh-Jabbari, Tureanu, PRL 86, 2716(2001)
CP violaton	Chang, Xing, hep-ph/0204255 Hinchliffe, Kersting, PR D64, 116007 (2001)
$(g-2)_\mu$	Kersting, hep-ph/0109224
Hyperfine structure	Mocioiu, Pospelov, Roiban, PL B489, 390 (2000)
$Z \to \gamma\gamma$, gg	Mocioiu, Pospelov, Roiban, PL B489, 390 (2000) Behr, *et al*, hep-ph/0202121
$\pi \to \gamma\gamma\gamma$	Grosse, Liao, PL B520, 63 (2001)
$Z \to l^+l^-$, $W \to l^+n$	Iltan, hep-ph/0204332

Consider e^+e^- colliders:

$\sqrt{s}$= 0.5, 0.8, 1.0, 1.5, 3, 5, 8 TeV

for TESLA/LC/CLIC

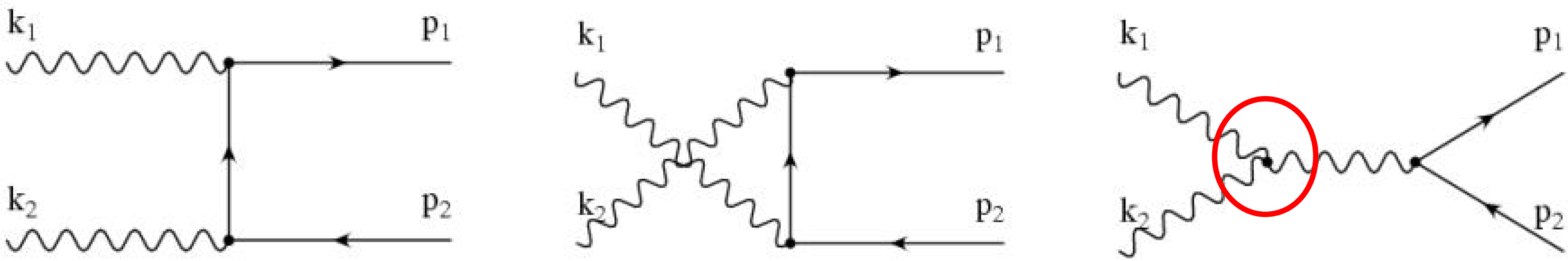
Assume L=500 fb^{-1}

$10^o \leq \theta \leq 170^o$

$p_T > 10$ GeV

Start with pair production: $\gamma\gamma \to e^+e^-$

SG + M Doncheski, PR D65, 015005 (2002) [hep-ph/0108268]



Differential Cross Section given by:

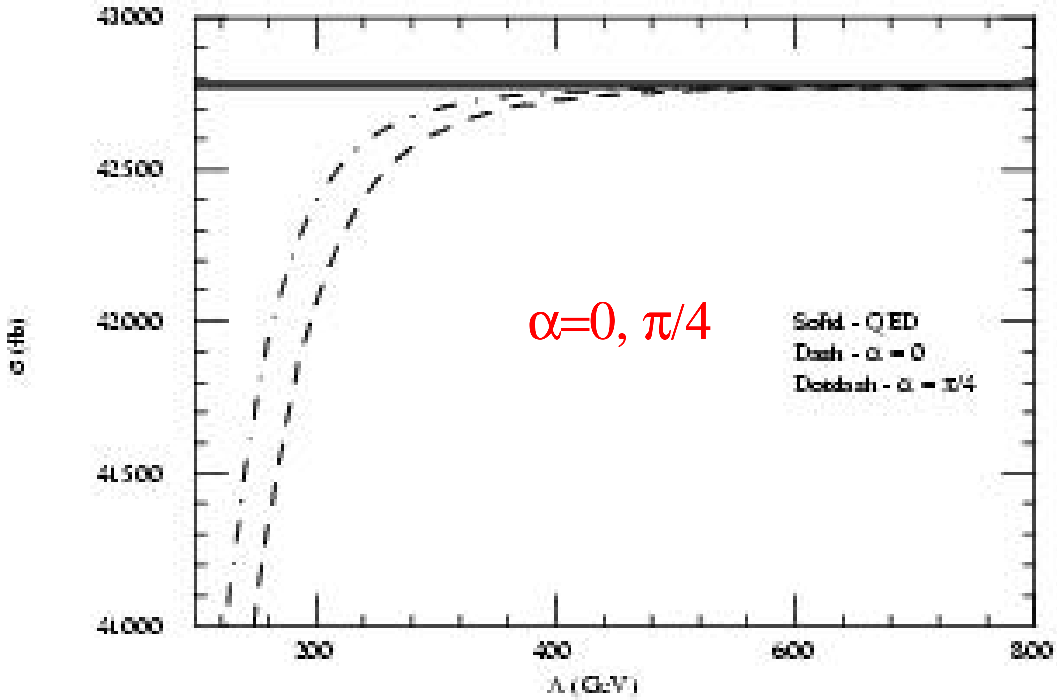
$$\frac{ds\,(gg \to e^+e^-)}{d\cos q\, df} = \frac{a^2}{2s}\left\{\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - 4\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\sin^2\left(\tfrac{1}{2}k_1 \cdot q \cdot k_2\right)\right\}$$

k_1 and k_2 are the momenta of the incoming photons

$$\frac{1}{2}k_1 \cdot q \cdot k_2 = \frac{\hat{s}}{4\Lambda_{NC}^2}C_{03} = \frac{\hat{s}}{4\Lambda_{NC}^2}\cos a$$

- Not Lorentz invariant, No φ dependence
- For $\Lambda_{NC} \to \infty$ phase angle goes to 0 and SM is recovered

$\sqrt{s}$=500 GeV L_{int}=500 fb^{-1}



High event rate

±1 σ bands

- To quantify sensitivity to NCQED calculate χ^2
- Take $d = \sqrt{d_{stat}^2 + d_{sys}^2}$ where $d_{sys} = 2\%$
- Conservative as TESLA TDR calls for 1%
- Bin in cosθ and ϕ (20 bins in each)

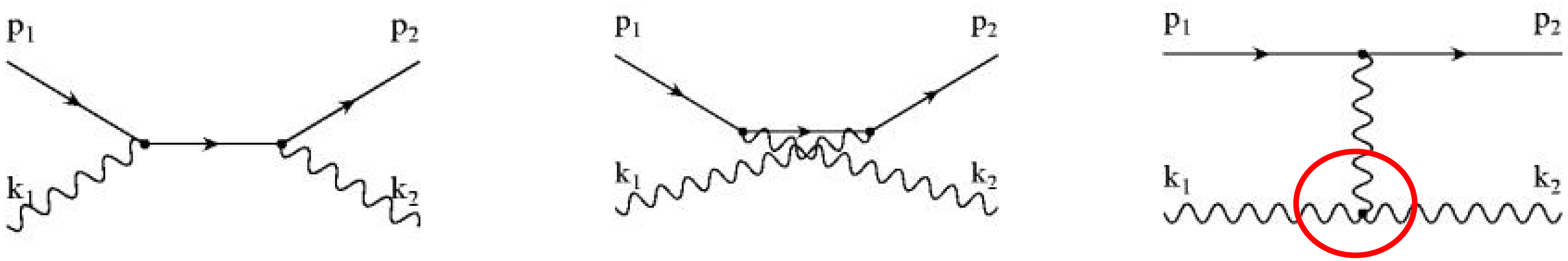
$$c_O^2(\Lambda) = \sum_i \left(\frac{O_i(\Lambda) - O_i^{QED}}{dO_i} \right)^2$$

Find limits of $\Lambda_{NC} > (0.5\text{-}1)\sqrt{s}$

Compton scattering $e^- g \to e^- g$

- Consider e+e- collider operating at TESLA/NLC/CLIC energies

√s = 0.5 – 5 TeV with L=500 fb^{-1}



- Differential Cross Section given by:

$$\frac{ds\,(e^- g \to e^- g)}{d\cos q\, df} = \frac{a^2}{2s}\left\{ -\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}} + 4\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\sin^2\left(\frac{k_1 \cdot q \cdot k_2}{2}\right)\right\}$$

- k_1 and k_2 are photon momenta

$$k_1 = x\frac{\sqrt{s}}{2}(1,0,0,-1)$$

$$k_2 = k(1, \sin q \cos f, \sin q \sin f, \cos q)$$

$$\frac{1}{2} k_1 \cdot q \cdot k_2 = \frac{xk\sqrt{s}}{4\Lambda_{NC}^2}\left[(C_{01} - C_{13})\sin q \cos f + (C_{02} + C_{23})\sin q \sin f + C_{03}(1+\cos q)\right]$$

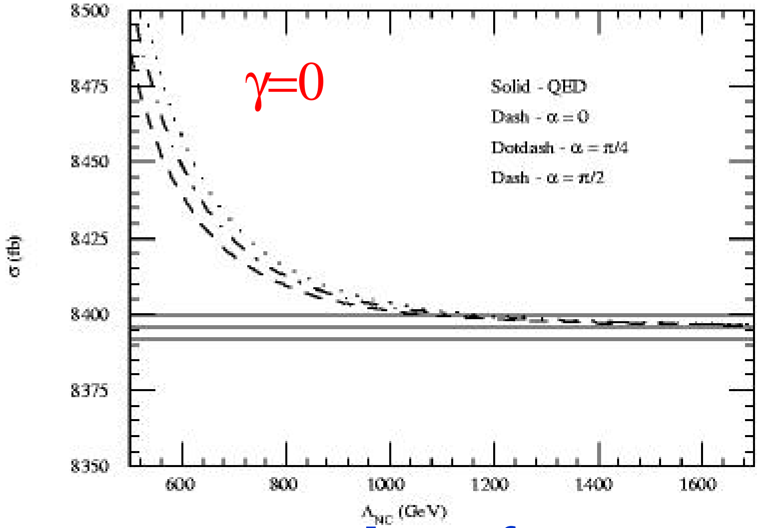
Both space-space and space-time NC contributions

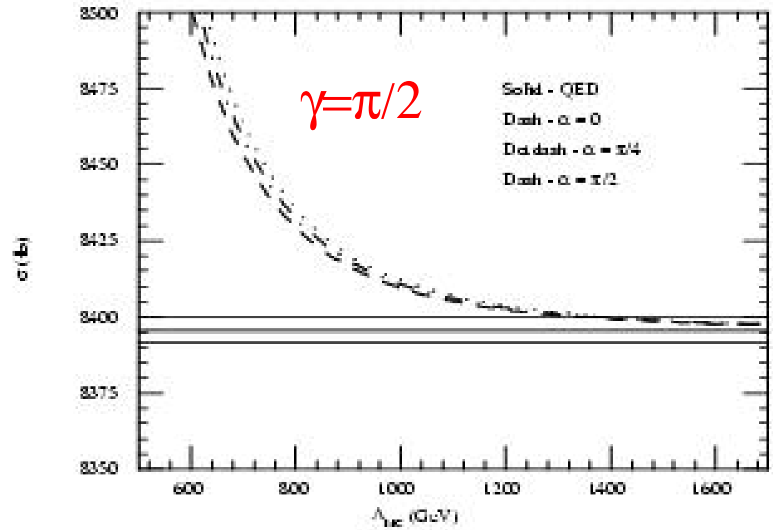
Take $\beta = \pi/2$

$$\frac{1}{2} k_1 \cdot q \cdot k_2 = \frac{xk\sqrt{s}}{4\Lambda_{NC}^2}\left[-\sin g \, \sin q \cos f + \sin a \, \sin q \sin f + \cos a (1+\cos q)\right]$$

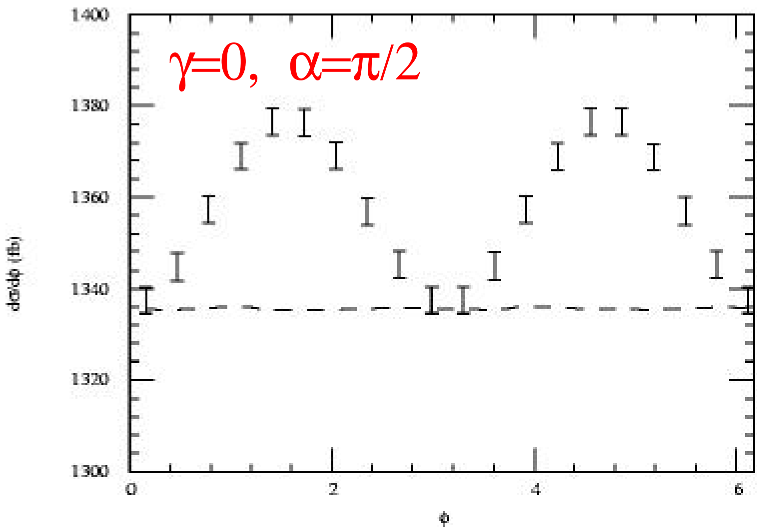
Because Compton scatter is sensitive to both γ and α it complements $\gamma\gamma \rightarrow e^+e^-$

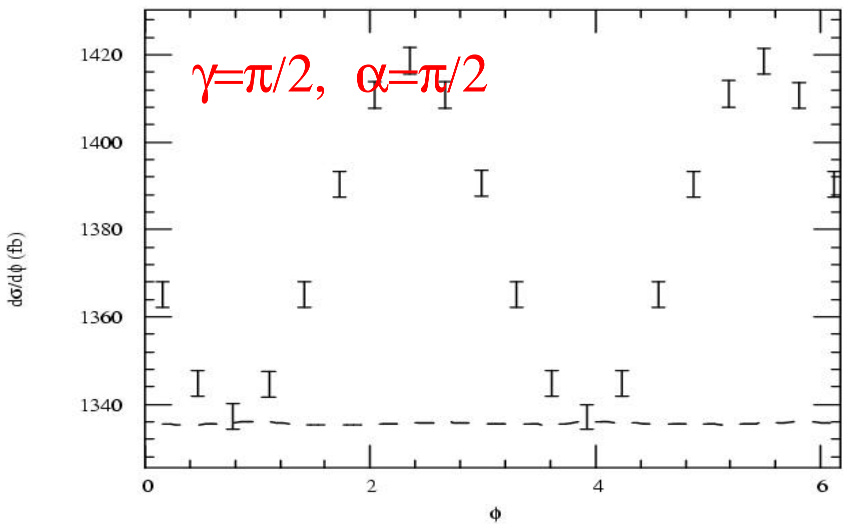
$\sqrt{s}$=500 GeV L_{int}=500 fb^{-1}





Most striking feature is structure in ϕ angular distribution





Find limits of $\Lambda_{NC} > (1\text{-}2)\sqrt{s}$

Bhabba scattering

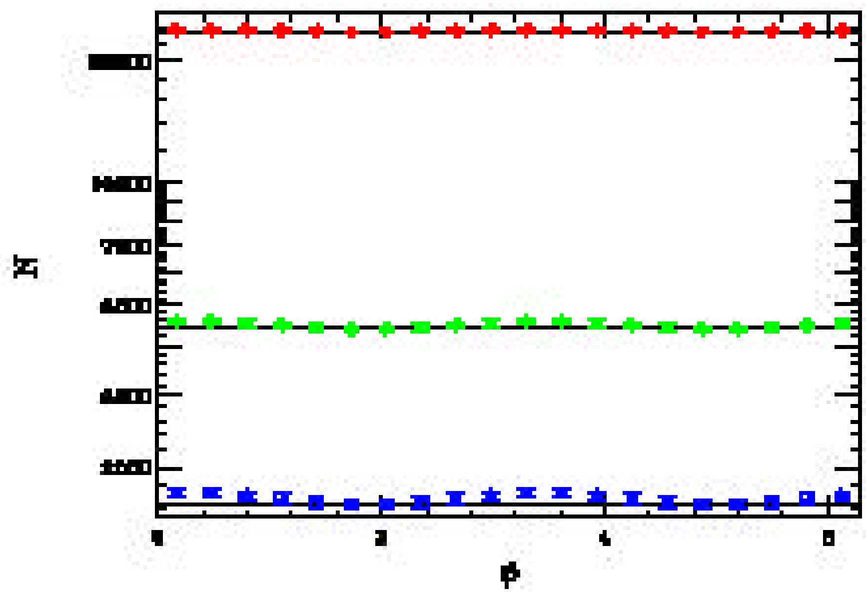
[Hewett Petriello Rizzo PR D64 075012 (2001)]

- Proceeds via both s- and t-channel gauge boson exchange
- No new amplitudes but vertices pick up kinematic dependent phases
- NC modifications appear in interference term

$$\Delta_{Bhabba} = f_s - f_t = \frac{-1}{\Lambda_E^2}\left[C_{01}t + \sqrt{ut}\left(C_{02}c_f + C_{03}s_f\right)\right]$$

- No cosθ modification
- Pick up azimuthal dependence
- More pronounced with stiffer cuts on scattering angle

  |cosθ|<0.9, 0.7, 0.5



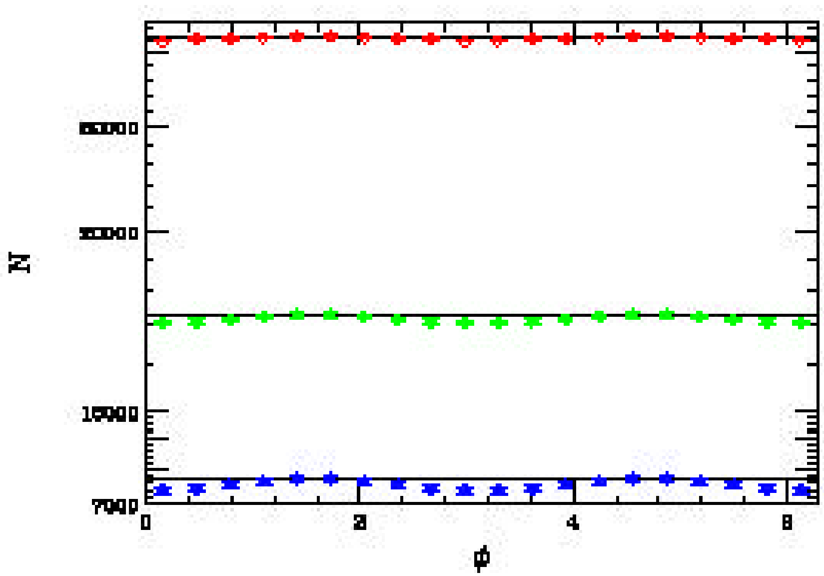
3 TeV CLIC, L=1 ab^{-1}, C_{02}=1, Λ=3 TeV

Moller scattering

- NC effects appear in interference between t- and u-channel amplitudes

$$\Delta_{Moller} = f_u - f_t = \frac{-\sqrt{ut}}{\Lambda_B^2}\left[C_{12}c_f - C_{31}s_f\right]$$

- Only sensitive to Λ_B
- If C_{12} or C_{31} is nonzero have azimuthal dependence



3 TeV CLIC, L=1 ab^{-1},

C_{12}=1, Λ=3 TeV

Higgs Pair Production

Grosse, Liao, PR D64, 115007 (2001)

In NCQED Higgs interact directly with the photon:

$e^+e^- \to H\ H$

- Proceeds via s-channel photon exchange
- results in larger σ than SM
- Different polarization dependences
- Azimuthal dependence

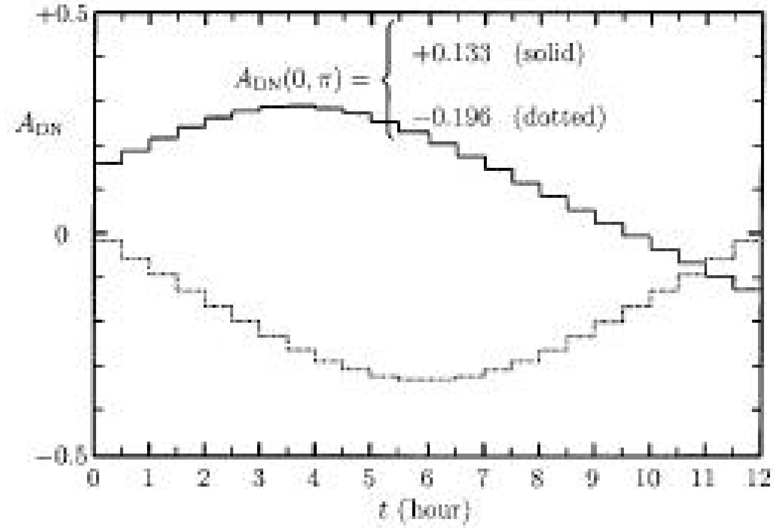


FIG. 3. Histograms of the day-night asymmetry A_{DN} as a function of time t. The solid and dotted curves are for the parameter sets (3) and (4) respectively.

Summary

- NCQFT phenomenology leads to novel probes of structure of space-time manifold
- Hallmark signature is appearance of azimuthally dependent cross section in 2→2 processes
- Different processes complement each other

Process	Probes	Reach
eγ→eγ	Space-space Space-time	~ 1.5 √s
gg ® e^+e^-	Space-time	~ 0.5 √s
e^+e^- ® gg	Space-time	~ 1.0 √s
e^+e^- ® e^+e^-	Space-time	~ 1.5 √s
e^-e^- ® e^-e^-	Space-space	~ 2.0 √s
gg ® gg	Space-space Space-time	~ 1.5 √s