Energy spectra and angular distributions of charged particles backscattered from solid targets

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Received 24 October 2007, in final form 2 January 2008
Published 14 February 2008
Online at stacks.iop.org/JPhysD/41/055505

Abstract
In this study, the EGSnrc (Electron Gamma Shower) Monte Carlo radiation transport code is used to simulate the energy spectra and the angular distributions of charged particles backscattered from solid targets. The study covers the energy range 10–70 keV, which is of interest to applied physics fields such as scanning electron microscopy, microprobe analysis and x-ray imaging. Simulation results are compared with experimental data from 11 different published experiments (1954–2002). Comparisons include electrons and positrons, low- and high-Z targets, normal and oblique incidence, different backscatter angles and backscatter planes, and backscatter from thin films. EGSnrc simulation results show excellent agreement with the majority of the published experimental data. Possible experimental and computational uncertainties explaining the few noted discrepancies are discussed. This study concludes that EGSnrc produces accurate backscatter data in the kilovoltage energy range. A documented EGSnrc user-code customized for backscatter calculations is available from the authors at http://www.physics.carleton.ca/clrp/backscatter.

1. Introduction
Accurate knowledge of the energy spectra and angular distributions of kilovoltage charged particles (electrons and positrons) backscattered from solid targets is important to many applied physics fields. In the field of analytical surface science, this knowledge is used in studying image contrast in scanning electron microscopy [1], developing correction factors for quantitative electron probe microanalysis [2, 3], evaluating layered structures and surface defects in surface spectroscopy [4], diffraction studies [5] and thin-film applications [6]. In the field of medical physics, this knowledge is used in predicting dose perturbations due to high-Z inhomogeneities in tissue [7], studying the performance of x-ray tubes when placed in the magnetic field of a magnetic resonance scanner [8, 9] and quantifying the effect of off-focal radiation on the output of x-ray systems [10]. From an academic perspective, this knowledge helps in understanding the processes involved in charged particle backscatter, and in evaluating the merits of various theories available to explain them. Over the years, there have been a number of Monte Carlo studies of the energy spectra and the angular distributions of backscattered charged particles in the kilovoltage energy range [3, 11–19]. However, there are many simplifying approximations in the models used, and these in-house Monte Carlo codes are not usually available to the researchers of the applied physics community with appropriate manuals and source-code documentation. In addition, a few investigators did not present comparisons of their Monte Carlo simulation results with experimental data. An alternative approach is to perform backscatter calculations using the currently available general-purpose Monte Carlo codes such as PENelope [20, 21], ETRAN-based [22] codes (i.e. MCNP [23] and ITS [24]), GEANT4 [25] or EGSnrc [26, 27]. The EGSnrc (Electron Gamma Shower) Monte Carlo code has sophisticated low-energy physics comparable to those in PENelope;
however, PENELope is considerably slower [28] and requires careful tuning of its simulations parameters [29], unlike EGSnrc.

This study tests the capability of EGSnrc to predict the energy spectra and the angular distributions of backscattered charged particles. In a related study [30] (called paper I henceforth), EGSnrc is shown to predict charged particle backscatter coefficients within \( \pm 4\% \) of the average of the majority of published experimental data, which is considered excellent given the scatter in the experimental data and the large uncertainties associated with them. In our current study, a new EGSnrc user-code, dedicated to backscatter calculations, is developed. Simulation results are compared with experimental data from 11 different published experiments. The comparisons cover the energy range 10–70 keV and include electrons and positrons, low- and high-\( Z \) targets, normal and oblique incidence, different backscatter angles and backscatter planes and backscatter from thin films.

2. Definitions

There is a large variation between different investigators in their definition of the parameters used to characterize the backscatter of charged particles. Because this study compares data from more than one experiment on the same graph, consistent terminology is needed. Figure 1 and the remainder of this section define the parameters used throughout this study. \( E_0 \) (in keV) is the kinetic energy of the incident monoenergetic charged particle beam. \( E \) (in keV) is the kinetic energy of the backscattered charged particle. \( Z \) is the atomic number of the target material. The incidence angle, \( \alpha \) (in degrees, \( 0 \leq \alpha < 90^\circ \)), is the angle between the vector of the incident beam and the vector normal to the target surface; the two vectors define the plane of incidence, \( P_0 \). The backscatter angle, \( \theta \) (in degrees, \( -90^\circ < \theta < 90^\circ \)), is the angle between the vector of the backscattered charged particle and the vector normal to the target surface; the two vectors define the plane of backscatter, \( P_\phi \), where \( \phi \) (in degrees, \( 0 \leq \phi < 90^\circ \)) is the angle between the plane of backscatter and the plane of incidence.

For this study, three backscatter planes (\( P_0 \), \( P_{90} \) and \( P_{45} \)) are investigated. The backscatter angle \( \theta \) is positive in the forward half of \( P_\phi \) and negative in the backward half. \( \Delta \Omega \) (in sr, \( 0 < \Delta \Omega \leq 2\pi \)) is the solid angle subtended by the detector. If the detector is a hemispherical collector then \( \Delta \Omega = 2\pi \) sr. The charged particle backscatter coefficient, \( \eta \) (in \%), is the number of charged particles that backscatter into the hemisphere above the target surface divided by the number of incident particles. By definition, \( \eta \) does not include electrons from secondary electron emission which is the generation of very low-energy surface electrons (conventionally with kinetic energy \(< 50 \) eV). The backscatter coefficients for electrons and positrons are \( \eta_- \) and \( \eta_+ \), respectively. For energy spectra, to differentiate between the spectra measured by a small detector at a fixed point in space and those measured by a hemispherical collector, the terms ‘local’ and ‘overall’ are used, respectively. The local energy spectrum, \( d^2 \eta(P_\phi, E, \theta)/dE d\Omega \) (in \( \text{keV}^{-1} \text{sr}^{-1} \)), is the number of charged particles backscattered in a given plane, \( P_\phi \), with energy between \( E \) and \( E + dE \), at a backscatter angle between \( \theta \) and \( \theta + d\theta \), per unit energy, per unit solid angle. The overall energy spectrum, \( d\eta(E)/dE \) (in \( \text{keV}^{-1} \)), is the number of charged particles backscattered in the hemisphere above the target surface, with energy between \( E \) and \( E + dE \), per unit energy. Finally, the angular distribution, \( d\eta(P_\phi, \theta)/d\Omega \) (in \( \text{sr}^{-1} \)), is the number of charged particles backscattered in a given plane, \( P_\phi \), at a backscatter angle between \( \theta \) and \( \theta + d\theta \), per unit solid angle. These definitions should be used to interpret the graphs in section 4.

3. Methods

3.1. Experimental measurements

This study includes experimental data from 11 different published experiments reported between 1954 and 2002 [16, 17, 31–39]. Most of the experimental data were digitized electronically from their original sources because they are not available in tabular form. The only mathematical manipulations performed on the experimental data are standard conversions of angles, bins, units and coordinate systems. Examples of such manipulations are the conversion of the integral energy spectra in Darlington [32] into their differential equivalent, and the conversion of the polar data in Darlinski [33] into their Cartesian equivalent. Additional notes on specific experiments are mentioned in section 4.

Measurements of charged particle backscatter characteristics are not trivial, and they are fraught with many sources of uncertainty. In paper I, a description of typical backscatter experiments and the uncertainties associated with the measurements of \( \eta_- \) and \( \eta_+ \) are discussed. The reasons for these uncertainties include secondary electron emission (defined in section 2), grid correction factors, voltage bias effects, surface contamination, setup and operational constraints and fluctuations, alignment issues, incident beam masking, detector entrance window, detector masking, etc.—see paper I for a more complete discussion. Most of these uncertainties affect the measurements of energy spectra and angular distributions as well. In addition, the following uncertainties are specific to the current study. Small incident beam movements can cause fluctuations in the energy response of the detector [12]. Converting the recorded energy fluence spectrum, i.e. \( E d\eta(E)/dE \), into a fluence spectrum, i.e. \( d\eta(E)/dE \), causes the lower portion

\[ \text{Figure 1. Definition of angles and planes used in this study.} \]
Figure 2. Overall energy spectra of electrons backscattered from semi-infinite targets of aluminium, copper, silver and gold for normal incidence. See section 2 for definitions of the parameters used. Experimental data are from [32] (□), [34] (×), [38] (○) and [39] (△). Data below $E/E_0 \sim 0.1$ are either unavailable or too noisy. In panel a, the anomalous point labelled with an arrow could be a typo in table 3 from Darlington [32], or it could be due to experimental instabilities in the energy recorder.

of the spectrum to have poorer resolution [31, 40]. Detector pulse pileup can distort the true energy spectrum if not carefully accounted for [16]. Deconvolving the true energy spectrum from the detector response always has varying degrees of uncertainty associated with it [16, 41]. Detector energy calibration and the exact point of zero energy loss can cause energy shifts in the recorded energy spectrum [31, 40]. The exact positioning of the detector in high gradient regions affects the accuracy of the angular distribution measurements, introducing uncertainties up to 12% for extreme angular distribution gradients [33]. The reader is referred to the original articles of the experiments for a more complete description of the experiments and their associated uncertainties.

3.2. Monte Carlo simulations

This study uses the EGSnrc Monte Carlo code [26, 27] for the simulations. The code has been in development—through its predecessors—for the past three decades, and it is currently the most heavily used Monte Carlo code in medical physics. EGSnrc provides accurate and artefact-free simulation of electron, positron and photon transport in all media, both in the kilovoltage and megavoltage range. The EGSnrc user-codes can simulate various geometry configurations commonly encountered in applied physics fields. In addition, there is a C++ interface that supports general geometry, scoring and source routines [42]. EGSnrc offers accurate low-energy physics for photons and for charged particles with kinetic energy $\geq 1$ keV. For photons, in addition to the standard photon physics, the code includes accurate coherent scattering form factors, binding effects in Compton scattering, photoelectron angular sampling, and accurate relaxation cascades after the creation of atomic vacancies (including Auger and Coster–Kronig electrons). For charged particles, the code uses the condensed history technique, which speeds up the calculations by many orders of magnitude while agreeing within 0.1% with single scattering calculations [26]. Other charged particle physics in EGSnrc include: explicit account of the differences in the stopping powers between positrons and electrons [43], electron impact ionization [44] whereby electrons directly create vacancies in the inner atomic shells, spin effects in electron elastic scattering, improved bremsstrahlung angular sampling, comprehensive treatment of inelastic collisions and inclusion of the density effect to take into account the polarization of the medium by the passing charged particle. For cross-section data, EGSnrc uses the most accurate data available for photons and charged particles [45–47]. For
more details on the code, the reader is referred to the EGSnrc manual [27] and the references therein.

As part of this study, a new EGSnrc user-code, dedicated to backscatter calculations, is developed. The inputs to the user-code are: the target specifications, the type, energy and angle of the incident charged particle beam, the cut-off energies for photons and charged particles below which their transport is terminated and their energy is deposited locally, the backscatter planes of interest (through the angle \( \phi \)), the detector locations specified by the user, and the desired bin widths for the output histograms. The output of the user-code is the charged particle backscatter coefficient, the energy spectra at the detector locations specified by the user, and the angular distributions in the backscatter planes specified by the user. In this study, a typical simulation includes a pencil beam of monoenergetic charged particles \((10 \leq E_0 \leq 70 \text{ keV})\) incident on a sample with thickness larger than the range of the charged particles in the sample material, except when thin-film targets are simulated. Three backscatter planes are investigated \((P_0, P_{45}, P_{90})\) to compare with available experimental data. Our sensitivity analysis shows that both energy spectra and angular distributions are sensitive to the choice of \(\Delta \Omega\), with the latter being more sensitive. For this reason, \(\Delta \Omega\) is varied between simulations to match the specific experiment being simulated, and its value is reported in all graphs in section 4. The bin widths chosen for the energy spectra and angular distributions are \((E_0/120)\text{ keV}\) and \(3^\circ\), respectively. Charged particles and photons are tracked down to a kinetic energy of 1 keV which introduces \(3\%\) systematic uncertainty in EGSnrc results—see paper I for a full discussion. All simulations are analogue, i.e. no variance reduction techniques

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**Figure 3.** Variation of the overall energy spectra of backscattered electrons with the angle of incidence (\(\alpha\)) for 70 keV electrons incident on aluminium. Experimental data are from [35]. For \(\alpha = 80^\circ\), both experimental data and EGSnrc simulation results are scaled down by a factor of five for clarity of the other curves. For \(\alpha = 0^\circ\), it was hard to accurately digitize the experimental curve due to poor resolution.

**Figure 4.** Variation of the local energy spectra of backscattered electrons with the backscattering angle (\(\theta\)) for 30 keV electrons normally incident on copper. Experimental data are from [31].

**Figure 5.** Local energy spectrum of 20 keV electrons backscattered from a thin copper film on an aluminium substrate. Experimental data are from [17]. Experimental data start at \(E/E_0 \approx 0.375\). Normalization is arbitrary (i.e. not largest-peak to largest-peak).

**Figure 6.** Comparison between simulation results using EGSnrc (our study) and EGS4 [9] for the overall energy spectrum of 65 keV electrons backscattered from a semi-infinite tungsten target.
are employed. Using a single 3.0 GHz Intel® Woodcrest 64-bit processor, the CPU time needed to reach an average uncertainty of \( \sim 1\% \) on the output profiles ranges from a few minutes for overall energy spectra and for angular distributions using relatively large detectors, to a few tens of hours for local energy spectra using very small detectors with high-energy incident charged particles.

**4. Results and discussion**

In this section, comparisons are presented between EGSnrc simulation results and experimental data for the energy spectra and angular distributions of backscattered charged particles. In any given figure, the highest peak in the EGSnrc results and the highest peak in the experimental data are both normalized to unity unless otherwise stated. Other data in the figure are normalized by the same factor to preserve the relative shapes and areas under different curves. No error bars are shown on the experimental data because in most experiments only the overall uncertainties are discussed without providing individual error bars. Experimental uncertainties of \( \sim 10\% \) are not uncommon. For the EGSnrc histograms, as discussed in section 3.2, the systematic and statistical uncertainties are estimated to be \( \leq 3\% \) and \( \sim 1\% \), respectively, and they are not shown for clarity of the graphs.

**4.1. Energy spectra**

Figure 2 shows the overall energy spectra of electrons backscattered from semi-infinite targets of aluminium, copper, silver and gold for normal incidence. As \( Z \) increases, the spectra peak closer to the original incident energy because backscattered electrons undergo more large angle elastic scattering deflections. The slight energy shift among the spectra from different experiments (more noticeable at the peaks) could be due to energy calibration uncertainties in the experiments. There are large variations among different experiments in the lower portions of the spectra, exceeding the uncertainties reported by investigators, e.g. \( \sim 10\% \) in Bishop [31]. These differences are mainly due to the limited detector resolution in that energy range [31]. Given the variations between experiments, it can be said that the spectra generated by EGSnrc are well within the average of the experimental measurements.

Figure 3 shows the variation of the overall energy spectrum of backscattered electrons with the angle of incidence (\( \alpha \)) for 70 keV electrons incident on aluminium. As \( \alpha \) increases, the electrons penetrate less and lose less energy. This is demonstrated by the broad spectrum at \( \alpha = 0^\circ \), which gets narrower and peaks towards higher energies as \( \alpha \) increases. The area under the curves also increases with \( \alpha \) because the portion of the differential elastic scattering distribution that
The discrepancy at $\theta = 67.5^\circ$ could be partially due to the increased experimental uncertainties as near-grazing incidence is approached, e.g. detector masking, surface contamination, etc—see paper I, and partially due to the increased importance of electron diffraction (i.e. collective wave-like elastic scattering of electrons by the atoms of a crystalline array) at near-grazing incidence which is not modelled in EGSnrc. Overall, EGSnrc simulation results in figure 4 reproduce the experimental measurements reasonably well. In separate comparisons of the mean and most probable energies of the spectra of backscattered electrons (not shown), the EGSnrc-calculated values for 30 keV electrons normally incident on aluminium, copper, silver and gold are found to be within 5% of those from four different experiments [31, 32, 37, 38] which is well within the uncertainties of the experimental data discussed above.

For backscatter from thin films, figure 5 shows the local energy spectrum of electrons backscattered at $\theta = 25^\circ$ in plane $P_0$ from a 0.1 $\mu$m copper film (equivalent to 6% of the CSDA range of 20 keV electrons in copper) deposited on an aluminium substrate. EGSnrc reproduces the features of the energy spectrum very well, except for the magnitude of the high-energy peak which is produced by the copper thin film. Rau et al [17], who measured the spectrum in figure 5, showed the same over-estimate of the high-energy peak in their Monte Carlo simulation. This discrepancy could be partially due to experimental uncertainties in the film thickness and partially due to the lack of energy loss straggling in the Monte Carlo simulations.

Before closing the discussion on energy spectra, it is worth noting the difference between EGSnrc and its predecessor EGS4 [48] in evaluating these spectra. A recent study [9] used EGS4 to generate the energy spectrum of backscattered electrons when 65 keV monoenergetic electrons normally impinge onto a semi-infinite tungsten target. Figure 6 shows a comparison between the spectrum calculated using EGS4 [9] and our EGSnrc-calculated spectrum. In addition to the slight energy shift, the features exhibited by the spectrum calculated using EGS4 are not observed in any of the experimental data presented above.

### 4.2. Angular distributions

Figure 7 shows the angular distribution in plane $P_0$ for normally incident electrons after they backscatter from semi-infinite targets of copper and gold. For normal incidence, the angular distribution exhibits a cosine-like distribution,
which shows as a circle in the polar plots. At large \( \theta \) (close to \( 90^\circ \) and \( -90^\circ \)), the angular distribution deviates slightly from a perfect cosine. Also for normal incidence, the angular distribution does not vary much with \( Z \) (the profiles for copper and gold are almost the same). The figure shows that for both medium- and high-\( Z \) targets, EGSnrc simulation results reproduce the experimental data very well. Figure 8 shows the variation of the angular distribution with the atomic number of the target material (\( Z \)) for oblique incidence. Unlike the normal incidence case, the angular distribution at oblique incidence varies dramatically with \( Z \). There are discrepancies between the EGSnrc simulation results and experimental data for carbon and uranium. A normalization scheme other than the one adopted in this study could improve the agreement for those elements but worsen it for others. Figure 9 shows the variation of the angular distribution with the angle of incidence (\( \alpha \)) for 30 keV electrons incident on aluminium and silver. The number of backscattered electrons per unit solid angle peaks near the ‘reflection’ or the ‘mirror’ angle of \( \alpha \), which becomes more obvious as \( \alpha \) increases. EGSnrc simulation results reproduce the experimental data very well, except for silver at \( \alpha = 30^\circ \) in the backward portion of the distribution. The discrepancy at \( \alpha = 80^\circ \) for both aluminium and silver can be attributed to the experimental and computational uncertainties associated with near-grazing incidence as discussed for figure 4 above. Figure 10 shows the variation of the angular distribution with the plane of backscatter (\( P_\theta \)) for oblique incidence (\( \alpha = 55^\circ \)) of 30 keV electrons on aluminium and gold. For aluminium, the angular distribution in plane \( P_0 \) peaks at the ‘mirror’ angle of \( \alpha \). As \( P_0 \) rotates \( 45^\circ \) to become \( P_{45} \), the angular distribution gets broader. As \( P_{45} \) rotates another \( 45^\circ \) to become \( P_{90} \) (perpendicular to \( P_0 \)), the angular distribution becomes symmetric around \( \theta = 0^\circ \) as expected due to symmetry. In the polar plots, as the plane of backscatter rotates from \( P_0 \) to \( P_{90} \), the cigar-like angular distribution gradually changes to a pancake-like one. The figure shows that EGSnrc simulation results reproduce the experimental data well, with the agreement being better for aluminium than it is for gold.

Figure 11 shows the angular distributions of 10 and 15 keV electrons backscattered in plane \( P_0 \) from a composite thin film made of 20 nm of gold followed by 20 nm of aluminium oxide (\( Al_2O_3 \)). At such low energies, the disagreement could partially be due to the limitations of the underlying theoretical models used in EGSnrc—see paper I for a full discussion. Because only one experiment is available and with a very limited number of data points, it can only be concluded that EGSnrc qualitatively reproduces angular distributions of backscattered electrons from composite thin films.

Figure 12 shows the variation of the angular distribution with the angle of incidence (\( \alpha \)) for 35 keV positrons incident on aluminium and gold targets. The profiles resemble those of backscattered electrons, and arguments similar to those made for electrons can also be made for positrons. As with electrons, EGSnrc reproduces the experimental data very well.

5. Conclusions

This study used the EGSnrc Monte Carlo radiation transport code to simulate the energy spectra and the angular
distributions of charged particles backscattered from solid targets in the kilovoltage energy range. Comparisons with experimental data show excellent agreement in most cases. The noted discrepancies are most likely due to systematic experimental uncertainties as indicated by the fluctuations between different experiments measuring the same profiles. For energies \( \sim 10\,\text{keV} \) and below, the discrepancies can partially be attributed to the limitations of the underlying theoretical models used in EGSnrc. Overall, EGSnrc can be used to generate accurate catalogues of kilovoltage backscatter data, which are needed in many applied physics fields. A documented EGSnrc user-code customized for backscatter calculations is available from the authors at http://www.physics.carleton.ca/clrp/backscatter.

Acknowledgments

The authors would like to thank Olaf Nairz for his help with the articles in German and Iwan Kawrakow for the discussions on EGSnrc limitations at very low energies. This work is partially funded by the Natural Sciences and Engineering Research Council of Canada (NSERC), the Canada Research Chairs program (CRC), the Canada Foundation for Innovation (CFI) and the Ontario Innovation Trust (OIT).

References


Figure 12. Variation of the angular distribution of backscattered positrons with the angle of incidence (α) for 35 keV positrons incident on aluminium and gold targets. Experimental data are from [16]. The number of experimental data points reported in [16] is limited.


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