3. Heavy Quarkonia

1. Spectroscopy
2. em decays
3. decays

2. The November Revolution:

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1974

Experimental Observation of a Heavy Particle J†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973
(Received 12 November 1974)

We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \to e^+ + e^- + x$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

Discovery of a Narrow Resonance in $e^+ e^-$ Annihilation*

J.-E. Augustin,† A. M. Boyarski, M. Breidenbach, F. Bulos, J. T. Dakin, G. J. Feldman, G. E. Fischer, D. Fryberger, G. Hanson, B. Jean-Marie,† R. R. Larsen, V. Lüth, H. L. Lynch, D. Lyon, C. C. Morehouse, J. M. Paterson, M. L. Perl, B. Richter, P. Rapidis, R. F. Schwitters, W. M. Tanenbaum, and F. Vannucci‡
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

G. S. Abrams, D. Briggs, W. Chinowsky, C. E. Friedberg, G. Goldhaber, R. J. Hollebeek, J. A. Kadyk, B. Lulu, F. Pierre,§ G. H. Trilling, J. S. Whitaker, J. Wiss, and J. E. Zipse
Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720
(Received 13 November 1974)

We have observed a very sharp peak in the cross section for $e^+e^- \to$ hadrons, e^+e^-, and possibly $\mu^+\mu^-$ at a center-of-mass energy of 3.105 ± 0.003 GeV. The upper limit to the full width at half-maximum is 1.3 MeV.

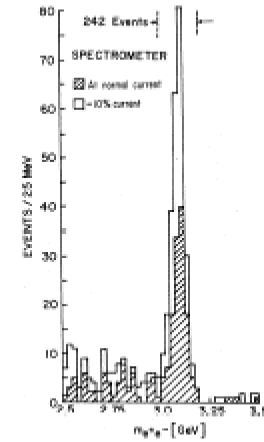


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

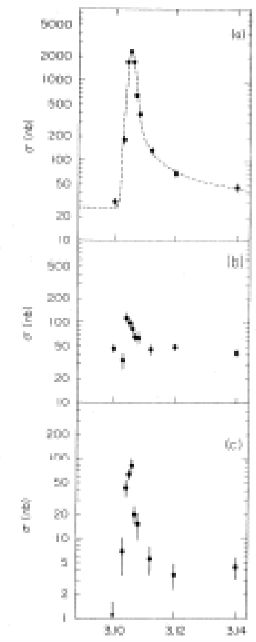


FIG. 1. Cross section versus energy for (a) multihadron final states, (b) e^+e^- final states, and (c) $\mu^+\mu^-$, $\pi^+\pi^-$, and K^+K^- final states. The curve in (a) is the expected shape of a δ-function resonance folded with the Gaussian energy spread of the beams and including radiative processes. The cross sections shown in (b) and (c) are integrated over the detector acceptance. The total hadron cross section, (a), has been corrected for detection efficiency.

VOLUME 34, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JANUARY 1975

Is Bound Charm Found?*

A. De Rújula
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

S. L. Glashow†
Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 27 November 1974)

We argue that the newly discovered narrow resonance at 3.1 GeV is a $^3S_1$ bound state of charmed quarks and we show the consistency of this interpretation with known meson systematics. The crucial test of this notion is the existence of charmed hadrons near 2 GeV.

Spectroscopy of the New Mesons*

Thomas Appelquist,† A. De Rújula, and H. David Politzer‡
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
and
S. L. Glashow§
Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 021
(Received 11 December 1974)

The interpretation of the narrow boson resonances at 3.1 and 3.7 GeV as charmed quark-antiquark bound states implies the existence of other states. Some of these should be copiously produced in the radiative decays of the 3.7-GeV resonance. We estimate the masses and decay rates of these states and emphasize the importance of γ-ray spectroscopy.

Two earlier papers[1,2] present our case that the recently discovered[3,4] and confirmed[5] resonance at 3.105 GeV is the ground state of a charmed quark bound to its antiquark, by colored gauge gluons: orthocharmonium I. More recently, a second state at 3.695 GeV has been reported[6] with an estimated width of 0.5–2.7 MeV and a partial decay rate ~2 keV into e^+e^-. We interpret this state as an S-wave radial excitation, orthocharmonium II, with $J^P = 1^-$ and $I^G = 0^-$. Here are three indications of the correctness of our interpretation: (1) Much of the time, orthocharmonium II decays into orthocharmonium I and two pions. This behavior suggests that orthocharmonium II is an excited state of orthocharmonium I.[7] (2) The leptonic width of orthocharmonium II is about half that of orthocharmonium I, not unexpected for an excited state whose wave function at the origin is smaller. (3) Orthocharmonium II is not seen in the Brookhaven National Laboratory–Massachusetts Institute of Technology experiment.[8] In a thermodynamic model,[9] the production cross section of a hadron of 3.7 GeV is suppressed by $\sim 10^{-2}$ relative to that of a hadron of 3.1 GeV. Moreover, the leptonic branching ratio of orthocharmonium II is smaller than that of orthocharmonium I by a factor of 10.

We predict the existence of other states of charmonium with masses less than 3.7 GeV, a

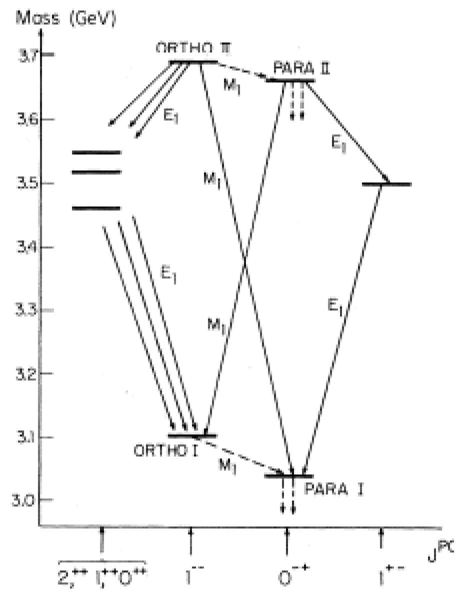


FIG. 1. Masses and radiative transitions of charmonium.

Spectrum of Charmed Quark-Antiquark Bound States*

E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan†
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
(Received 17 December 1974)

The discovery of narrow resonances at 3.1 and 3.7 GeV and their interpretation as charmed quark-antiquark bound states suggest additional narrow states between 3.0 and 4.3 GeV. A model which incorporates quark confinement is used to determine the quantum numbers and estimate masses and decay widths of these states. Their existence should be revealed by γ-ray transitions among them.

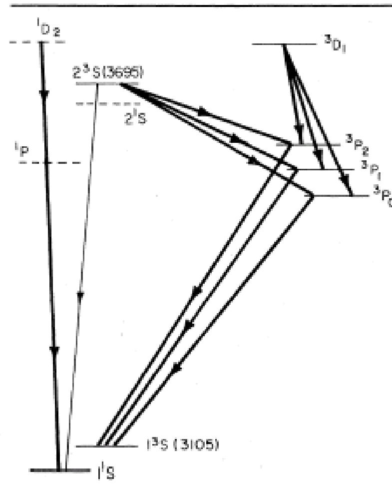


FIG. 1. The spectrum of charmonium. The vertical scale is schematic; our predictions of masses for the P and D levels are given in the text. The $^3D_2$ and $^3D_3$ levels are not shown as their position relative to $^3D_1$ is sensitive to $2\,^3S_1$–$^3D_1$ mixing. Heavy lines are allowed $E1$ γ transitions; the $2\,^3S \to 1\,^1S$ decay is a highly suppressed $M1$ transition. Dashed levels are unlikely to be produced or fed from above at an e^+e^- storage ring. Transitions among levels of an LS multiplet are probably unobservable, while γ transitions between states having the same value of $C = (-1)^{L+S}$ are rigorously forbidden.

TABLE I. γ ray widths.[a]

Transition	Γ_γ	Γ_γ (keV)
$2\,^3S \to {}^3P_2$	$5I_1\alpha k^3$	120
$\to {}^3P_1$	$3I_1\alpha k^3$	70
$\to {}^3P_0$	$1I_1\alpha k^3$	25
${}^3P_2 \to 1\,^3S$	$I_2\alpha k^3$	240
${}^3P_1 \to 1\,^3S$	$I_2\alpha k^3$	240
${}^3P_0 \to 1\,^3S$	$I_2\alpha k^3$	240
${}^1P_1 \to 1\,^1S$	$I_2\alpha k^3$	240
${}^3D_1 \to {}^3P_2$	$1I_3\alpha k^3$	7
$\to {}^3P_1$	$15I_3\alpha k^3$	110
$\to {}^3P_0$	$20I_3\alpha k^3$	150
$2\,^3S \to 1\,^1S$	$I_4\alpha k^7$	~1

[a]In the second column $1/\alpha = 137$, k is the energy of the transition, and I_n is a radial integral. The last column is based on our wave functions and energy differences, with fine-structure splittings and S–D mixing ignored.

P multiplet lies about 230 MeV below that of the $2S$ levels. This energy difference is not very sensitive to our choice of parameters: It decreases to 160 MeV if α_s and m_c assume the unreasonable values of 0.8 and 0.9 GeV, respectively.

(b) The c.o.g. of the lowest D multiplet is 70 MeV above that of the $2S$ levels. These D levels may therefore lie below the threshold M_c.

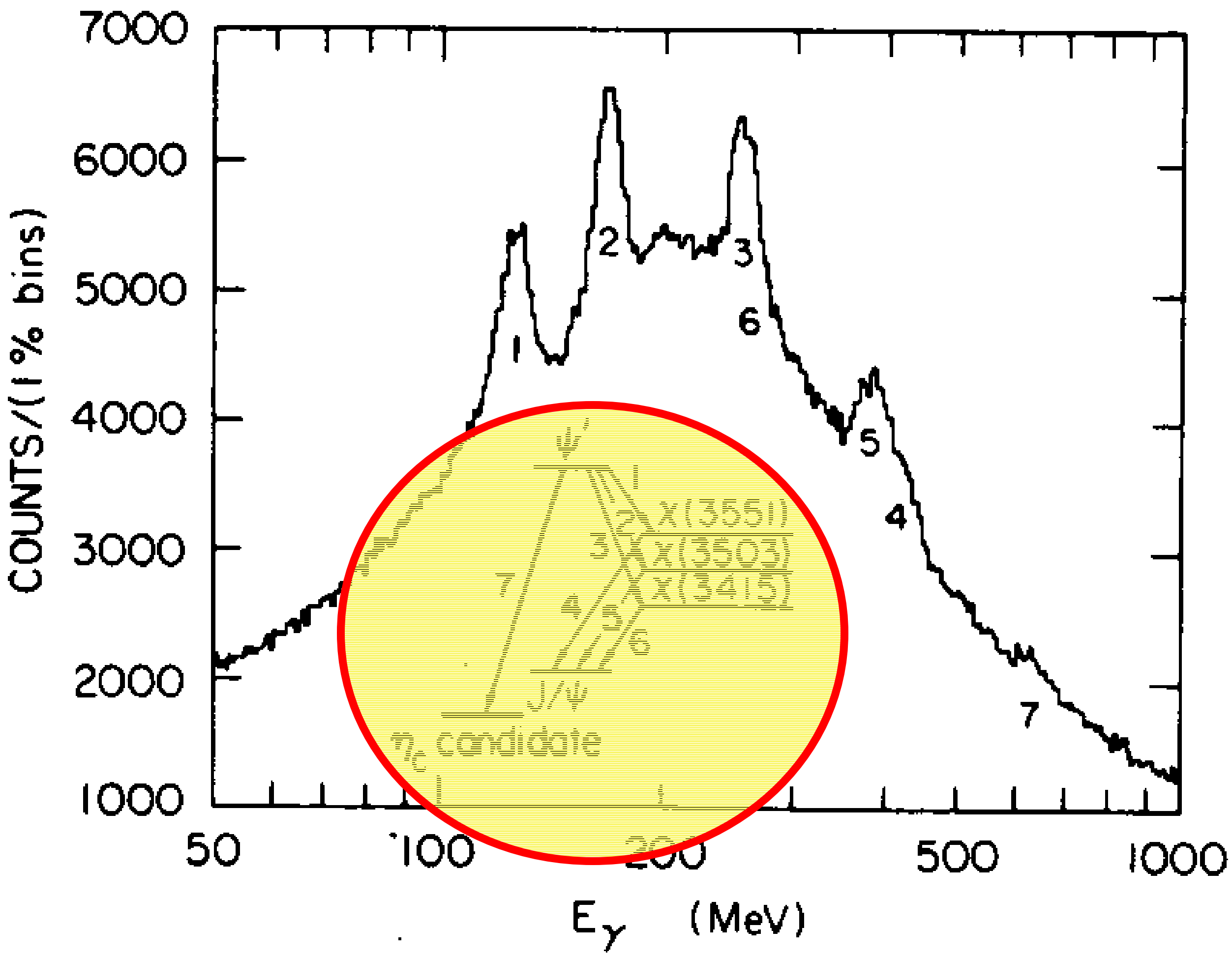
(c) The $3S$ level lies at ~4.2 GeV. As no sharp resonance has been found in this region,[2] this implies that $M_c < 4.2$ GeV.

(d) The almost inevitable presence of tensor forces has an intriguing consequence, for it guar-

The Charmonium Spectrum

Volume 45, Number 14 PHYSICAL REVIEW LETTERS 6 October 1980

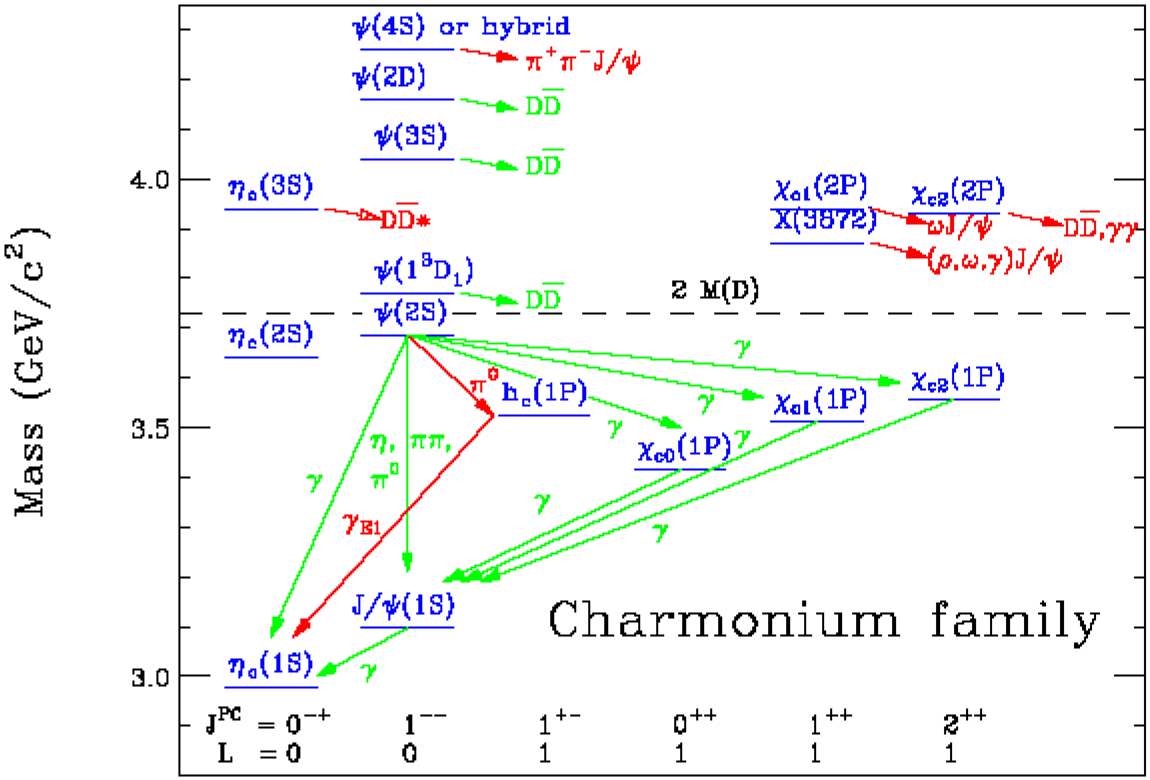
Observation of an η_c Candidate State with Mass 2978 ± 9 MeV



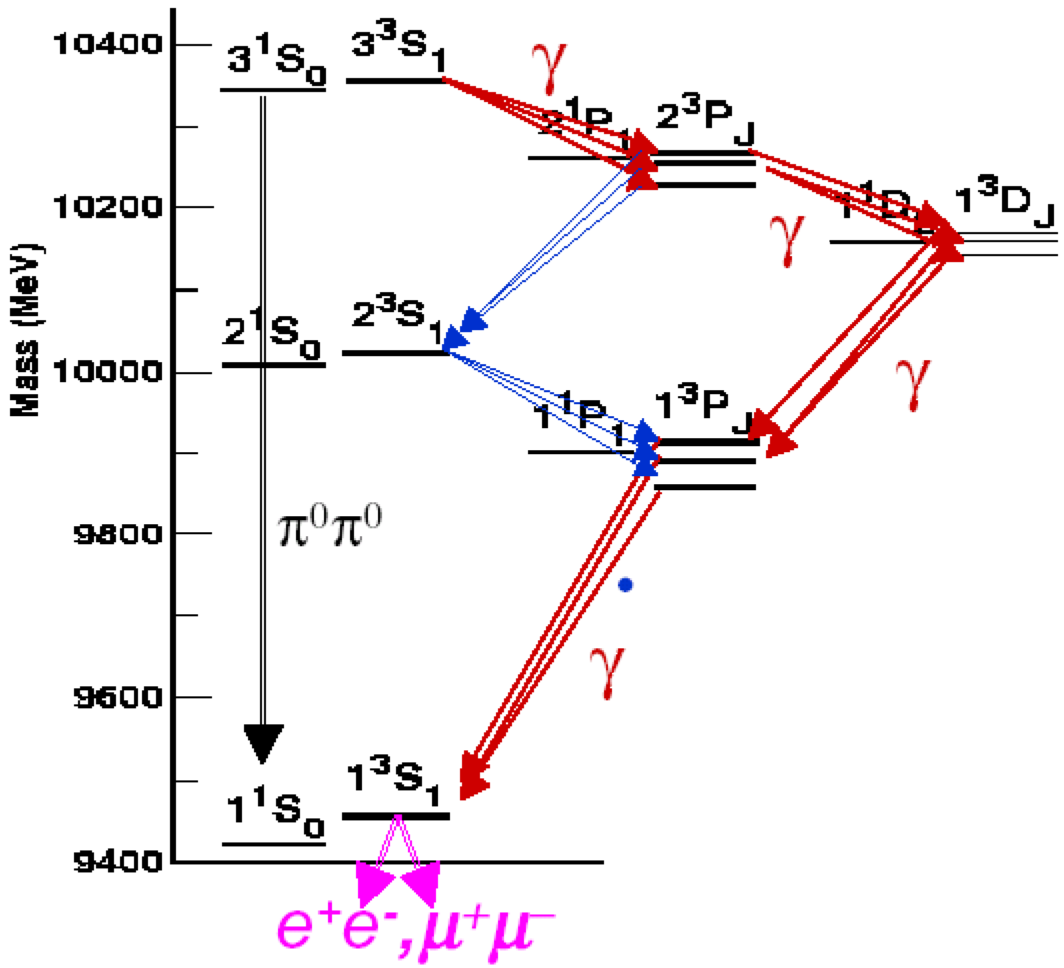
Richter Ting

"New" Spectroscopy of Mesons

Spectroscopy convinced us that quarks were real

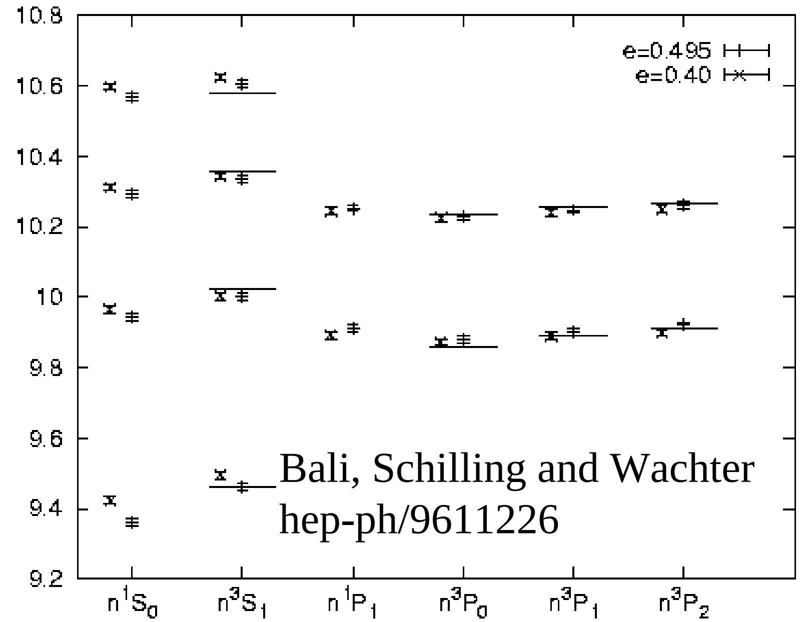


"New" Spectroscopy of Mesons



Why is this important?

- Much theoretical progress:
- Lattice QCD is a first principles calculation starting from the QCD lagrangian
  - Gives a good description of the observed spectrum or heavy quarkonium



- NRQCD
- Quark Models
  - Potential description works well
- ***Absolutely necessary to test theory against experiment***
- Use the (venerable) Quark Model to point the way
- Recent interest due to
  - Observation of many new states
  - CLEO/CESR + BESIII + B-factories

1. Potential Models:

- Spin independent potentials
- Relativistic corrections
- Spin dependent effects
- Coupled channel effects

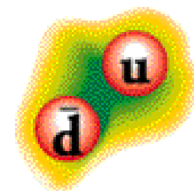
Reviews:

Kwong and Rosner, Ann. Rev. Nucl. Part. Sci. 37, 325 (1987)
Buchmuller and Cooper, Adv.Ser.Direct.High Energy Phys. 1, 412 (1988)
Konigsmann, Phys. Rept. 139, 243 (1986).

Thomas as has recent review and maybe quigg?

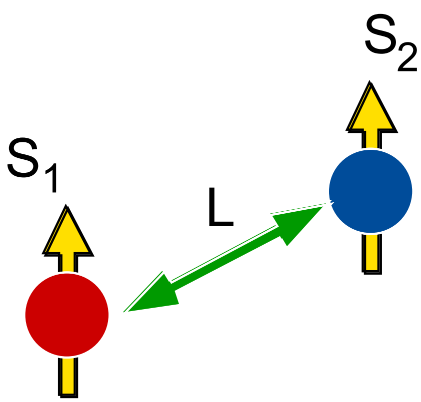
Mesons are composed of a quark-antiquark pair

Combine u,d,s,c,b quark and antiquark to form various mesons:



π meson

Meson quantum numbers characterized by given J^{PC}



$S = S_1 + S_2$

$J = L + S$

$P = (-1)^{L+1}$

$C = (-1)^{L+S}$

Allowed:

$J^{PC} = 0^{-+}\ \ 1^{--}\ \ 1^{+-}\ \ 0^{++}\ \ 1^{++}\ \ 2^{++}\ \dots$

Not allowed: exotic combinations:

$J^{PC} = 0^{--}\ \ 0^{+-}\ \ 1^{-+}\ \ 2^{+-}\ \dots$

4.1 The Spin-Independent Potential

Previously gave qualitative arguments why the spin-independent potential is linear + Coulomb

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + br \qquad b \simeq 0.18 \text{ GeV}^2$$

We also saw how this potential is consistent with results from Lattice QCD

However, Historically this form was arrived at through trial and error (Although Appelquist and Politzer got it right in an early paper ~ 1975)

Emperically, the Schrodinger eqn was solved for a given potential which was modified until agreement was achieved between theory and experiment.

$$M = m_1 + m_2 + E_{nl}$$

$$\left[\frac{p^2}{2\mu} + V(r)\right]\psi = E_{nl}\psi \qquad \left(\mu = \frac{m_1 m_2}{m_1+m_2}\right)$$

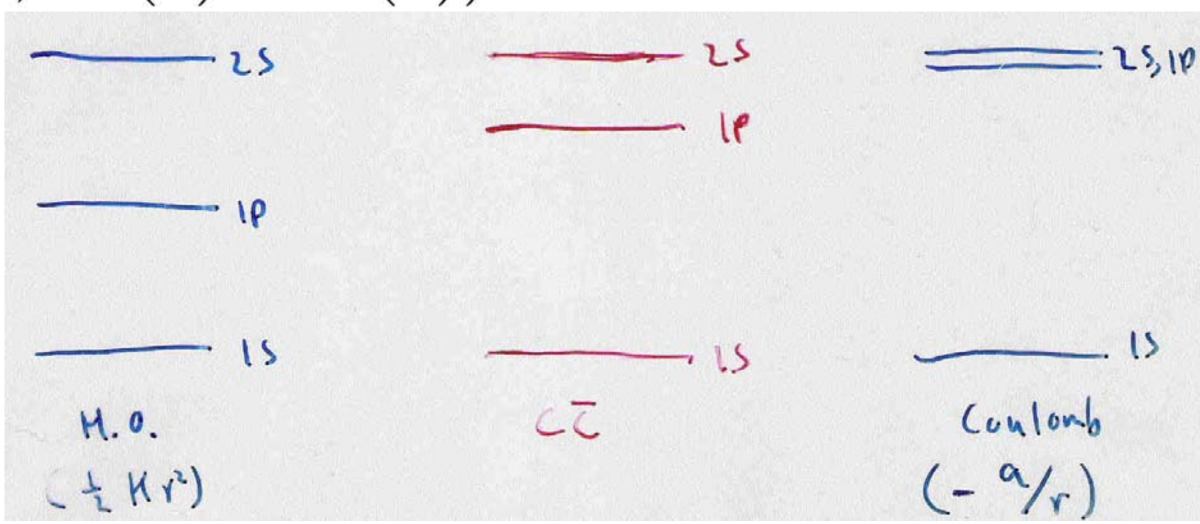
$$\left[\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right]\psi = E_{nl}\psi$$

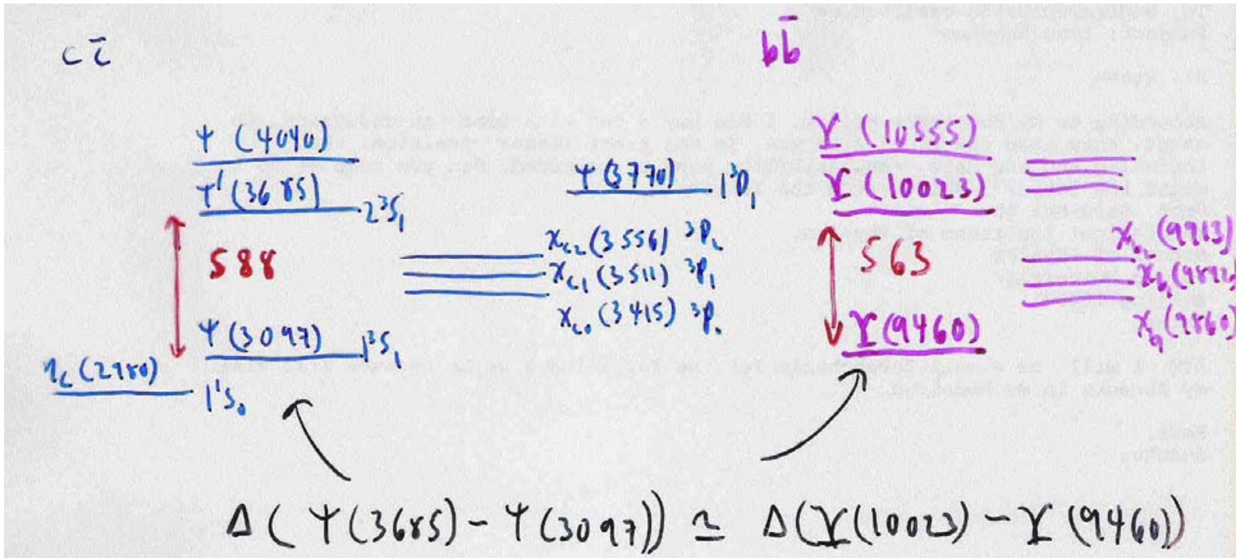
$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

$$\psi(r,\theta,\phi) = R(r)Y_{\ell m}(\theta,\phi) \qquad U(r) \equiv rR(r)$$

$$\frac{\hbar^2}{2\mu}\frac{d^2U}{dr^2} + \left[V(r) + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}\right]U = E_{n\ell}U$$

$$(U(0) = 0,\ U'(0) = R(0))$$

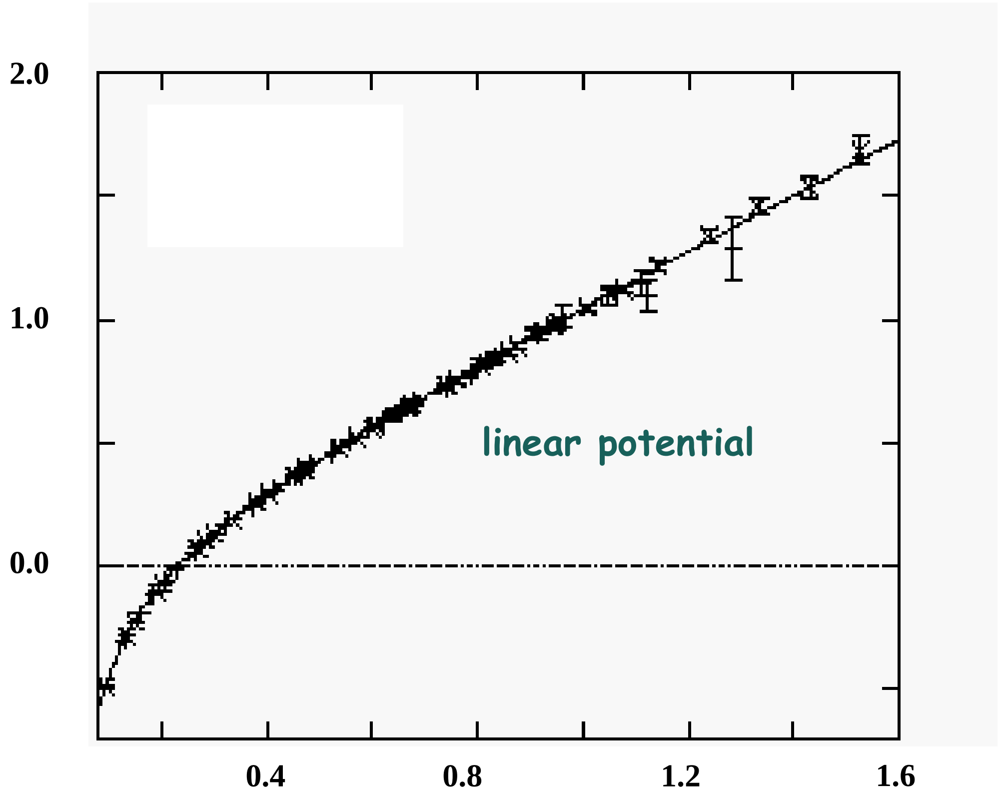




$$\Rightarrow \begin{cases} V(r) = \lambda r^{\nu} & r \simeq 0.1 \\ V(r) = c\ln(r/r_0) & c \simeq 0.73 \end{cases}$$

Also $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br$ for suitable α_s, b

Lattice QCD gives qq potential:



From G. Bali

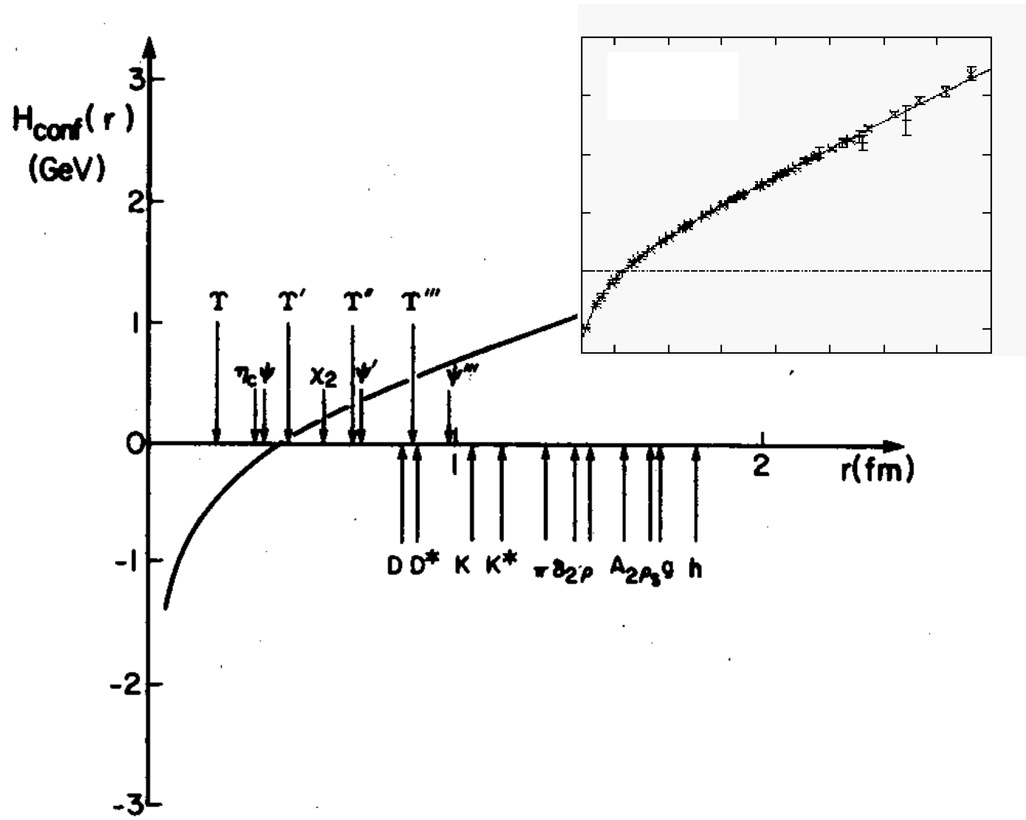
Quark-antiquark Potential

For given spin and orbital angular momentum configurations
& radial excitations generate our known spectrum of light quark mesons

$$H_{ij}^{conf} = -\frac{4}{3}\frac{\alpha_s(r)}{r} + br$$

$$M = m_1 + m_2 + E_{nl}$$

$$\left[\frac{p^2}{2\mu} + V(r)\right]\psi = E_{nl}\psi$$



Solve Schrodinger eqn
for meson masses

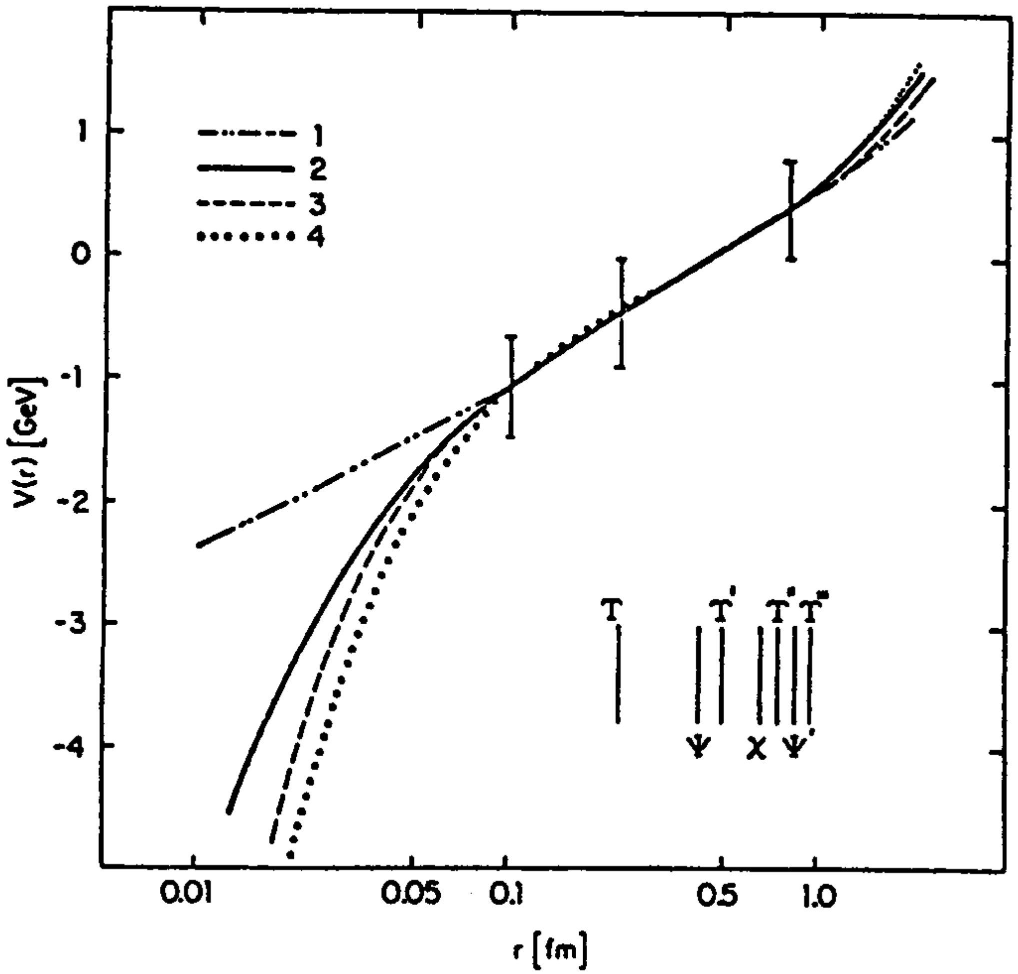


Figure 21: Various $Q\bar{Q}$ potentials. The potentials have been shifted to agree at r=0.5 fm. The numbers refer to the following references: 1: Martin [101], 2: Buchmüller, Grunberg and Tye [99], 3: Bhanot and Rudaz [102], 4: Cornell group [97].

From Buchmuller & Tye
PR D24, 132 (1981)

Quark potential models are strongly supported by emperical agreement with quarkonium spectroscopy and with lattice QCD

Could also use position of P-waves

Spin averaged ^3P_J gives

$$\bar{M} = (5M_{^3P_2} + 3M_{^3P_1} + M_{^3P_0})/9$$

For $c\bar{c}$ $\bar{M} = 3522$ MeV

$$\frac{M(2S) - M(1P)}{M(2S) - M(1S)} \begin{cases} 1/2 & \text{H.O.}(\nu = 2) \\ \simeq 1/4 & \text{for } \nu = 0 \\ 0 & \text{Coulomb}(\nu = -1) \end{cases}$$

$$c\bar{c} \Rightarrow \nu \simeq 0.15$$

Spin-dependent potentials:

Generally expect spin-dependent Interactions:

$$\vec{S}_1 \cdot \vec{S}_2 \qquad \vec{L} \cdot \vec{S} \qquad S_{12}$$

Start by looking at spin-dependent interactions of QED in hydrogen atom

Spin-Orbit: electron sees the proton circling around

- The orbital motion creates a magnetic field at the centre:

$$B = \frac{ev}{cr^2}$$

- In terms of L=mvr

$$\vec{B} = \frac{e}{mcr^3}\vec{L}$$

- The spinning electron constitutes a magnetic dipole

$$\vec{\mu} = -\frac{e}{mc}\vec{S}$$

- The interaction energy is

$$W = -\vec{\mu} \cdot \vec{B}$$

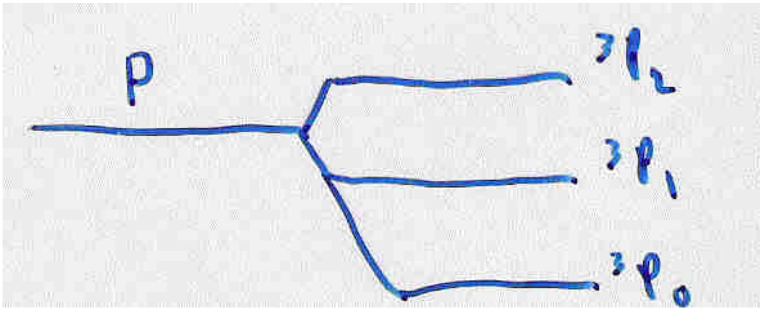
More rigorously (derived as a succession of infinitesimal Lorentz transformations) leads to the Thomas precession with a factor of 1/2

$$\Delta H_{S.O.} = \frac{e^2}{2m^2c^2r^3}\vec{L}\cdot\vec{S}$$

$$\vec{J}^2 = \vec{L}^2 + \vec{S}^2 + 2\vec{L}\cdot\vec{S}$$

$$\begin{aligned} \Rightarrow \vec{L}\cdot\vec{S} &= \frac{1}{2}[\vec{J}^2 - \vec{L}^2 - \vec{S}^2] \\ &= \frac{1}{2}[J(J+1) - L(L+1) - S(S+1)] \end{aligned}$$

$$\text{For} \quad \begin{cases} {}^3P_2 & \vec{L}\cdot\vec{S} = 1 \\ {}^3P_1 & \vec{L}\cdot\vec{S} = -1 \\ {}^3P_0 & \vec{L}\cdot\vec{S} = -2 \end{cases}$$

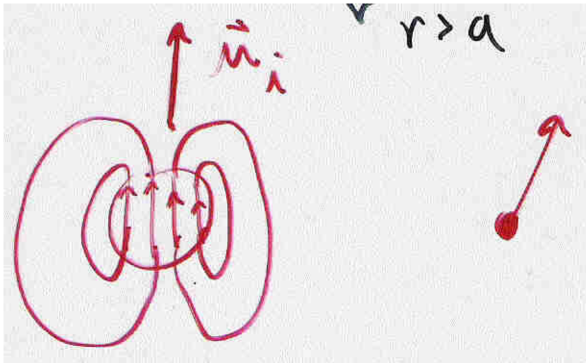


Hyperfine: Again in hydrogen, the proton has dipole moment:

$$\vec{\mu}_P = \gamma_P \frac{e}{m_P c} \vec{S}_P \quad (\gamma_P = 2.73)$$

The magnetic dipole has a field:

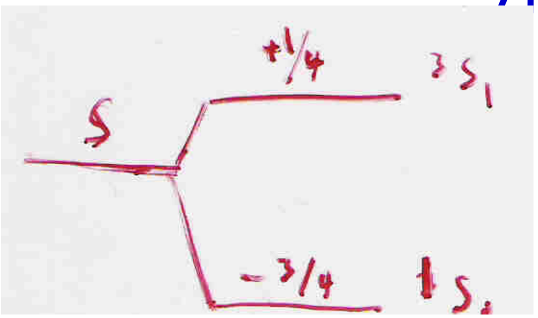
$$\vec{B}(\vec{r}) = \underbrace{\frac{1}{r^3}\left[\frac{3(\vec{\mu}\cdot\vec{r})\vec{r}}{r^2} - \vec{\mu}\right]}_{r>a} + \underbrace{\frac{8\pi}{3}\vec{\mu}}_{r<a}$$



The energy of the electon in the presence of μ_i

$$\Delta H_{SS} = \frac{\gamma_P e^2}{m m_P c^2}\left\{\frac{1}{r^3}[3(\vec{S}_P\cdot\hat{r})(\vec{S}_e\cdot\hat{r}) - \vec{S}_P\cdot\vec{S}_e)] + \frac{8\pi}{3}\vec{S}_P\cdot\vec{S}_e\delta^3(\vec{r})\right\}$$

Gives rise to the hyperfine structure of hydrogen



$$\vec{S}_1\cdot\vec{S}_2 = \frac{1}{2}[\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2] = \frac{1}{2}[s(s+1) - \frac{3}{2}]$$

21 cm line in hydrogen

One can take this over to 1-gluon interaction of QCD:

$$\Delta H_{ij}^{hyp} = -\frac{\alpha_s(r)}{m_i m_j}\left\{\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\,\delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3}[3(\vec{S}_i\cdot\hat{r}_{ij})(\vec{S}_j\cdot\hat{r}_{ij}) - \vec{S}_i\cdot\vec{S}_j)]\right\}\ \vec{F}_i\cdot\vec{F}_j$$

$$\Delta H_{ij}^{S.O.(c.m.)} = -\frac{\alpha_s(r)}{r_{ij}^3}\left(\frac{1}{m_i}+\frac{1}{m_j}\right)\left(\frac{\vec{S}_i}{m_i}+\frac{\vec{S}_j}{m_j}\right)\cdot\vec{L}\ \vec{F}_i\cdot\vec{F}_j$$

$$\Delta H_{ij}^{S.O.(TP)} = -\frac{1}{2r_{ij}}\frac{\partial V(r)}{\partial r_{ij}}\left(\frac{\vec{S}_i}{m_i^2}+\frac{\vec{S}_j}{m_j^2}\right)\cdot\vec{L}\ \vec{F}_i\cdot\vec{F}_j$$

For mesons $\langle\vec{F}_i\cdot\vec{F}_j\rangle = -\frac{4}{3}$

Systematic treatment starts with Wilson loop

Eichten and Feinberg, PR D23, 2724 (1981)
Gromes, Yukon Advanced Study Inst.

- Expanding in $1/m_Q$ write spin-dependent Hamiltonian in terms of static potential and correlation functions of colour electric and magnetic fields
- With some assumptions one obtains:

$$V_{spin}(r) = \frac{1}{m^2}\left(\frac{-k}{2r} + \frac{2\alpha_s}{3r^3}\right)\vec{L}\bullet\vec{S} + \frac{1}{m^2}\frac{4\alpha_s}{3r^3}S_{12} + \frac{1}{m^2}\frac{32\pi\alpha_s}{9}\delta^3(\vec{r})\vec{S}_1\bullet\vec{S}_2$$

Which corresponds to short range vector and long range scalar exchange

Observation of $^1P_1$ states is important test

Spin-dependent potentials:

- Need some sort of reduction to find spin dependent terms
- Depends on Lorentz nature of potential

    we find phenomenologically

    short range Lorentz Vector 1-gluon exchange

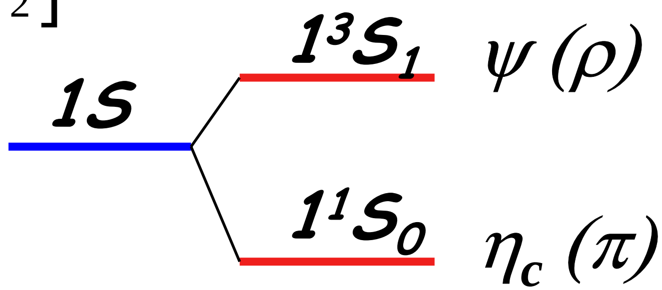
    + long range Lorentz scalar confining potential

- Use Breit-Fermi Hamiltonian
- Spin-dependent interactions are $(v/c)^2$ corrections

Spin-spin interactions:

$$H_{ij}^{hyp} = \frac{4\alpha_s(r)}{3m_i m_j}\left\{\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\,\delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3}\left[\frac{3\vec{S}_i\cdot\vec{r}_{ij}\vec{S}_j\cdot\vec{r}_{ij}}{r_{ij}^2} - \vec{S}_i\cdot\vec{S}_j\right]\right\}$$

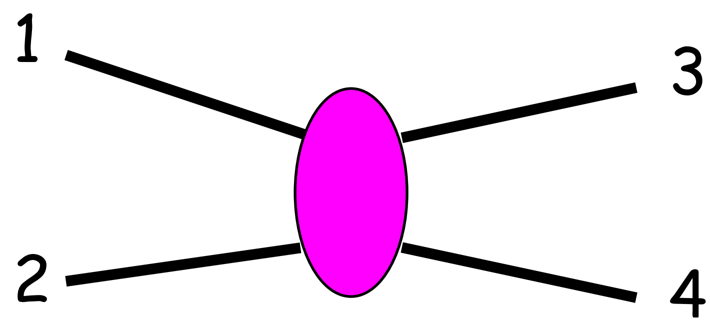
$$\vec{S}_1\cdot\vec{S}_2 = \tfrac{1}{2}\left[S^2 - S_1^2 - S_2^2\right] = \tfrac{1}{2}\left[s(s+1) - \tfrac{3}{2}\right]$$



Useful to look at more rigorous derivation

2 approaches: Bethe-Salpeter equation
equate potential to scattering amplitude
(Berestetskii, Lifshitz, and Pitaevski,
Relativistic Quantum Theory, Volume 1, Pergamon Press

Expand in powers of inverse quark mass an interaction of the form:



(in weak binding limit)

$$U(q^2) = V(q^2)[\bar{U}(p_3)\Gamma^i U(p_1)][\bar{U}(p_4)\Gamma_i U(p_2)]/[\prod_{i=1}^{4}(2E_i)]^{1/2}$$

$$\text{where } V(q^2) = \int d^3r e^{-i\vec{q}\cdot\vec{r}} V(r)$$

The interaction $\Gamma^i \otimes \Gamma_i$ is arbitrary

$\gamma^\mu \otimes \gamma_\mu$	vector exchange (1-gluon exchange)
$I \otimes I$	scalar (linear?)
$\gamma_5 \otimes \gamma_5$	pseudoscalar
etc	

e.g. Hyperfine Splitting

For interactions of the form $[\bar{U}\Gamma^i U][\bar{U}\Gamma_i U]$
in the static limit only $\Gamma^i=\gamma^0$ contibutes so $U(q^2)=V(q^2)$

We are interested in the $\mathcal{O}(q^2)$ corrections that contributes
To S-wave states of the form $\vec{\sigma}_1 \cdot \vec{\sigma}_2$

$$\bar{U}(p_3)\gamma^0 U(p_1) = \sqrt{E_3+m_3}\sqrt{E_1+m_1}\left(\chi_3^\dagger,\ \chi_3^\dagger \frac{\vec{\sigma}_3\cdot\vec{p}_3}{E_3+m_3}\right)\left(\begin{array}{c}\chi_1 \\ \frac{\vec{\sigma}_1\cdot\vec{p}_1}{E_1+m_1}\chi_1\end{array}\right)$$

To order 1/m this does contribute to H_I

$$\begin{aligned}\bar{U}(p_3)\gamma^i U(p_1) &= \sqrt{E_3+m_3}\sqrt{E_1+m_1} \\ &\times \left(\chi_3^\dagger,\ \chi_3^\dagger \frac{\vec{\sigma}_1\cdot\vec{p}_3}{E_3+m_3}\right)\left(\begin{array}{cc} 0 & \sigma^i \\ \sigma^i & 0\end{array}\right)\left(\begin{array}{c}\chi_1 \\ \frac{\vec{\sigma}_1\cdot\vec{p}_1}{E_1+m_1}\chi_1\end{array}\right) \\ &\simeq \chi_3^\dagger[\vec{\sigma}_1\cdot\vec{p}_3\sigma^i + \sigma^i\vec{\sigma}_1\cdot\vec{p}_1]\chi_1 + \ldots\end{aligned}$$

Where we set $m_3=m_1$

$p_3 = p_1 - q$ so

$$\begin{aligned} \vec{\sigma}_1 \cdot \vec{p}_3 \sigma^i + \sigma^i \vec{\sigma}_1 \cdot \vec{p}_1 &= \{\vec{\sigma}_1 \cdot \vec{p}_1,\ \sigma_{1i}\} - \vec{\sigma}_1 \cdot \vec{q}\ \sigma_1^i \\ &= 2p_1^i - q^i - i\epsilon^{kim} q^k \sigma_1^m \end{aligned}$$

We discard the first 2 terms because they don't contain σ

Similarly: $\bar{U}(p_4)\gamma^i U(p_2) \simeq \chi_4^\dagger [\vec{\sigma}_2 \cdot \vec{p}_4 \sigma^i + \sigma^i \vec{\sigma}_2 \cdot \vec{p}_4] \chi_2 + \ldots$

$p_4 = p_2 + q$

$$\vec{\sigma}_2 \cdot \vec{q}\ \sigma_{2i} = q_i + i\epsilon_{jil} q_j \sigma_{2l}$$

and $(-i\epsilon^{kim} q^k \sigma_1^m)(i\epsilon_{jil} q_j \sigma_{2l}) = -\vec{q}^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \vec{q}$

For S-waves we average over all angles to obtain: $-\frac{2}{3}\vec{q}^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$

$$\begin{aligned} \therefore U(\vec{q}^2) &= V(\vec{q}^2)[1 - \frac{2}{3}\vec{q}^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{1}{2m_1 2m_2}] \\ U(r) &= \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} U(\vec{q}^2) \\ &= [1 - \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{6m_1 m_2} \vec{\nabla}^2] V(r) \end{aligned}$$

One obtains:

Interaction Potential	$\gamma^\mu \otimes \gamma_\mu$ $V(r)$	$I \times I$ $S(r)$	$\gamma_5 \otimes \gamma_5$ $P(r)$
Spin-Orbit	$\frac{3}{2m^2}\frac{1}{4}\frac{\partial V}{\partial r}\vec{L}\cdot\vec{S}$	$-\frac{1}{2m^2}\frac{1}{4}\frac{\partial S}{\partial r}\vec{L}\cdot\vec{S}$	0
Tensor	$\frac{S_{12}}{12m^2}\left[\frac{1}{r}\frac{dV}{dr}-\frac{d^2V}{dr^2}\right]$	0	$-\frac{S_{12}}{12m^2}\left[\frac{1}{r}\frac{dP}{dr}-\frac{d^2P}{dr^2}\right]$
Hyperfine	$\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{6m^2}\nabla^2 V$	0	$\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{12m^2}\nabla^2 P$

Spin-orbit interactions:

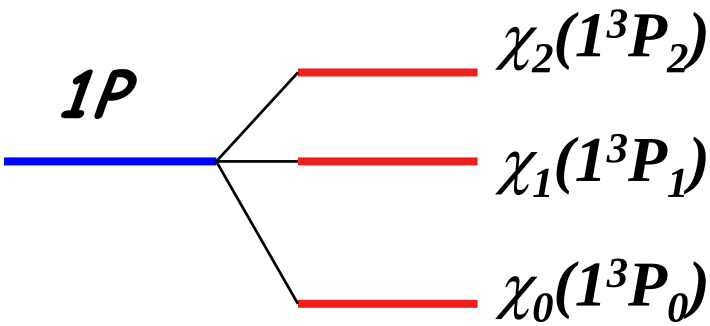
$$H_{ij}^{s.o.(cm)} = \frac{4\alpha_s(r)}{3r_{ij}^3}\left(\frac{1}{m_i}+\frac{1}{m_j}\right)\left(\frac{\vec{S}_i}{m_i}+\frac{\vec{S}_j}{m_j}\right)\cdot\vec{L}$$

$$H_{ij}^{s.o.(tp)} = \frac{-1}{2r_{ij}}\,\frac{\partial V(r)}{\partial r_{ij}}\left(\frac{\vec{S}_i}{m_i^2}+\frac{\vec{S}_j}{m_j^2}\right)\cdot\vec{L}$$

$${}^3P_2: \quad \vec{L}\cdot\vec{S} = 1$$

$${}^3P_1: \quad \vec{L}\cdot\vec{S} = -1$$

$${}^3P_0: \quad \vec{L}\cdot\vec{S} = -2$$



But numerous variations exist:

eg. Ebert Faustov & Galkin introduce Lorentz vector piece of confining potential: Phys.Rev. D67, 014027 (2003); D62, 034014 (2000)

$$V_V(r) = (1-\varepsilon)Ar + B$$

$$V_S(r) = \varepsilon Ar$$

also include anomalous chromomagnetic moment of the quark in V_V:

$$\Gamma_\mu(k) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu$$

Long range magnetic contributions vanish from choice of Parameters (which is equivalent to scalar confinement)

Also included spin independent relativistic effects

Let us examine the spin-dependent splittings in charmonium

- Using H.O. wavefunctions simplifies the calculations
- Fitting the oscillator parameter to the r.m.s. radii of exact solutions is a good approximation:

$$\psi_{1S} = \frac{2}{\pi^{1/4}}\beta^{3/2}e^{-\beta^2 r^2/2}Y_{00} \qquad \langle r^2\rangle_{1S} = \frac{3}{2}\frac{1}{\beta^2} = 2.5 \Rightarrow \beta = 0.77$$

$$\psi_{2S} = \sqrt{\frac{8}{3}}\frac{\beta^{3/2}}{\pi^{1/4}}(\frac{3}{2} - \beta^2 r^2)e^{-\beta^2 r^2/2}Y_{00} \qquad \langle r^2\rangle_{2S} = \frac{7}{2}\frac{1}{\beta^2} = 11 \Rightarrow \beta = 0.564$$

$$\psi_{1P} = \sqrt{\frac{8}{3}}\frac{\beta^{5/2}r}{\pi^{1/4}}e^{-\beta^2 r^2/2}Y_{1m} \qquad \langle r^2\rangle_{1P} = \frac{5}{2}\frac{1}{\beta^2} \simeq 7 \Rightarrow \beta = 0.598$$

$$\langle 1/r\rangle_{1P} = \frac{4}{3}\frac{\beta}{\pi^{1/2}} = 0.45$$

$$\langle 1/r^3\rangle_{1P} = \frac{4}{3}\frac{\beta^3}{\pi^{1/2}} = 0.16$$

Hyperfine Effects:

$$H_{ij}^{hyp} = \frac{32\pi}{9}\frac{\alpha_s}{m^2}\vec{S}_1 \cdot \vec{S}_2\, \delta^3(r_{ij})$$

$$\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}[s(s+1) - 3/2]$$

$$\Rightarrow \begin{cases} \langle {}^3S_1|\vec{S}_1 \cdot \vec{S}_2|{}^3S_1\rangle & = +1/4 \\ \langle {}^1S_0|\vec{S}_1 \cdot \vec{S}_2|{}^1S_0\rangle & = -3/4 \end{cases}$$

$$\begin{aligned} \therefore M({}^3S_1) - M({}^1S_0) &= \frac{32\pi}{9}\frac{\alpha_s}{m^2}\langle\delta^3(r_{ij})\rangle \\ &= \frac{32\pi}{9}\frac{\alpha_s}{m^2}|\psi(0)|^2 \\ &= \frac{32\pi}{9}\frac{\alpha_s}{m^2}\frac{\beta^3}{\pi^{3/2}} \\ &= 115 \text{ MeV (where } \beta = 0.77 \text{ GeV}, \alpha_s = 0.32, m_c = 1.6 \text{ GeV)} \\ &\text{vs 115 MeV from experiment} \qquad (1) \end{aligned}$$

$$M(2^3S_1) - M(2^1S_0) = 67 \text{ MeV}$$

Fine Structure:

We can write the ^3P_J Masses as:

$$M = M(1P) + a\langle \vec{L}\cdot\vec{S}\rangle + b\langle S_{12}\rangle$$
$$M(^3P_2) = M(1P) + a - \frac{2}{5}b = 3556$$
$$M(^3P_1) = M(1P) - a - 2b = 3511$$
$$M(^3P_0) = M(1P) - 2 - 4b = 3415$$
$$M(1P) = 3525$$

Lorentz Vector 1-gluon exchange gives:

$$a = \frac{3}{2m^2}\frac{4}{3}\frac{\alpha_s}{r^3} = 40 \text{ MeV}$$
$$b = \frac{1}{4m^2}\frac{4}{3}\frac{\alpha_s}{r^3} = 7 \text{ MeV}$$

If confining piece is br

(a) Lorentz Vector: a′ = a + 47 MeV
b′ = b + 3 MeV

(b) Lorentz Scalar: a′ = a - 16 MeV
b′ = b

(c) Lorentz Pseudoscalar: a′ = a
b′=b – 3 MeV

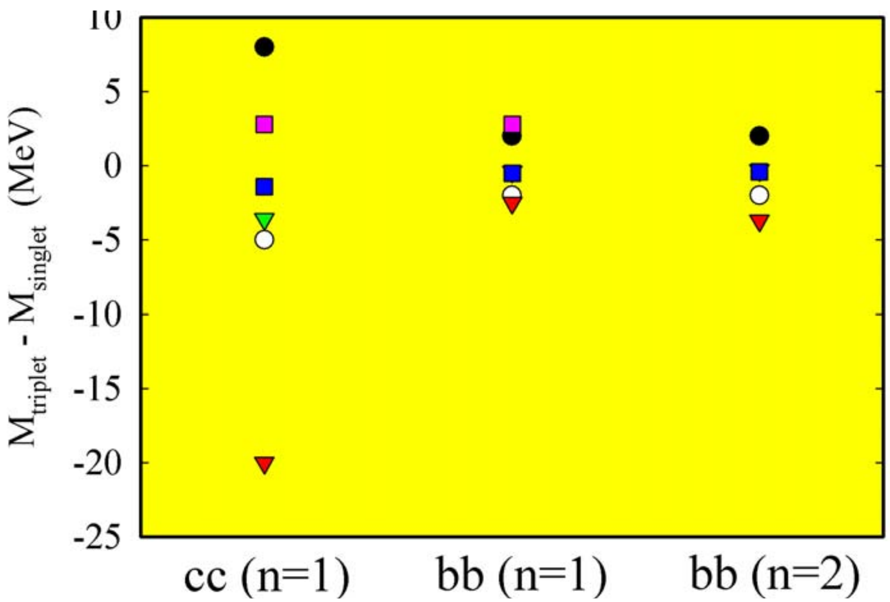
Experiment favours Lorentz Scalar Confining

$^1P_1$ vs $^3P_{cog}$ mass – distinguish models

- Important to distinguish models

- In QM triplet-singlet splittings test
  - the Lorentz nature of the confining potential
  - Relativistic effects

- important validation of
  - lattice QCD calculations
  - NRQCD calculations



Observation of $^1P_1$ states is an important test of theory

Wide variation of theoretical predictions:

TABLE I. Predictions for hyperfine splittings $M(n\,{}^3P_{\rm cog}) - M(n\,{}^1P_1)$ for $c\bar{c}$ and $b\bar{b}$ levels.

	Reference	Approach	$n=1$ $c\bar{c}$ (MeV)	$n=1$ $b\bar{b}$ (MeV)	$n=2$ $b\bar{b}$ (MeV)
QM	GI85 [14]	a	8	2	2
	MR83 [15]	b	0	0	1
	LPR92 [16]	c	4	2	1
	OS82 [17]	d	10	3	3
QM	MB83 [18]	e	-5	-2	-2
	GRR86 [19]	f	-2	-1	-1
	IO87 [20]	g	24.1 ± 2.5	3.73 ± 0.1	3.51 ± 0.02
	GOS84 $\eta_s=1$ [21]	h	6	3	2
	GOS84 $\eta_s=0$ [21]	h	17	8	6
QM	PJF92 [22]	i	-20.3 ± 3.7	-2.5 ± 1.6	-3.7 ± 0.8
	HOOS92 [23]	j	-0.7 ± 0.2	-0.18 ± 0.03	-0.15 ± 0.03
PQCD	PTN86 [25]	j	-3.6	-0.4	-0.3
	PT88 [26]	j	-1.4	-0.5	-0.4
lattice	SESAM98 [31]	k	–	~-1	–
	CP-PACS00 [33]	l	1.7–4.0	1.6–5.0	–
	EFG		0	-1	-1

Quark Potential Models with 1-gluon exchange:

$$H_{q\bar{q}}^{hyp} = \frac{32\pi}{9} \frac{\alpha_s}{m_q m_{\bar{q}}} \vec{S}_q \cdot \vec{S}_{\bar{q}} \delta^3(\vec{r})$$

δ function is short range but smeared by relativistic effects modeled by a Gaussian.

- gives $M(^3P_{cog}) > M(^1P_1)$

Godfrey & Isgur, PR D32, 189 (1985)

- but with spin-independent relativistic corrections
  McClary & Byers find
  $M(^3P_{cog}) < M(^1P_1)$

McLary & Byers, PR D28, 1692 (1983)

- Introducing long range Lorentz Vector Franzini finds:
  $M(^3P_{cog}) < M(^1P_1)$

Franzini, PL B296, 199 (1992)

Perturbative QCD: $M(^3P_{cog}) < M(^1P_1)$ Pantaleone and Tye, PR D37, 3337 (1988)

$$[\overline{M}(n\,^3P_j) - M(n\,^1P_1)] = \frac{8}{9}\left[\frac{1}{4} - \frac{N_f}{3}\right]\frac{\alpha_s^2}{\pi}\frac{1}{m^2}\left\langle\frac{1}{r^3}\right\rangle$$

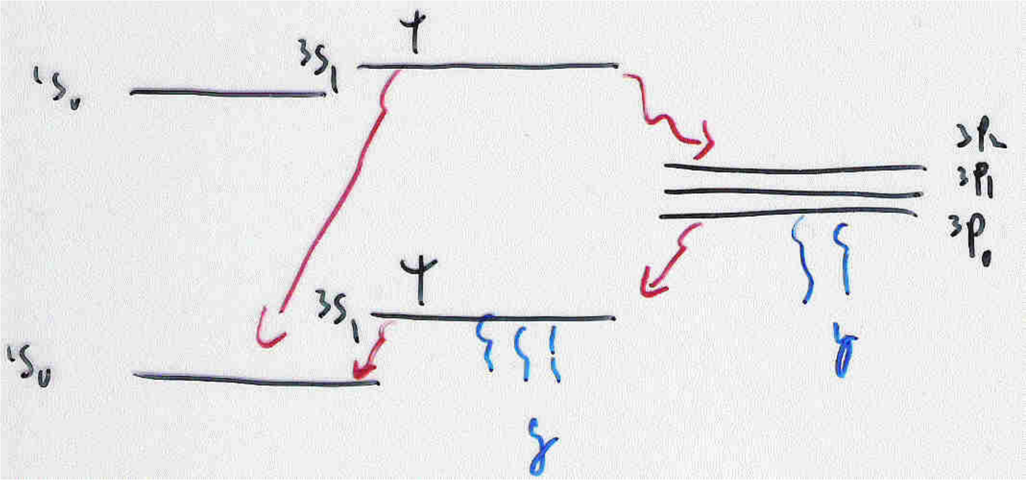
-ve for $N_f > 0$ but other possible contributions;
long-range, relativistic, coupled channel..

Lattice QCD: $M(^3P_{cog}) > M(^1P_1)$

- Ultimately the definitive answer
- Need more precise results.

wide variation in predictions indicates need for experimental data

Decays and Transitions



- To calculate Decays and Transitions we need to calculate hadronic matrix elements.
- Define a “Mock” meson which we equate with the wavefucntion of the physical meson

$$|M(\vec{K}) = \sqrt{2E_M}\int d^3p\Phi(\vec{p})\chi_{s\bar{s}}\ \phi_{q\bar{q}}\ \phi_{colour}|q(\frac{m_q}{m_q+m_{\bar{q}}}\vec{K}+p,\ s)\bar{q}(\frac{m_{\bar{q}}}{m_q+m_{\bar{q}}}\vec{K}-p,\ \bar{s})\rangle$$

There are two generic types of matrix elements:

$$\langle 0|A|M_i\rangle \text{ like in } J/\psi \to e^+e^-$$

$$\langle M_f|A|M_i\rangle \text{ like in } \chi_{c2} \to J/\psi + \gamma$$

A is some sort of transition operator like:

$$j^\mu_{em} = \bar{q}\gamma^\mu q$$

Crystal Ball

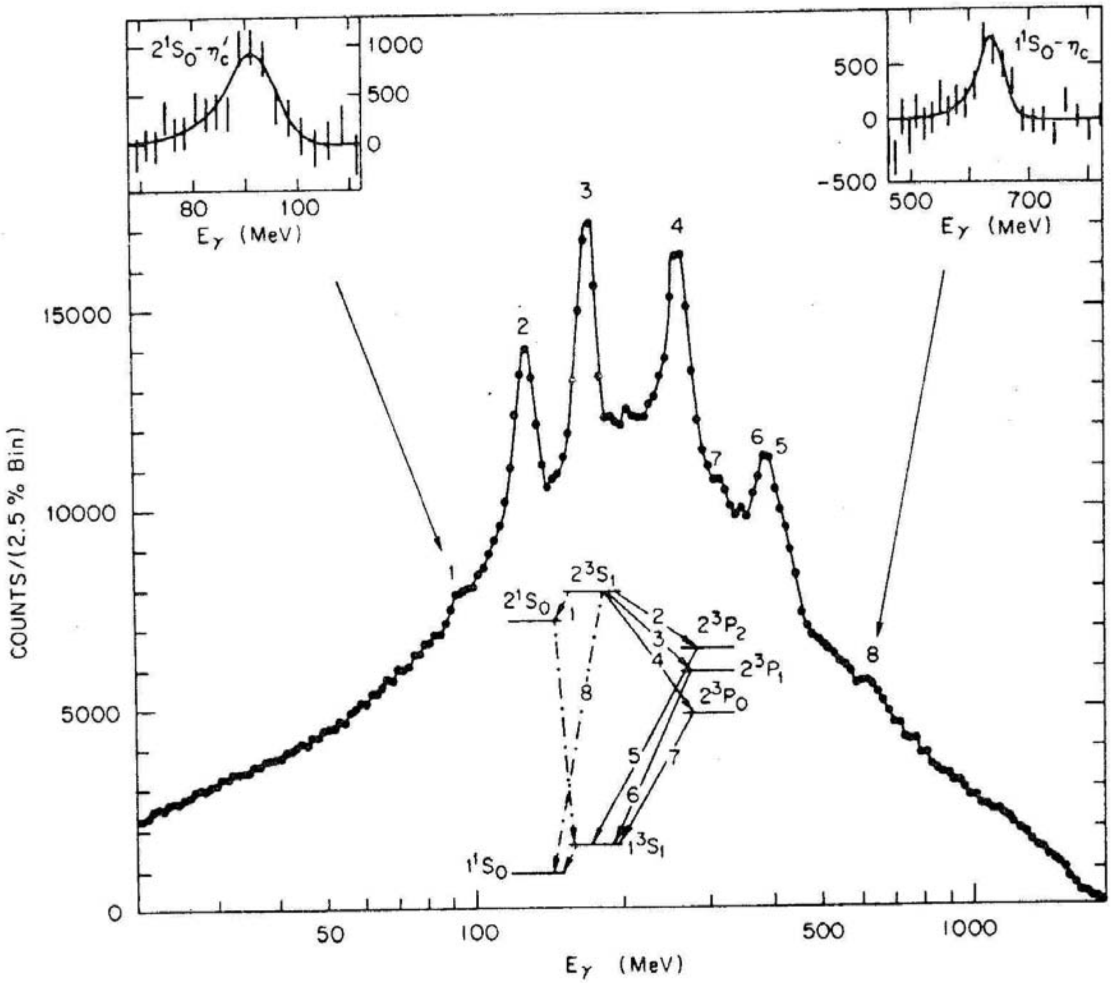


Fig. 5. Inclusive photon spectrum at the ψ′ obtained by the Crystal Ball experiment. Note that the logarithmic energy scale yields bin sizes approximately proportional to photon energy resolution. The numbers over the spectrum correspond to the expected radiative transitions shown in the spectrum inset.

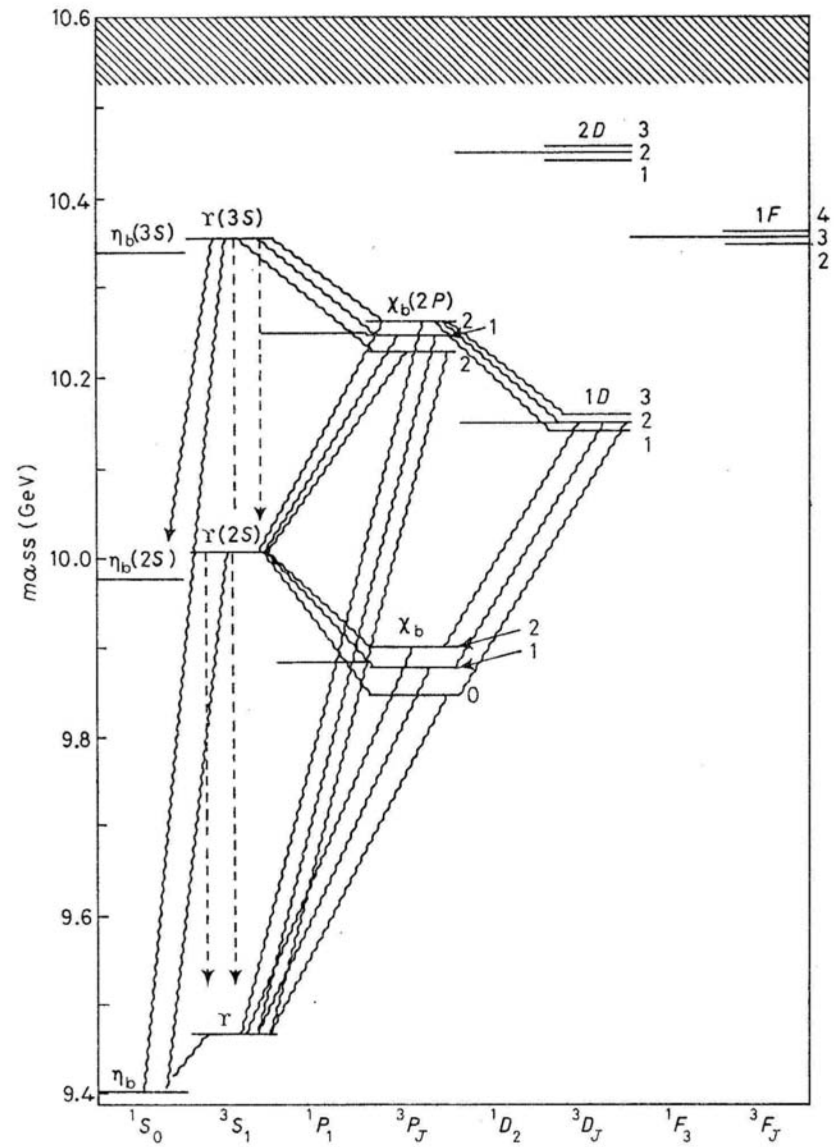


Fig. 4. – The $b\bar{b}$ level diagram showing transitions between states. We use the familiar spectroscopic notation $n^{2S+1}L_J$, where n is the principal quantum number (with the convention that n is one plus the number of nodes in the wavefunction), and L, S, and J are the orbital angular momentum, total spin, and total angular momentum. The parity and C-parity are given by $P=(-)^{L+1}$ and $C=(-)^{L+S}$. Note that not all states and transitions shown have been observed and not all possible transitions are shown. 〰 γ-transitions, --- hadronic transitions.

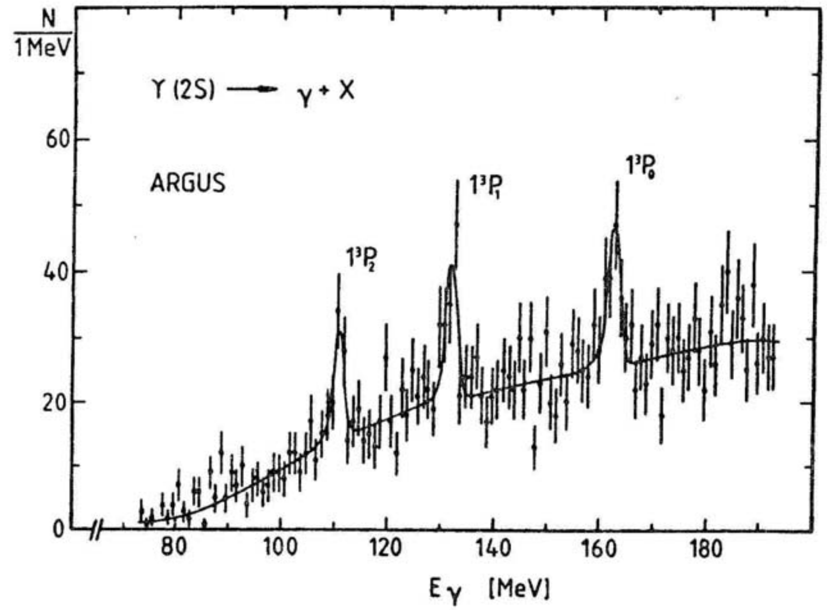


Figure 11: ARGUS [74] $\Upsilon(2S)\to\gamma$+hadrons with $\gamma\to e^+e^-$.

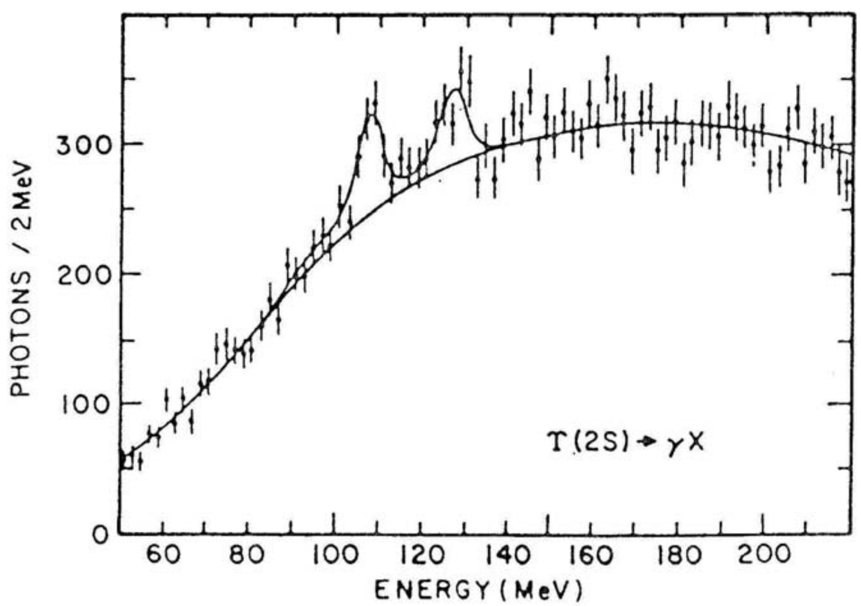


Figure 12: CLEO [75] $\Upsilon(2S)\to\gamma$+hadrons with $\gamma\to e^+e^-$. The data do not re
line, and it is not included in the fit shown.

PRL 94, 032001 (2005) PHYSICAL REVIEW LETTERS week ending 28 JANUARY 2005

Photon Transitions in $\Upsilon(2S)$ and $\Upsilon(3S)$ Decays

(CLEO Collaboration)

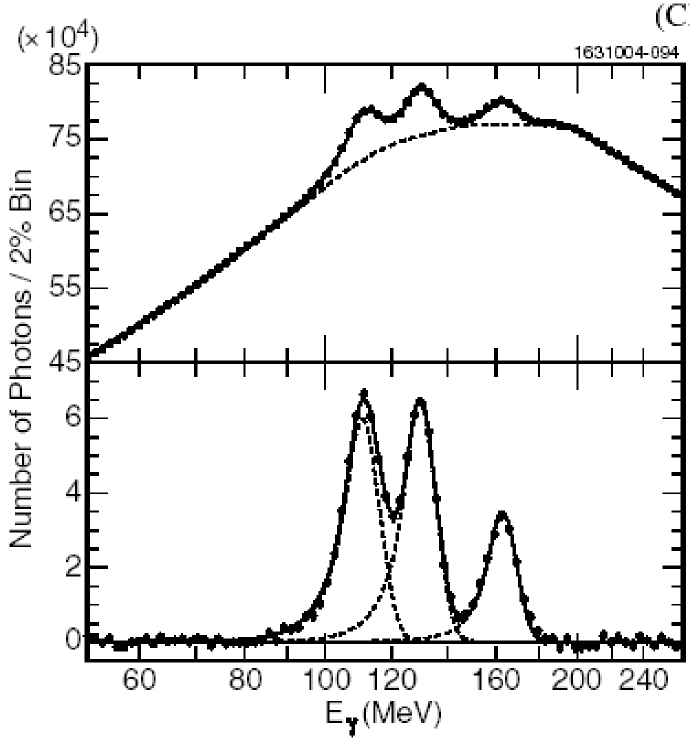


FIG. 2. Fit to the $\Upsilon(2S) \rightarrow \gamma\chi_{bJ}(1P)$ $(J = 2, 1, 0)$ photon lines in the data. The points represent the data (top plot). Statistical errors on the data are smaller than the point size. The solid line represents the fit. The dashed line represents total fitted background. The background subtracted data (points with error bars) are shown at the bottom. The solid line represents the fitted photon lines together. The dashed lines show individual photon lines.

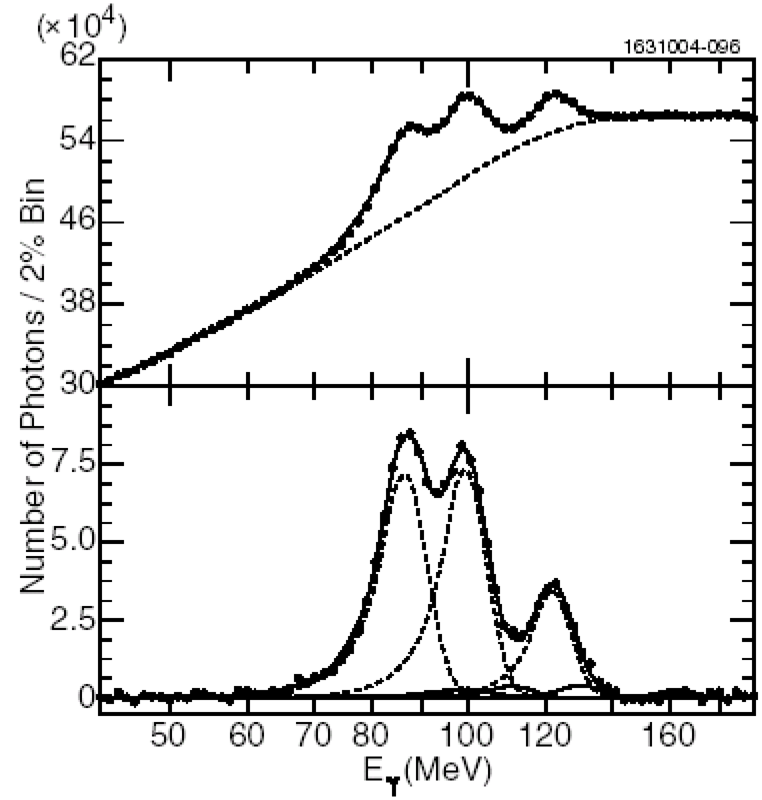
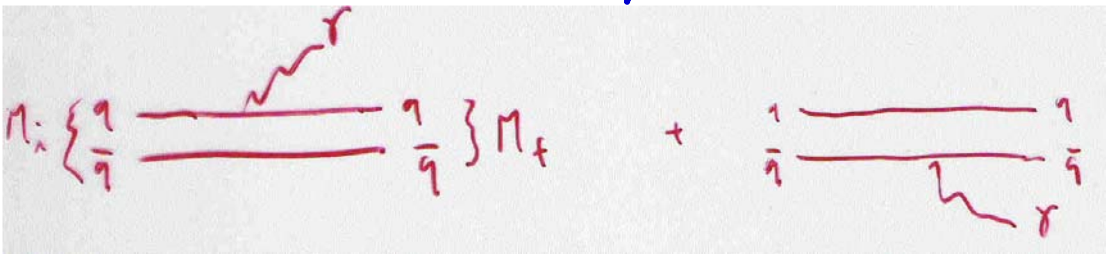


FIG. 3. Fit to the $\Upsilon(3S) \rightarrow \gamma\chi_{bJ}(2P)$ $(J = 2, 1, 0)$ photon lines in the data. See caption of Fig. 2 for the description. Small solid line peaks in the bottom plot show the $\chi_{bJ}(2P) \rightarrow \gamma\Upsilon(1D)$ and $\Upsilon(2S) \rightarrow \gamma\chi_{bJ}(1P)$ contributions.

Radiative (e.m.) Transitions

Same physics as in atomic and nuclear systems

An e.m. transition is described by:



For 2 body decay $M_i \to M_f \gamma$

$$d\Gamma = \frac{(2\pi)^4\, \delta^4(P_f + p_\gamma - p_i)}{2M_i} |M_{fi}|^2 \frac{d^3 p_f}{(2\pi)^3 (2E_f)} \frac{d^3 p_\gamma}{(2\pi)^3 (2E_\gamma)}$$

$$\Gamma = \frac{1}{2\pi M^2} |M_{if}|^2 p$$

$$\text{where } p = \frac{(M_i^2 - M_f^2)}{2M_i}$$

$$= \frac{|M_{if}|^2}{8\pi M} (1 - M_f^2/M_i^2)$$

$$\frac{d\Gamma}{d\cos\theta} = \frac{|M_{if}|^2}{16\pi^2 M_i} (1 - M_f^2/M_i^2) = \frac{|M_{if}|^2}{8\pi^2 M_i^2} k_\gamma$$

Start with E1 Transitions:

$$\frac{p^2}{2m} \to \frac{(\vec{p}-e\vec{A})^2}{2m} = \frac{p^2}{2m} - \frac{e\vec{p}\cdot\vec{A}}{2m} - \frac{e\vec{A}\cdot\vec{p}}{2m} + e^2\frac{\vec{A}^2}{2m}$$

$p^2/2m$ is the original kinetic energy term

drop higher order $e^2\vec{A}^2 terms$

Interested in:

$$H_I = -\frac{e}{2m}(\vec{A}\cdot\vec{p}+\vec{p}\cdot\vec{A})$$

$$\vec{A}(x) = \frac{1}{\sqrt{2\omega}}\vec{\epsilon}(\vec{k})e^{i\vec{k}\cdot\vec{x}}$$

$$e^{i\vec{k}\cdot\vec{x}} \simeq 1 + i\vec{k}\cdot\vec{x} + \ldots$$

in the long wavelength limit $\frac{1}{k} >> r$

$$\Rightarrow \vec{A}(x) \simeq \frac{1}{\sqrt{2\omega}}\vec{\epsilon}(\vec{k})$$

$$H_I = -\frac{e}{2m}(\vec{\epsilon}\cdot\vec{p}+\vec{p}\cdot\vec{\epsilon})$$

To evaluate $\langle A|\vec{p}|B\rangle \cdot \vec{\epsilon}$

Start with $[p_i, r_j] = -i\delta_{ij}$

$$\Rightarrow [\vec{p}^2, r_j] = p_i[p_i, r_j] + [p_i, r_j]p_i = -2ip_j$$

$$\begin{aligned}
\langle A|p_i|B\rangle &= i\langle A|[\vec{p}^2/2, r_j]|B\rangle \\
&= i\mu\langle A|[H, r_j]|B\rangle \\
&\quad (H = p^2/2\mu + V(r) \text{ but } [V(r), r] = 0) \\
&= i\mu\langle A|Hr_j - r_jH|B\rangle \\
&= i\mu(E_A - E_B)\langle A|r_j|B\rangle \\
&= i\frac{m}{2}\omega\langle A|r_j|B\rangle \\
\langle A|H_I|B\rangle &= -\frac{iem\omega}{2m}\langle A|r_i|B\rangle\epsilon_i \\
&= -\frac{ie\omega}{2}\langle A|r_i|B\rangle\epsilon_i \\
&= -\frac{ie\omega}{2}\langle A|\vec{r}|B\rangle \cdot \vec{\epsilon}
\end{aligned}$$

There are two methods for evaluating the matrix element

Method 1:

The sum over final polarizations is:

$$\sum_{pol} \vec{\epsilon}_i(k)\vec{\epsilon}^*(k) = \delta_{ij} - k_i k_j/\vec{k}^2$$

So:

$$\sum_{pol} |\langle B|H_I|A\rangle|^2 = \omega^2 e^2 Q^2 \left\{ |\langle B|\vec{r}|A\rangle|^2 - |\langle B|\vec{r}\cdot\hat{k}|A\rangle|^2 \right\}$$

Averaging over directions:

$$= \omega^2 e^2 Q^2 \frac{2}{3} |\langle B|\vec{r}|A\rangle|^2$$

Start with $^3P_J \to {}^3S_1$

- The orbital angular momentum is zero in the final state
- We may choose any J_Z since we averaged over the photon directions

  Convenient to choose $J_Z = J$

Start by writing down the meson wavefunction: $|M\rangle = \sqrt{2M}\psi(r)$

where $\sqrt{2M}$ is introduced to normalize the wavefunction when integrating over relativistic phase space.

$^3P_2(J_z = 2) : |J = J_Z = 2\rangle = |L = L_Z = 1\rangle \otimes |S = S_Z = 1\rangle = |Y_{11} \uparrow\uparrow\rangle$

Only $J'_Z = S'_Z = 1$ contributes since H_I does not flip spin.

$$\langle f|\vec{r}|i\rangle = \langle f|r|i\rangle \int \langle Y_{00} \uparrow\uparrow | \sqrt{\frac{4\pi}{3}} Y_{1-1} | Y_{11} \uparrow\uparrow\rangle d\Omega = \langle f|r|i\rangle \sqrt{\frac{1}{3}}$$

where $\langle f|r|i\rangle = \int r^2 dr R_f(r) r R_i(r) \sqrt{2M_i}\sqrt{2M_f}$

$$\begin{aligned}
\Gamma(^3P_2 \rightarrow ^3S_1) &= \frac{1}{8\pi M^2}|M_{if}|^2\omega \\
&= \frac{\omega}{8\pi M^2}\omega^2 e^2 Q^2 |\langle f|r|i\rangle|^2 (sM_i)(2M_f) \times \frac{2}{3} \times \frac{1}{3} \\
&= \frac{4\pi\alpha\omega^3 e_q^2}{8\pi}\frac{8}{9}|\langle f|r|i\rangle|^2 \left(\frac{M_i M_f}{M_i M_i}\right) \\
&= \frac{4}{9}\alpha\omega^3 e_q^2 |\langle f|r|i\rangle|^2 \left(\frac{M_f}{M_i}\right)
\end{aligned}$$

For ${}^3P_1 \rightarrow {}^3S_1$

$$|J = J_Z = 1\rangle = \frac{1}{\sqrt{2}}|Y_{11}\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) - Y_{10}\uparrow\uparrow\rangle$$

so that

$$\langle Y_{00}|\vec{r}|J = J_Z = 1\rangle = \frac{1}{\sqrt{2}}\langle Y_{00}|\vec{r}|Y_{11}\rangle - \frac{1}{\sqrt{2}}\langle Y_{00}|\vec{r}|Y_{10}\rangle$$

$$= \left[\frac{1}{\sqrt{2}}\langle Y_{00}|\frac{1}{\sqrt{3}}(\frac{-\hat{x}+i\hat{y}}{\sqrt{2}})|Y_{11}\rangle - \frac{1}{\sqrt{2}}\langle Y_{00}|\frac{1}{\sqrt{3}}\hat{z}|Y_{10}\rangle\right]\langle 1S|r|1P\rangle$$

$$\Rightarrow |\langle{}^3S_1|\vec{r}|{}^3P_1\rangle|^2 = [\frac{1}{2}\frac{1}{3} + \frac{1}{2}\frac{1}{3}]|\langle 1S|r|1P\rangle|^2$$

${}^3P_0 \rightarrow {}^3S_1$

$$|J = J_Z = 0\rangle = \sqrt{\frac{1}{3}}|Y_{11}\downarrow\downarrow - Y_{10}\sqrt{\frac{1}{2}}(\uparrow\downarrow + \downarrow\uparrow) + Y_{1-1}\uparrow\uparrow\rangle$$

resulting in $|\langle{}^3S_1|\vec{r}|{}^3P_0\rangle|^2 = [\frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{1}{3}]|\langle 1S|r|1P\rangle|^2$

Summarizing all these results we obtain:

$$\Gamma(^3P_2 \to ^3 S_1\gamma) = \frac{\omega^3 e^2 Q^2}{3\pi}\frac{1}{3}|\langle 1S|r|1P\rangle|^2$$

$$\Gamma(^3P_1 \to ^3 S_1\gamma) = \frac{\omega^3 e^2 Q^2}{3\pi}\left\{\frac{1}{2}\frac{1}{3} + \frac{1}{2}\frac{1}{3}\right\}\frac{1}{3}|\langle 1S|r|1P\rangle|^2$$

$$\Gamma(^3P_0 \to ^3 S_1\gamma) = \frac{\omega^3 e^2 Q^2}{3\pi}\left\{\frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{1}{3}\right\}|\langle 1S|r|1P\rangle|^2$$

Comparing these expressions we see that in all cases

$$\Gamma(^3P_J \to ^3 S_1\gamma) = \frac{4\alpha\omega^3 Q^2}{9}|\langle 1S|r|1P\rangle|^2$$

Similarly we obtain:

$$\Gamma(^3S_1 \to ^3 P_J\gamma) = \frac{4\alpha\omega^3 Q^2(2J+1)}{27}|\langle 1S|r|1P\rangle|^2$$

Let us return to our *effective* wavefunctions:

$$\psi_{1S} = \frac{2}{\pi^{1/4}}\beta^{3/2}e^{-\beta^2 r^2/2}Y_{00} \qquad \beta = 0.77 \text{ GeV}$$

$$\psi_{1P} = \sqrt{\frac{8}{3}}\frac{\beta^{5/2}r}{\pi^{1/4}}e^{-\beta^2 r^2/2}Y_{1m} \qquad \beta = 0.598 \text{ GeV}$$

This gives:

$$\langle\psi_{1S}|r|\psi_{1P}\rangle = \frac{2}{\pi^{1/4}}\sqrt{\frac{8}{3}}\frac{1}{\pi^{1/4}}\beta_S^{3/2}\beta_P^{5/2}\int r^4 e^{-(\beta_S^2+\beta_P^2)r^2/2}dr$$

$$= \sqrt{\frac{8}{3}}15\frac{\beta_S^{3/2}\beta_P^{5/2}}{(\beta_S^2+\beta_P^2)^{5/2}}$$

$$= 5.2 \text{ GeV}^{-1}$$

$$\Rightarrow \Gamma(^3P_2 \rightarrow ^3S_1\gamma) = 0.59 \text{ MeV} \quad vs \quad \Gamma^{expt} = 0.351^{+.2}_{-.14} \text{ MeV}$$

$$\Gamma(^3P_1 \rightarrow ^3S_1\gamma) = \text{ MeV} \quad vs \quad \Gamma^{expt} < 0.355 \text{ MeV}$$

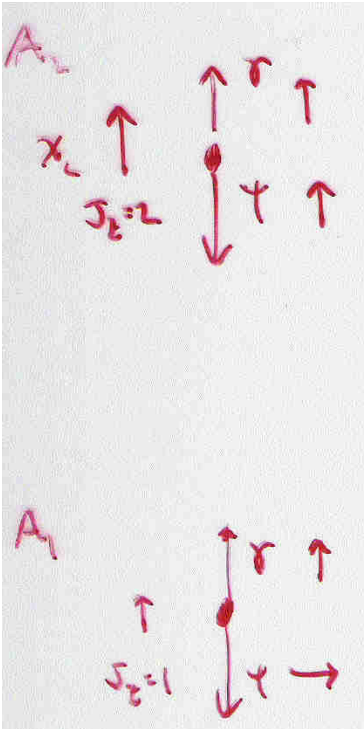
Another useful technique uses helicity amplitudes:

$$\Gamma = \frac{\omega}{2J+1}\frac{1}{\pi}\sum_{\lambda\geq 0}|A_\lambda|^2$$

$$M_{if} = ie_q k_\gamma \langle f|\vec{r}|i\rangle \cdot \vec{\epsilon}^* \sqrt{2M_i}\sqrt{2M_f}$$

$$\text{take } \vec{\epsilon} = -\frac{1}{\sqrt{2}}(1, i, 0)$$

$\chi_{2c} \to \psi\gamma \qquad ({}^3P_2 \to \gamma\, {}^3S_1)$



$$M_{if} = ie_q k_\gamma \langle f|\vec{r}|i\rangle \cdot \vec{\epsilon}^* \sqrt{2M_i}\sqrt{2M_f}$$

$$= \frac{-ie_q\omega}{2}\langle f|r|i\rangle \langle {}^3S_1|\sqrt{\frac{4\pi}{3}}Y_{1-1}|{}^3P_2\rangle$$

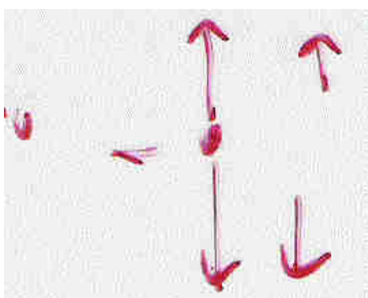
$$\langle {}^3S_1|\sqrt{\frac{4\pi}{3}}Y_{1-1}|{}^3P_2\rangle = \int \langle Y_{00}\uparrow\uparrow|\sqrt{\frac{4\pi}{3}}Y_{1-1}|Y_{11}\uparrow\uparrow\rangle\, d\Omega = \sqrt{\frac{1}{3}}$$

$$A_1 = \sqrt{2M_i}\sqrt{2M_f} e_q k_\gamma \langle f|r|i\rangle$$

$$\int d\Omega \int \langle Y_{00}\chi_{10}|\sqrt{\frac{4\pi}{3}}Y_{1-1}|\frac{1}{\sqrt{2}}Y_{11}\chi_{10} + \frac{1}{\sqrt{2}}Y_{10}\chi_{11}\rangle$$

$$= \sqrt{2M_i}\sqrt{2M_f} e_q k_\gamma \langle f|r|i\rangle \sqrt{\frac{1}{6}}$$

where $\chi_{10} = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$ and $\chi_{11} = \uparrow\uparrow$



$$A_0 = \sqrt{2M_i}\sqrt{2M_f} e_q k_\gamma \langle f|r|i\rangle$$

$$\int d\Omega \int \langle Y_{00}\chi_{1-1}|\sqrt{\frac{4\pi}{3}}Y_{1-1}|\frac{1}{\sqrt{6}}Y_{11}\chi_{1-1} + \frac{1}{\sqrt{3}}Y_{10}\chi_{10} + \frac{1}{\sqrt{6}}Y_{1-1}\chi_{11}\rangle$$

$$= \sqrt{2M_i}\sqrt{2M_f} e_q k_\gamma \langle f|r|i\rangle \sqrt{\frac{1}{18}}$$

Putting it all together we obtain:

$$\Gamma = \omega^3 e_q^2 \alpha \frac{4}{2J_i+1} \sum_{\lambda \geq 0} |A_\lambda|^2$$

$$= \alpha\omega^3 \left(\frac{e_q}{e}\right)^2 \frac{4}{2J_i+1} |\langle f|r|i\rangle|^2 \left[\frac{1}{3} + \frac{1}{6} + \frac{1}{18}\right]$$

(as before)

3. E1 transitions

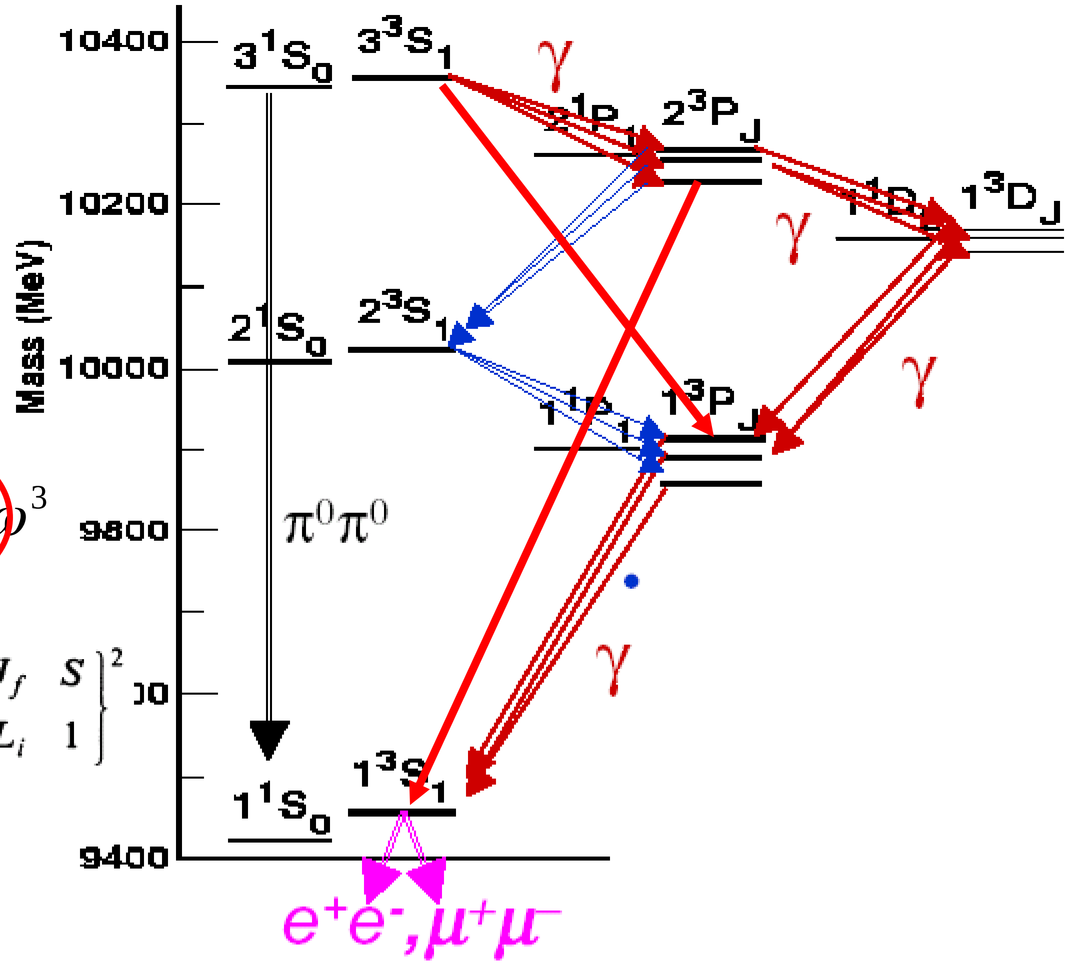
McClary and Byers, PR D28, 1692 (1983)

E1 decays sensitive to nodes in wavefunction

radiative transitions tests internal structure

$$\Gamma = \frac{4}{3} e_Q^2 \alpha C(J_i L_i J_f L_f S) |\langle P|r|S\rangle| \omega^3$$

$$C(J_i L_i J_f L_f S) = \max(L_i, L_f)(2J_f+1) \begin{Bmatrix} L_f & J_f & S \\ J_i & L_i & 1 \end{Bmatrix}^2$$



Including relativistic corrections corresponds to using eigenfunctions and eigenvalues of the Breit-Fermi Hamiltonian (Siegert's theorem)

	$\|<2P\|r\|3S>\|$		$\|<1P\|r\|2S>\|$		$\|<1P\|r\|3S>\|$		$\frac{\|<1S\|r\|2P>\|}{\|<2S\|r\|2P>\|}$	
	GeV^{-1}		GeV^{-1}		GeV^{-1}			
DATA	2.7±0.2		1.9±0.2		0.050±0.006		0.096±0.005	
	World Average				This measurement			
Model	NR	rel	NR	rel	NR	rel	NR	rel
Kwong,Rosner [13]	2.7		1.6		0.023		0.13	
Fulcher [14]	2.6		1.6		0.023		0.13	
Büchmuller et al.[15]	2.7		1.6		0.010		0.12	
Moxhay,Rosner [16]	2.7	2.7	1.6	1.6	0.024	0.044	0.13	0.15
Gupta et al.[17]	2.6		1.6		0.040		0.11	
Gupta et al.[18]	2.6		1.6		0.010		0.12	
Fulcher [19]	2.6		1.6		0.018		0.11	
Danghighian et al.[20]	2.8	2.5	1.7	1.3	0.024	0.037	0.13	0.10
McClary,Byers [21]	2.6	2.5	1.7	1.6			0.15	0.13
Eichten et al.[22]	2.6		1.7		0.110		0.15	
Grotch et al.[23]	2.7	2.5	1.7	1.5	0.011	0.061	0.13	0.19

Tomasz Skwarnicki, Syracuse U. ICHEP, Amsterdam July,2002

Relativistic effects gives differences between E1 matrix elements:

$$\langle 2P|r|3S\rangle = 2.7 \pm 0.2 \text{ GeV}^{-1}$$

$$\langle 2^3P_2|r|3^3S_1\rangle \approx -2.4 \text{ GeV}^{-1}$$

$$\langle 2^3P_1|r|3^3S_1\rangle \approx -2.3 \text{ GeV}^{-1}$$

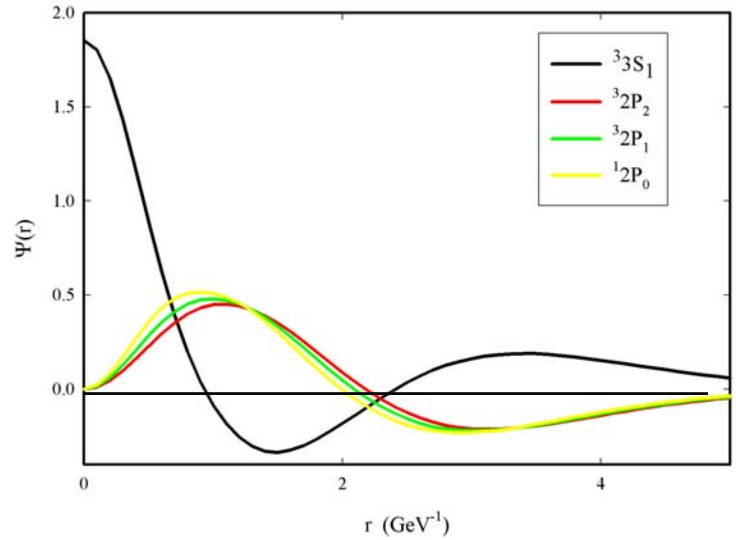
$$\langle 2^3P_0|r|3^3S_1\rangle \approx -2.2 \text{ GeV}^{-1}$$

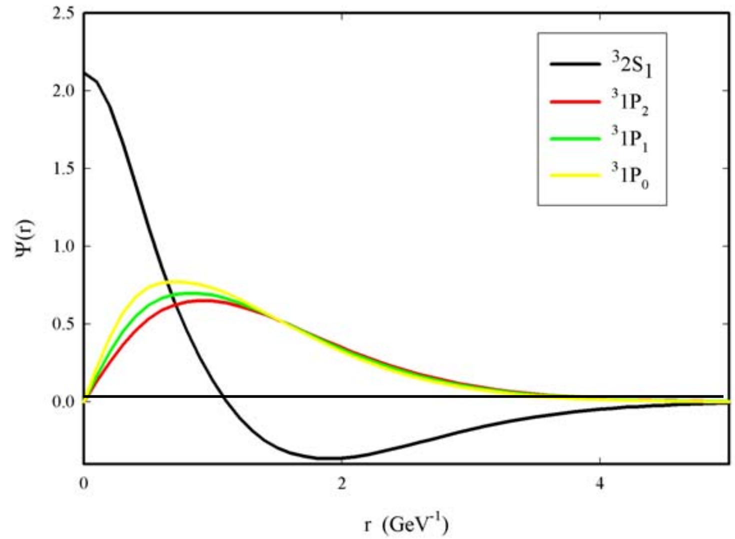
$$\langle 1P|r|2S\rangle \pm 1.9 \pm 0.2 \text{ GeV}^{-1}$$

$$\langle 1^3P_2|r|2^3S_1\rangle \approx -1.5 \text{ GeV}^{-1}$$

$$\langle 1^3P_1|r|2^3S_1\rangle \approx -1.4 \text{ GeV}^{-1}$$

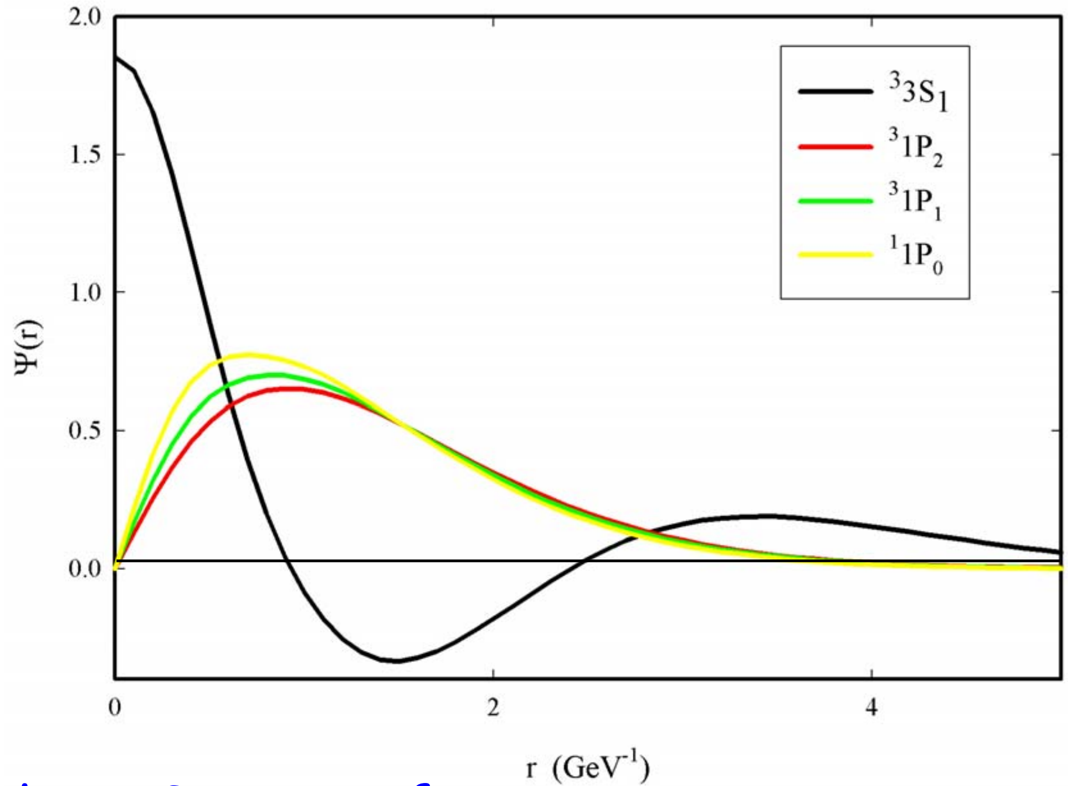
$$\langle 1^3P_0|r|2^3S_1\rangle \approx -1.3 \text{ GeV}^{-1}$$





see also McClary and Byers, PR D28, 1692 (1983)

$\lvert\langle 1P \lvert r \rvert 3S \rangle\rvert$	
GeV^{-1}	
0.050±0.006	
This meas	
NR	rel
0.023	
0.023	
0.010	
0.024	0.044
0.040	
0.010	
0.018	
0.024	0.037
0.110	
0.011	0.061



Node in 3S wavefunction near maximum in 1P wavefunction so large cancellation very sensitive to details of the wavefunctions

$$\langle 1^3P_2 | r | 3^3S_1 \rangle \approx +0.096 \text{ GeV}^{-1}$$

$$\langle 1^3P_1 | r | 3^3S_1 \rangle \approx +0.040 \text{ GeV}^{-1}$$

$$\langle 1^3P_0 | r | 3^3S_1 \rangle \approx -0.026 \text{ GeV}^{-1}$$

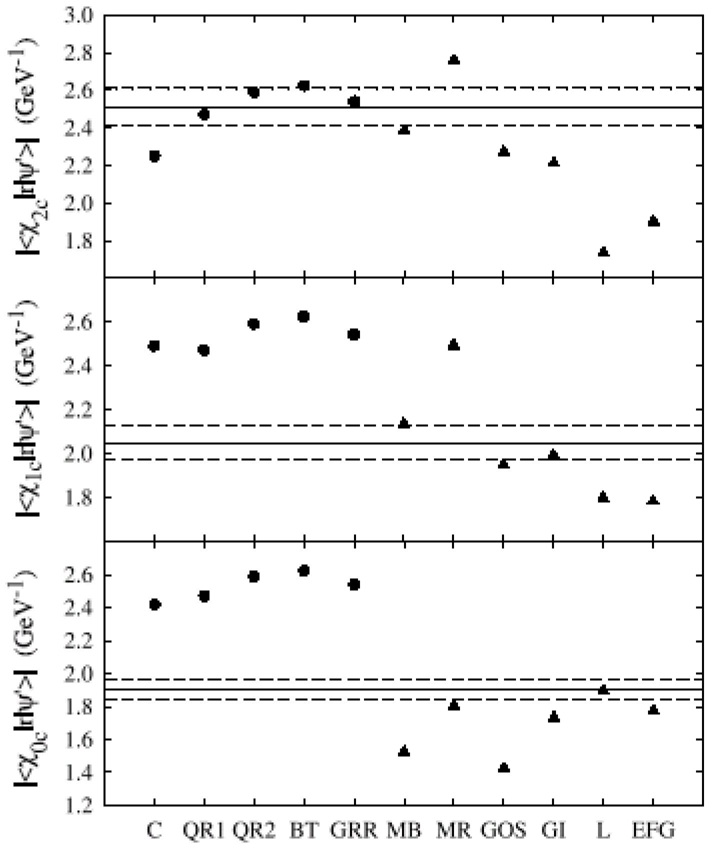


Table I: Properties of $\psi(2S) \to \gamma\chi_{cJ}$ decays, using results from Refs. [54] and [66] as well as Eq. (6).

J	k_γ (MeV)	$\mathcal{B}$ [66] (%)	$\Gamma[\psi(2S) \to \gamma\chi_{cJ}]$ (keV)	$\lvert\langle 1P \lvert r \rvert 2S\rangle\rvert$ (GeV^{-1})
2	127.60±0.09	9.33±0.14±0.61	31.4±2.4	2.51±0.10
1	171.26±0.07	9.07±0.11±0.54	30.6±2.2	2.05±0.08
0	261.35±0.33	9.22±0.11±0.46	31.1±2.0	1.90±0.06

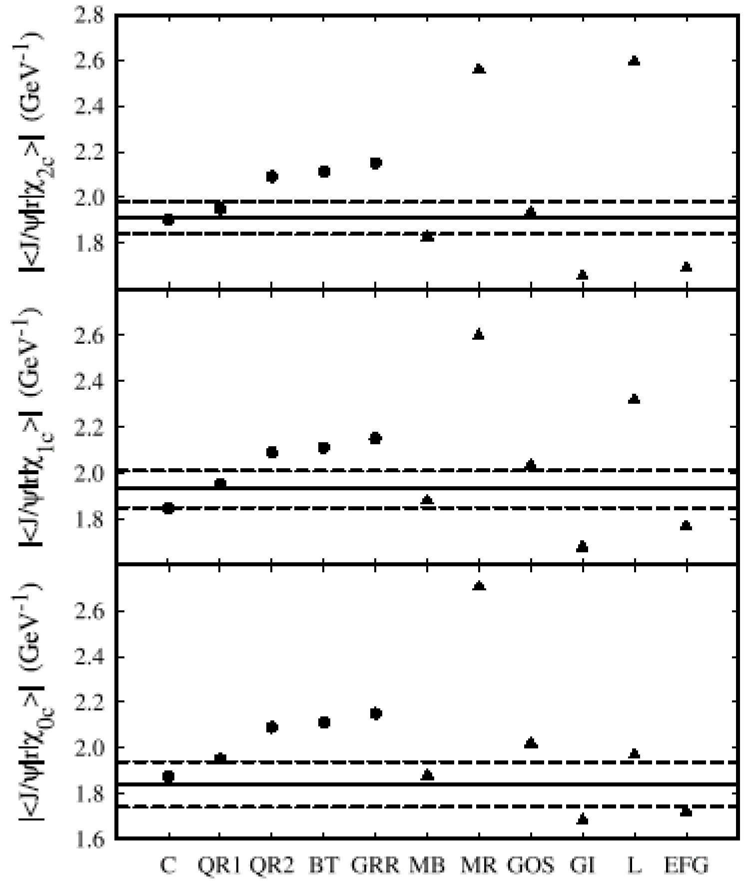
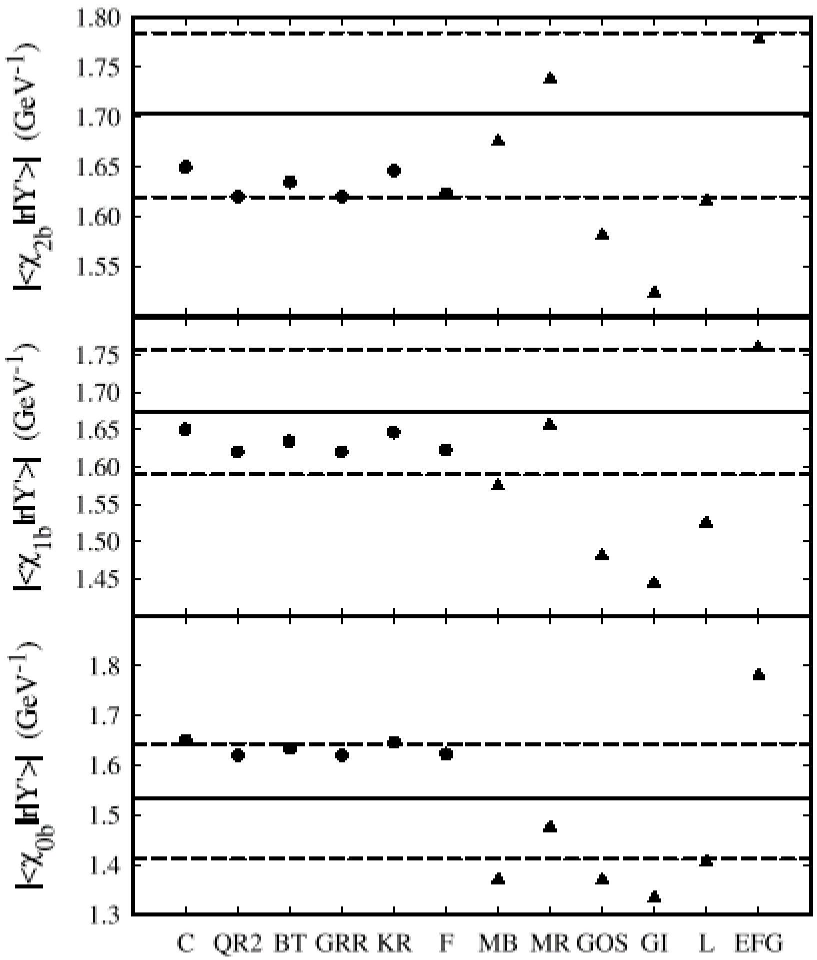
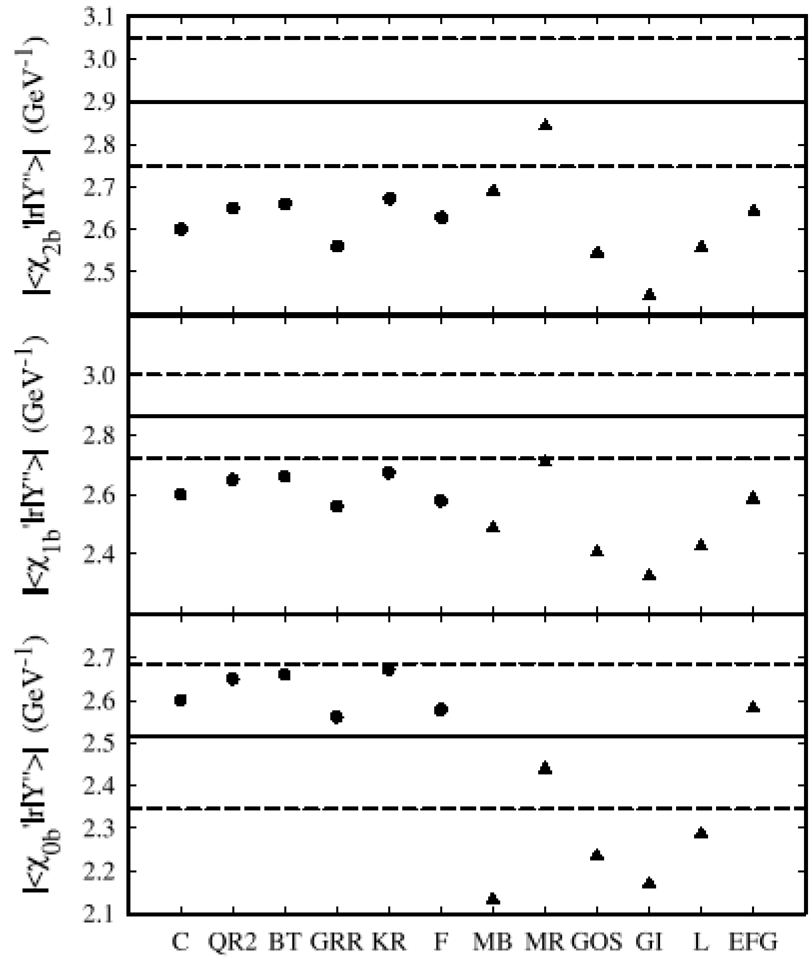


Table III: Properties of the transitions $\chi_{cJ} \to \gamma J/\psi$. (Ref. [54]; Eq. (6)).

J	k_γ (MeV)	$\Gamma(\chi_{cJ} \to \gamma J/\psi)$ (keV)	$\lvert\langle 1S \lvert r \rvert 1P\rangle\rvert$ $(\mathrm{GeV})^{-1}$
2	429.63±0.08	416±32	1.91±0.07
1	389.36±0.07	317±25	1.93±0.08
0	303.05±0.32	135±15	1.84±0.10



1.80
1.75
1.70
1.65
1.60
1.55
$|\langle\chi_{2b}|rY'\rangle|$ (GeV^{-1})
1.75
1.70
1.65
1.60
1.55
1.50
1.45
$|\langle\chi_{1b}|rY'\rangle|$ (GeV^{-1})
1.8
1.7
1.6
1.5
1.4
1.3
$|\langle\chi_{0b}|rY'\rangle|$ (GeV^{-1})
C QR2 BT GRR KR F MB MR GOS GI L EFG



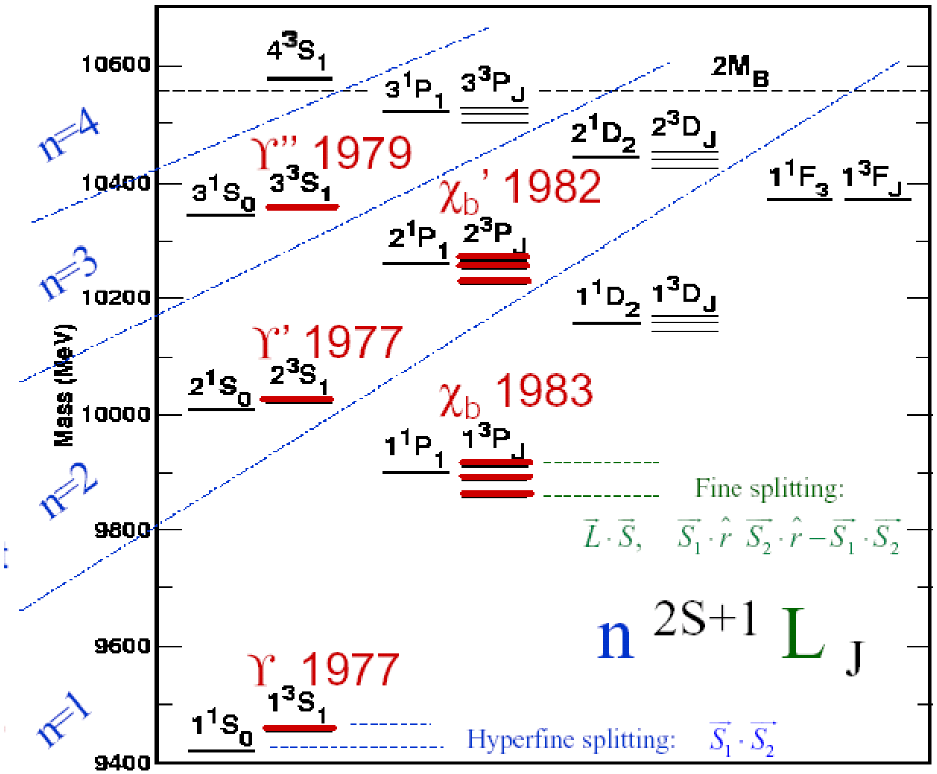
3.1
3.0
2.9
2.8
2.7
2.6
2.5
$|\langle\chi_{2b}'|rY''\rangle|$ (GeV^{-1})
3.0
2.8
2.6
2.4
$|\langle\chi_{1b}'|rY''\rangle|$ (GeV^{-1})
2.7
2.6
2.5
2.4
2.3
2.2
2.1
$|\langle\chi_{0b}'|rY''\rangle|$ (GeV^{-1})
C QR2 BT GRR KR F MB MR GOS GI L EFG

Matrix elements sensitive to relativistic corrections via shifts in nodes in wavefunctions

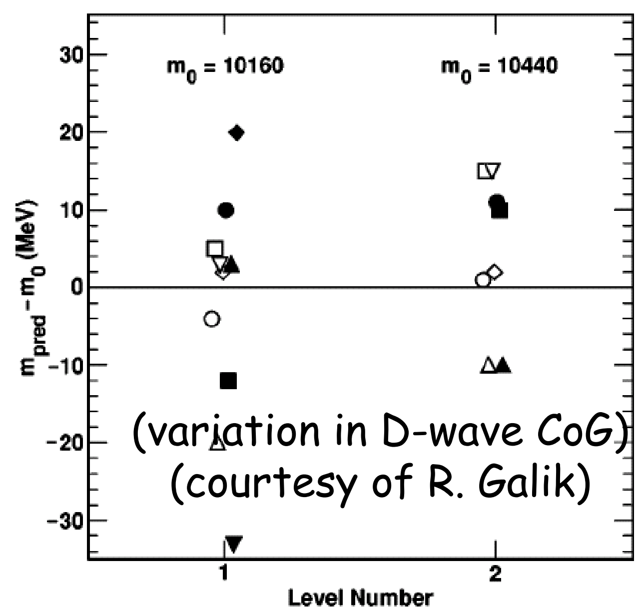
- there can big difference in matrix elements (not clear what exactly CLEO did)
- More useful to compare individual matrix elements to test relativistic corrections
- transitions involving D-waves would be interesting tests
- Angular distributions also provide additional information

Bottomonium

- Largest number of stable states
    - Numerous states below threshold
    - Only 9 out of 30 narrow states observed so far
    - No spin-singlet states observed
    - No new states observed in 19 years!



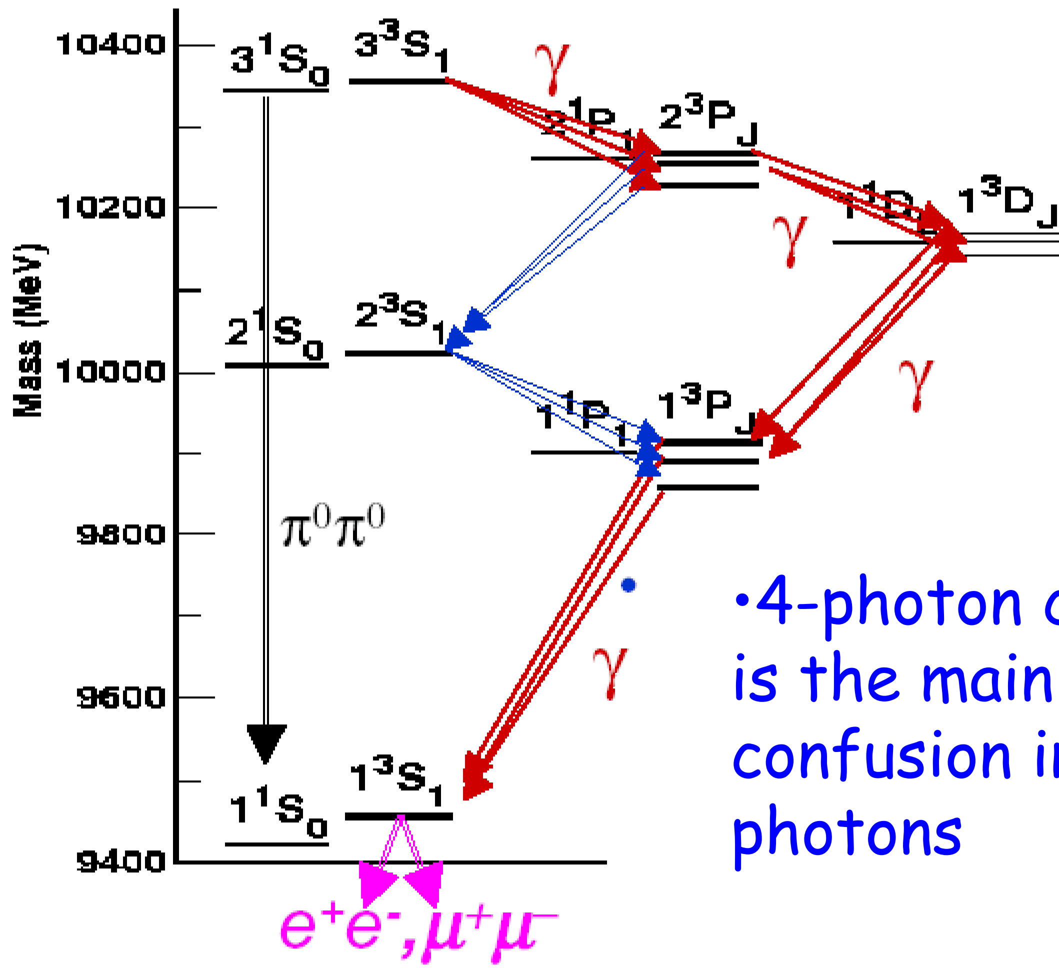
- Wide variation in splittings
- Their observation will test the various calculations
- Expect many of these states to be found in
    - The recent CESR/CLEO run
    - B-decays at B-factories
    - At future CLEO-c/CESR-c



(variation in D-wave CoG)
(courtesy of R. Galik)

Production of the D-wave states

- By direct scans in e^+e^- to produce $^3D_1$ **(J^{PC} = 1^{--})**
- Use for 4γ E1 cascade to search for $\Upsilon(1^3D_J)$



- 4-photon cascade via the $\Upsilon(2S)$ is the main background due to confusion in ordering the observed photons

• CESR/CLEO has completed a high statistics run at the $\Upsilon(3S)$

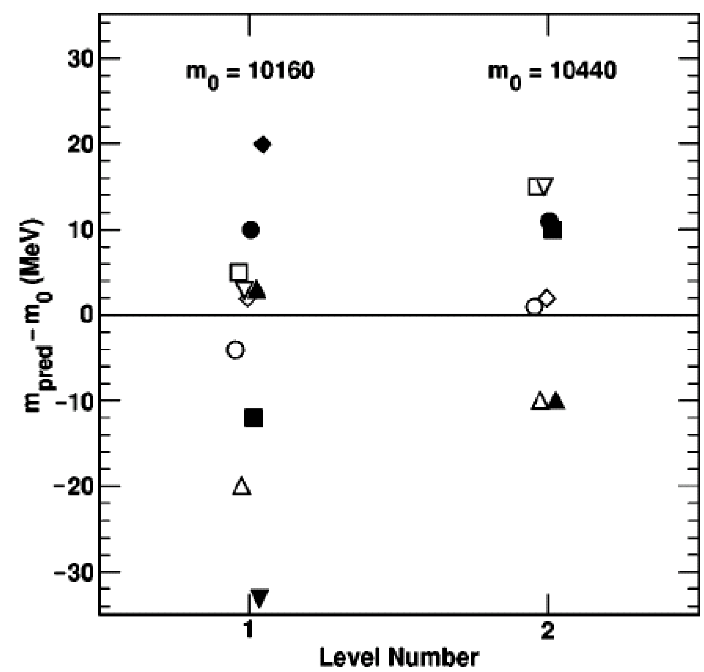
• Ran on $\Upsilon(2S)$ and running again at $\Upsilon(3S)$

• Expect very rich spectroscopy

• Estimate the radiative widths and BR using quark model

• ^3D_J masses - test spin dependent splittings

• Wide variation in masses:

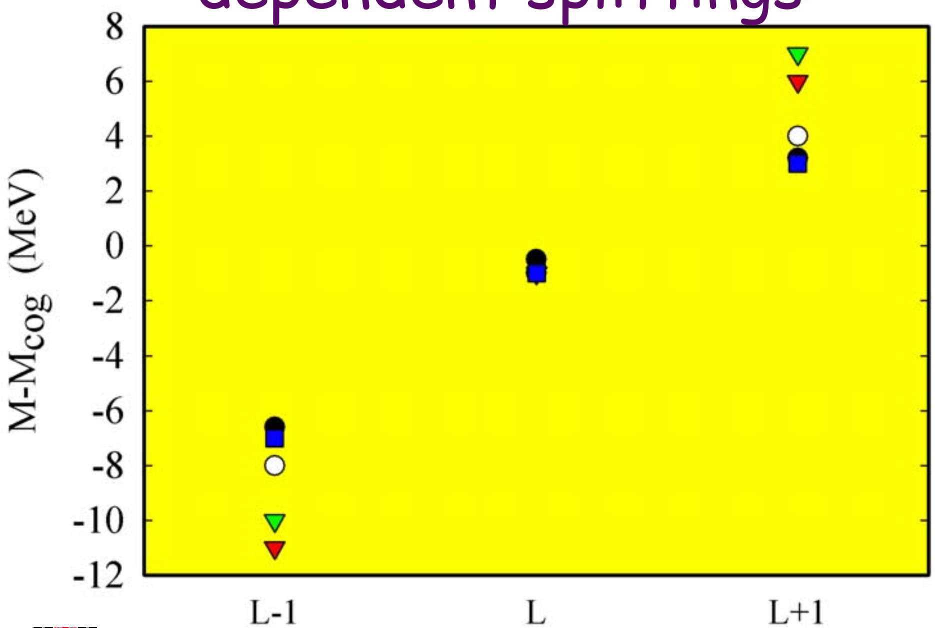


There is still some question about the Lorentz structure of the qq potential

see Eichten & Feinberg PRL 43, 1205 (1979)
Pantaleone Tye & Ng PR D33, 777 (1986);
Buchmuller Ng & Tye PR D24, 3003 (1981)
Gupta Radford & Repko PR D26, 3305 (1982);
Gromes, Z. Phys C22, 265 (1984)…..

- vector 1-gluon exchange + scalar confinement
- vector 1-gluon exchange + colour electric confinement
- + more complicated structures

Variation in the spin dependent splittings



because the D-waves are larger they will feel the long range spin-dependent potential more than the P-waves

observation of ^3D_J would be important in understanding the Lorentz structure of the confining potential

•In e.m. cascades: $\Upsilon(3S) \to \gamma\, \chi'_b \to \gamma\gamma\, {}^3D_J$

$$\Gamma = \frac{4}{3} e_Q^2 \alpha C(J_i L_i J_f L_f S) \left| \langle P | r | S \rangle \right| \omega^3 \qquad C(J_i L_i J_f L_f S) = \max(L_i, L_f)(2J_f+1) \begin{Bmatrix} L_f & J_f & S \\ J_i & L_i & 1 \end{Bmatrix}^2$$

•Some 4γ cascades with observable # of events/10^6 Υ(3S)'s:

Cascade	Events
$3^3S_1 \to 2^3P_2 \to 1^3D_3 \to 1^3P_2 \to 1^3S_1$	7.8
$3^3S_1 \to 2^3P_2 \to 1^3D_2 \to 1^3P_1 \to 1^3S_1$	2.7
$3^3S_1 \to 2^3P_1 \to 1^3D_2 \to 1^3P_1 \to 1^3S_1$	20
$3^3S_1 \to 2^3P_1 \to 1^3D_1 \to 1^3P_1 \to 1^3S_1$	3.3

S.G + J. Rosner, Phys Rev D64, 097501 (2001)

Expect ~38 events /10^6 Υ(3S) via ^3D_J

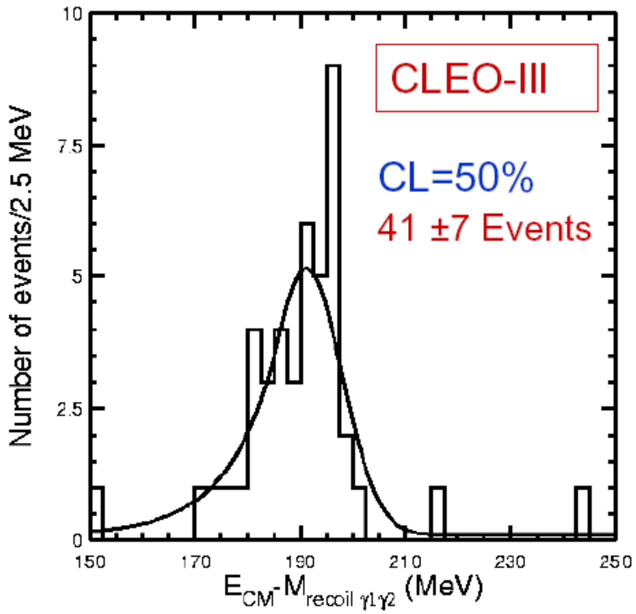
•The e^+e^- final states leads to less background

• $\mu^+\mu^-$ final states also contribute if μ's are identified

CLEO finds:

$B(\Upsilon(3S) \mapsto \gamma\gamma\Upsilon(1D) \mapsto \gamma\gamma\gamma\,\Upsilon(1S) \mapsto \gamma\gamma\gamma\ell^+\ell^-) = (3.3\pm0.6\pm0.5)\ 10^{-5}$

(vs GR prediction of 3.8×10^{-5})

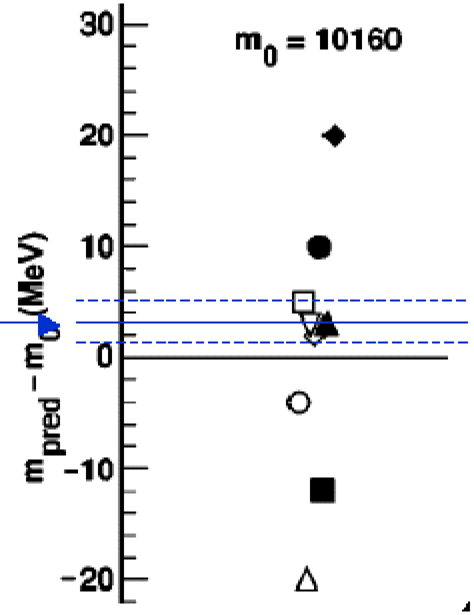


- Mass averaged over different fits: **10162.2 ±1.6 MeV**
- Inconsistent with the $\Upsilon(1D_3)$
- Could be the $\Upsilon(1D_2)$ or $\Upsilon(1D_1)$
- The theory predicts the rate ratio: $\Upsilon(1D_2)/\Upsilon(1D_1)=6$
- **Thus, the $\Upsilon(1D_2)$ is the most likely interpretation**

All calculations of the fine splitting predict the $\Upsilon(1D_2)$ mass from –0.5 to –1.0 MeV below the center-of-gravity of the triplet

↦Our mass measurement is consistent with the c.o.g.~10163 ±2 MeV

Spread in the predictions of the center-of-gravity of the triplet 1D states by various potential models



M1 Transitions

Because quarks have spin they may emit a photon via a spin flip
- The magnetic dipole transition

To obtain the interaction Hamiltonian we perform a non-relativistic reduction of

$$H_I = e \int dx j^{\mu}_{em}(x) A_{\mu}(x)$$

$$\text{where } j^{\mu}_{em}(x) = \bar{q}(x) Q \gamma^{\mu} q(x)$$

We expand the Dirac spinors to lowest order in p/m
Denoting the large and small components by q_1 and q_2

$$q_2(x) = -\frac{i\vec{\sigma} \cdot \vec{\nabla}}{2m} q_1(x)$$

$$\vec{j}_{em}(x) = \frac{-i}{2m}[q_1^\dagger Q(\nabla q_1) - (\nabla q_1^\dagger)Qq_1 + i\nabla \times q_1^\dagger Q\vec{\sigma}q_1]$$

So the interaction Hamiltonian is given by:

$$H_I = \frac{-eQ}{2m}[\vec{A}(\vec{r}) \cdot \vec{p} + \vec{p} \cdot \vec{A}(\vec{r}) + \vec{\sigma} \cdot [\vec{\nabla} \times \vec{A}(\vec{r})]$$

So:

$$\langle 0|H_I|\gamma(\vec{k},\epsilon)\rangle = -\frac{1}{(2\pi)^{3/2}}\frac{1}{(2\omega)^{1/2}}eQ\frac{1}{2m}[e^{i\vec{k}\cdot\vec{r}}\vec{\epsilon}\cdot\vec{p} + \vec{\epsilon}\cdot\vec{p}e^{i\vec{k}\cdot\vec{r}} + i\vec{\sigma}\cdot(\vec{k}\times\vec{\epsilon})e^{i\vec{k}\cdot\vec{r}}]$$

(For antiquarks change the sign of the charge)

$$\mu = \frac{e}{2m_1}$$ Is the magnetic dipole moment of the quark

For magnetic dipole transitions:

$$M_{if} = i\mu\langle f|\vec{\sigma}|i\rangle \cdot \vec{k} \times \vec{\epsilon}^{*}$$

$$\vec{\epsilon} = \frac{1}{\sqrt{2}}(1, \pm i, 0)$$

$$\begin{vmatrix} \sigma_x & \sigma_y & \sigma_z \\ k_x & k_y & k_z \\ 1 & i & 0 \end{vmatrix} = i\sigma_z(k_x + ik_y) - ik_z\sigma_x + k_z\sigma_y$$

Choosing z as the γ direction

$$M_{if} = -\frac{ie_q}{2m}k_\gamma\langle f|\sigma_x - i\sigma_y|i\rangle \text{ where } \sigma_x - i\sigma_y = \sigma_-$$

if instead take $\vec{k} = k_y$

$$M_{if} = -\frac{ie_q}{2m}k_\gamma\langle f|\sigma_z|i\rangle$$

$$= k_\gamma\sqrt{2M_i}\sqrt{2M_f}\int d^3r\psi_f^*(r)\psi_i(r) \times \langle f|\sum \mu_i\sigma_{zi}|i\rangle$$

e.g. $J/\psi \to \eta_c\gamma(^3S_1 \to ^1 S_0\gamma)$

$$A(^3S_1 \to ^1 S_0\gamma) = -\, ik_\gamma\sqrt{2M_i}\sqrt{2M_f}\langle f|i\rangle$$

$$\times \langle\sqrt{\frac{1}{2}}(\uparrow\downarrow - \downarrow\uparrow)|\frac{e_q}{2m_q}\frac{(\sigma_x - i\sigma_y)_q}{\sqrt{2}} + \mu_{\bar{q}}\frac{(\sigma_x - i\sigma_y)_{\bar{q}}}{\sqrt{2}}|\uparrow\uparrow\rangle$$

$$= -\, ik_\gamma\sqrt{2M_i}\sqrt{2M_f}\langle f|i\rangle \left[\frac{-e_q}{2m_q} + \frac{e_{\bar{q}}}{2m_{\bar{q}}}\right]$$

$$= -\, ik_\gamma\sqrt{2M_i}\sqrt{2M_f}\langle f|i\rangle \frac{ee_q}{m_c}$$

$$\Rightarrow \frac{d\Gamma}{d\Omega} = k_\gamma \frac{4\pi\alpha}{8\pi^2} k_\gamma^2 |\langle f|i\rangle|^2 \frac{e_c^2}{m_c^2}$$

averaging over angles gives the total width

$$\Gamma = \frac{k_\gamma^3}{3\pi}|\langle f|i\rangle|^2 \frac{e_c^2}{m_c^2}$$

Take $\langle f|i\rangle = 1 \qquad \omega = 115$

so $\Gamma = 0.19$ MeV vs 0.88 keV (expt)

What about? $2^3S_1 \to 1^1S_0$

$\langle f|i\rangle = 0$ since $2S \perp 1S$

The decay $\psi(2S) \to \gamma\eta_c(1S)$ is a forbidden magnetic dipole (M1) transition

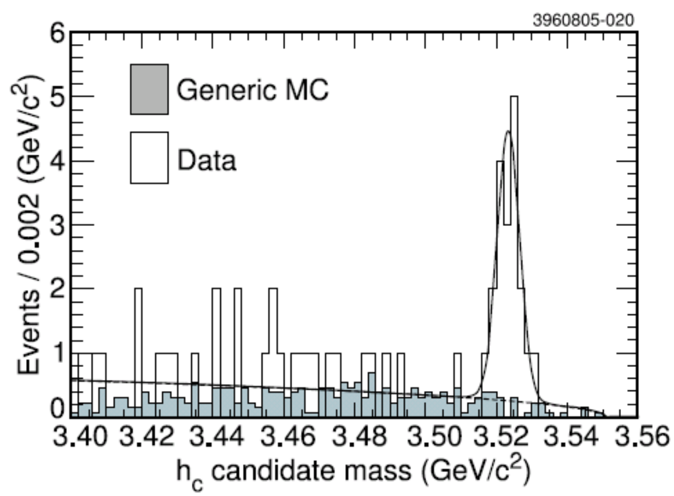
The photon energy is 638 MeV, leading to a non-zero matrix element $\langle 1S|j_0(kr/2)|2S\rangle$.

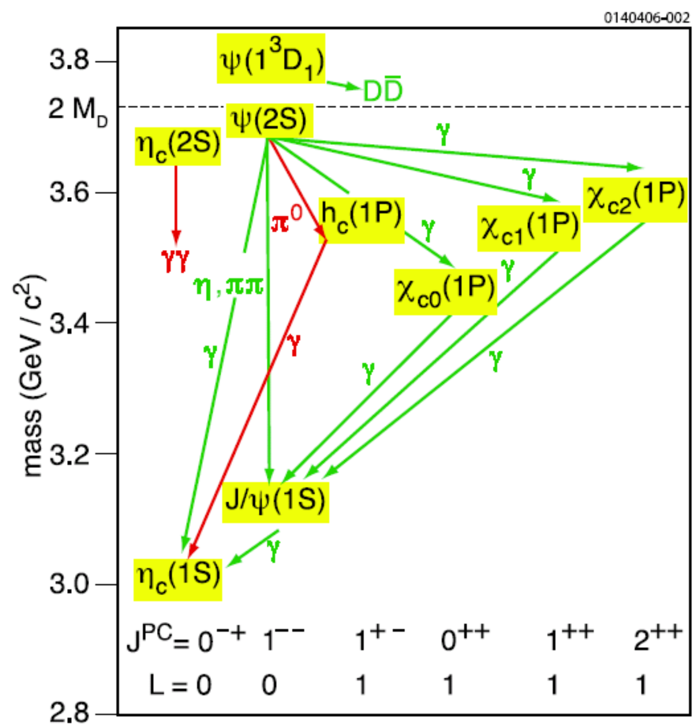
$$\Gamma[\psi(2S) \to \gamma\eta_c(1S) = (1.00 \pm 0.16)$$

$$|\langle 1S|j_0(kr/2)|2S\rangle| = 0.045 \pm 0.004.$$

The $h_c(1^1P_1)$

$$\mathcal{B}(h_c \to \gamma\eta_c) = 0.5.$$





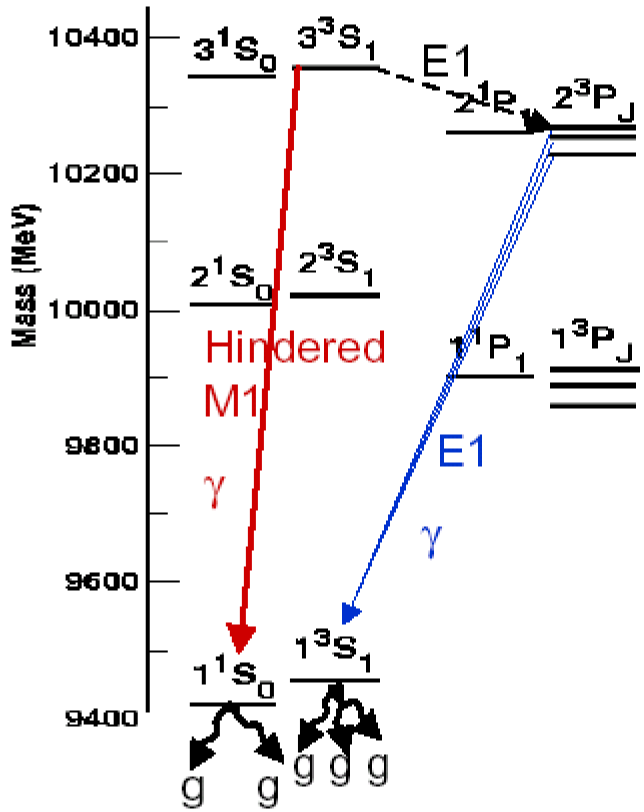
M1 transitions: production of $\eta_b(nS)$ states

S.G + J. Rosner, Phys Rev D64, 074011 (2001)

Proceeds via magnetic dipole (M1) transitions:

$$\Upsilon(nS) \to \eta(n'S) + \gamma$$

$$\Gamma(^3S_1 \to {}^1S_0 + \gamma) = \frac{4}{3}\alpha \frac{e_Q^2}{m_Q^2} \left| \langle f | j_0(kr/2) | i \rangle \right|^2 \omega^3$$



- Hindered transitions have large phase space
- Relativistic corrections resulting in differences in $^3S_1$ and $^1S_0$ wavefunctions due to hyperfine interaction

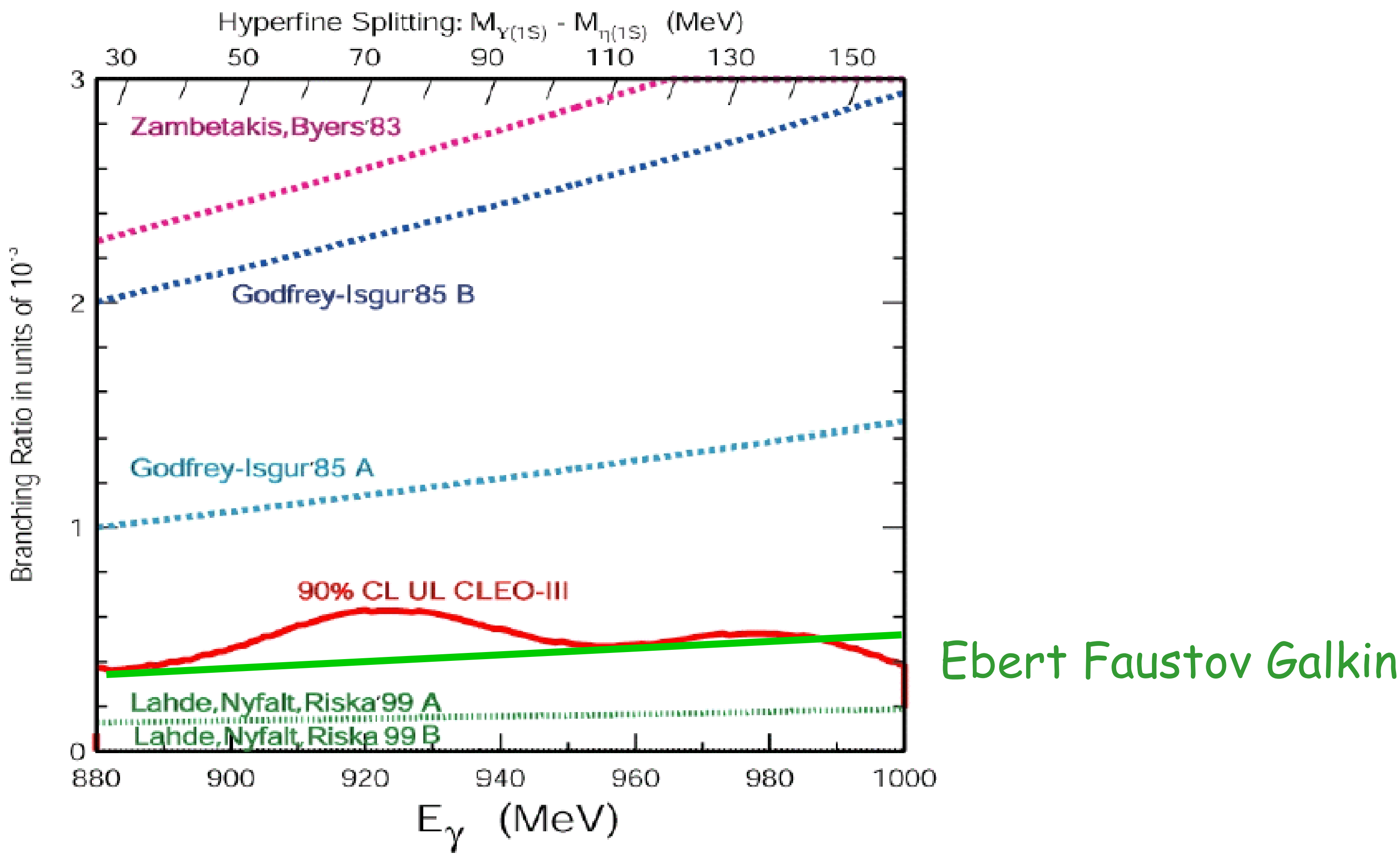
	Transition	BR (10^{-4})
Υ(3S)		
(Γ_{tot}=52.5 keV)	$\to 3^1S_0$	0.10
	$\to 2^1S_0$	4.7
	$\to 1^1S_0$	25
Υ(2S)	$\to 2^1S_0$	0.21
(Γ_{tot}=44 keV)	$\to 1^1S_0$	13
Υ(1S)	$\to 1^1S_0$	2.2
(Γ_{tot}=26.3 keV)		

- Expect substantial rate to produce η_b's
- Also $\Upsilon(3S) \to h_b(^1P_1)\,\pi\pi \to \eta_b + \gamma + \pi\pi$

BR=0.1-1% BR = 50%

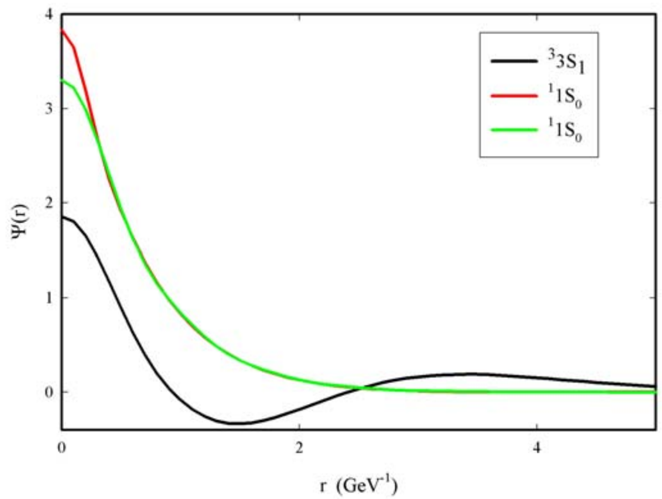
[Kuang & Yan PRD24, 2874 (1981); Voloshin Yad Fiz 43, 1571 (1986)]

But no signal found!



Is there a problem?

•Does not appear due to wavefunction effects like in E1 transitions:



BR=2.3 x 10^{-3}

BR=2.4 x 10^{-3}

Not much difference

•Most likely due to poorly understood relativistic effects:

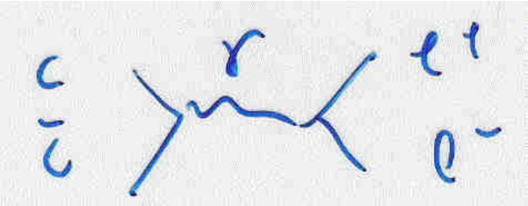
$$I = \left\langle 1 - \frac{k^2 r^2}{24} - \frac{2}{3}\frac{\vec{p}^2}{m_Q^2} - \frac{1}{6}\frac{\vec{p}^2}{m_Q^2} - \frac{V_S}{m_Q} \right\rangle$$

the last term is due to pair creation in the binding potential

see Sucher, Rep. Prog. Phys 41, 1781 (1978), Kang & Sucher PR D18, 2698 (1978), Feinberg & Sucher, PRL 35, 1740 (1975); Grotch Owen & Sebastian PR D30, 1924 (1984), Zabetakis & Byers PR D28, 2908 (1983)

Decays:

$$J/\psi \to e^+e^-$$
$$({}^3S_1 \to e^+e^-)$$



$$A(V_i \to e^+e^-) \equiv \langle e^+e^-|M|V_i\rangle$$
$$= \frac{4\pi\alpha e_q}{M^2}\langle e^+e^-|j_k^{(em)}|0\rangle\langle 0|j_k|V_i\rangle$$
$$= \frac{4\pi\alpha e_q}{M^2}\bar{U}_e(-p_+)\gamma_k U(p_-)\langle 0|j_k|V_i\rangle$$

$$\langle 0|j_k|V_i\rangle = \sqrt{3\times 2M}\int d^3p\phi_s(p)Y_{00}\langle 0|j^\mu_{em}|c\bar{c}\rangle$$

where $\sum_{colour}\sqrt{\frac{1}{3}}(r\bar{r}+b\bar{b}+g\bar{g}) = \frac{3}{\sqrt{3}} = \sqrt{3}$

Typically express the matrix element in the form:

$$\langle 0|j^\mu_{em}(0)|\psi(k,\lambda)\rangle = \frac{\epsilon^\mu(k,\lambda)}{(2\pi)^{3/2}}f_\psi$$

For the + polarization:

$$\langle 0|j^\mu_{em}(0)|\psi(k,\lambda)\rangle = \sqrt{6M}\int d^3p\,\phi_S(p)Y_{00}(\theta,\phi)\langle 0|j^\mu_{em}|\bar{c}(-\vec{p},\uparrow)c(\vec{p},\uparrow)\rangle$$

and $\langle 0|j^\mu_{em}|\bar{c}(-\vec{p},\uparrow)c(\vec{p},\uparrow)\rangle = \frac{\bar{V}(-\vec{p},\uparrow)}{(2\pi)^{3/2}}\gamma^\mu\frac{U(\vec{p},\uparrow)}{(2\pi)^{3/2}}$

By explicit evaluation:

$$\langle 0|j^0_{em}|\bar{c}(-\vec{p},\uparrow)c(\vec{p},\uparrow)\rangle = 0$$

$$\langle 0|j^1_{em}|\bar{c}(-\vec{p},\uparrow)c(\vec{p},\uparrow)\rangle = \frac{1}{(2\pi)^3}\frac{1}{m}\left[\frac{p_+p_x}{E+m} - E\right]$$

$$\langle 0|j^2_{em}|\bar{c}(-\vec{p},\uparrow)c(\vec{p},\uparrow)\rangle = \frac{1}{(2\pi)^3}\frac{1}{m}\left[\frac{p_+p_y}{E+m} - iE\right]$$

$$\langle 0|j^3_{em}|\bar{c}(-\vec{p},\uparrow)c(\vec{p},\uparrow)\rangle = \frac{1}{(2\pi)^3}\frac{1}{m}\frac{p_+p_z}{E+m}$$

$$p_+ = p_x + ip_y$$

$$\langle 0|j^{\mu}_{em}(0)|V(\uparrow)\rangle = \frac{\sqrt{6M}}{m}\int \frac{d^3p}{(2\pi)^3}\,\phi_S(p)Y_{00}(\theta,\phi)\left[\frac{-p^{+}p^{\mu}}{E+m} - E(g^{\mu 1}+g^{\mu 2})\right]$$

Integrand is symmetric except for p^+p^μ term

$$= -\frac{\sqrt{6M}}{m}\frac{1}{(2\pi)^3}\int d^3p\,\phi_S(p)Y_{00}(\theta,\phi)\left[\frac{2E+m}{3}\right](g^{\mu 1}+g^{\mu 2})$$

$$= \frac{\sqrt{6M}}{m}\sqrt{2}\,\frac{\epsilon^{\mu}(\uparrow)}{(2\pi)^3}\int d^3p\,\phi_S(p)Y_{00}(\theta,\phi)\left[\frac{2E+m}{3}\right]$$

In non-relativistic limit

$$= \sqrt{12M}\,\frac{\epsilon^{\mu}(\uparrow)}{m}\int d^3p\,\phi_S(p)Y_{00}(\theta,\phi)m\frac{e^{-i\vec{p}\cdot\vec{0}}}{(2\pi)^{3/2}}$$

$$\Rightarrow \langle 0|j^{\mu}_{em}(0)|V(\uparrow)\rangle = \sqrt{12M}\,\epsilon^{\mu}(\uparrow)\,\psi_S(0)$$

$$\equiv \epsilon^{\mu}(k,\lambda)\,f_V$$

$$f_V = \sqrt{12M}\,\psi_S(0)$$

$$
\begin{aligned}
\Gamma &= \frac{1}{2M} \int |M|^2 \frac{m_e}{E_{e^+}} \frac{m_e}{E_{e^-}} \frac{d^3p_+}{(2\pi)^3} \frac{d^3p_-}{(2\pi)^3} \, (2\pi)^4 \, \delta^4(P - p_+ - p_-) \\
&= \frac{e_Q^2 e^4}{12\pi M^3}(12M(2\pi)^3|\psi(0)|^2) \\
&= \frac{16\pi^2\alpha^2 e_Q^2}{\pi M^3} M|\psi(0)|^2 \\
&= \frac{16\pi\alpha^2 e_Q^2}{M^2} \, |\psi(0)|^2
\end{aligned}
$$

$$
\psi_S(0) = \frac{1}{\sqrt{4\pi}} R(0)
$$

What about ψ''(3770) ? $e^+e^- \rightarrow \psi''(3770)$

$^3D_1$ state so expect $\Gamma=0$ since $\psi_D(0)=0$ but not so

$$|V(\uparrow)\rangle = \sqrt{6M}\int d^3p\,\phi_D(p)\{\sqrt{3/5}Y_{2+2}(\theta,\phi)|q(\downarrow)\,\bar{q}(\downarrow)\rangle - \sqrt{3/10}Y_{2+1}(\theta,\phi)|q(\uparrow)\,\bar{q}(\downarrow)\rangle + \sqrt{1/10}Y_{20}(\theta,\phi)|q(\uparrow)\,\bar{q}(\uparrow)\rangle\}$$

After much work get:

$$\langle 0|j^\mu_{em}(0)|V(\uparrow)\rangle = \frac{\sqrt{12M}}{(2\pi)^3}\,\epsilon^\mu(\uparrow)\int d^3p\,\frac{\phi_D(p)}{\sqrt{32\pi}}\frac{4}{3}\frac{p^2}{E(E+m)}$$

$$\lim_{x\to 0}\int d^3p\,\phi_D(p)\,\frac{p^2}{2m^2}\frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}} = -\frac{1}{2m^2}\lim_{x\to 0}\frac{\partial^2}{\partial x_i^2}\int d^3p\,\frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}}\,\phi_D(p)$$

$$= -\frac{1}{2m^2}\frac{\partial^2 R_D(0)}{\partial r^2} = -\frac{1}{2m^2}R''_D(0)$$

In general, for state of angular momentum L get $R^{(L)}(0)$

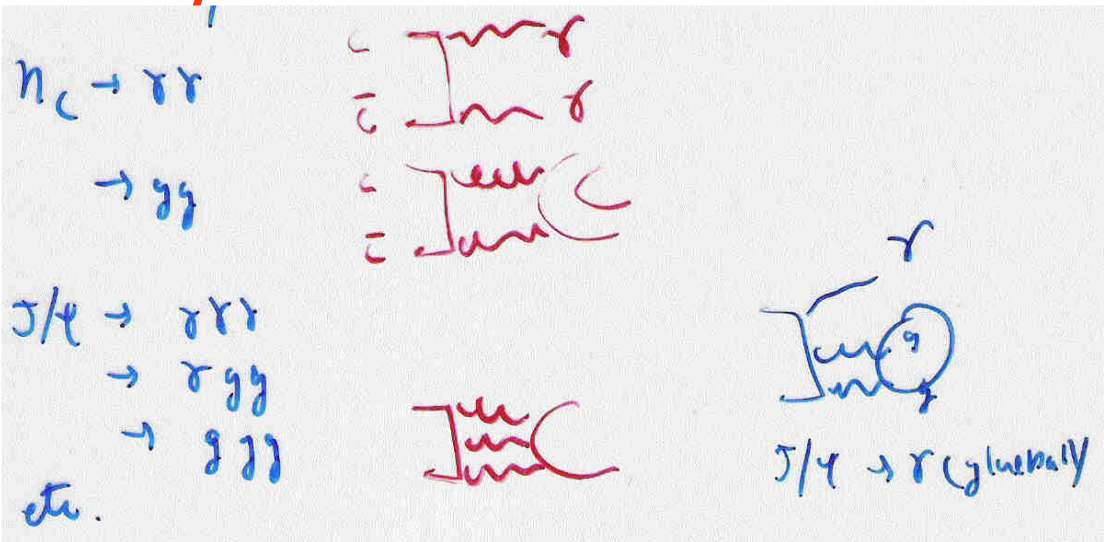
More carefully get:

$$\langle 0|j^{\mu}_{em}(0)|V(\uparrow)\rangle = \frac{\sqrt{12M}}{(2\pi)^{3/2}}\frac{5}{4}\;\frac{R''(0)}{m^2\sqrt{2\pi}}\epsilon^{\mu}(\uparrow)$$

and

$$\Gamma = \frac{\alpha^2(e_q/e)^2}{M_V^2}\frac{25}{2}\frac{|R''_D(0)|^2}{m_q^2}$$

Also have decays to hadronic final states:



Start with annihilation rates for positronium:

$$\Gamma(^1S_0 \to 2\gamma) = \frac{4\pi\alpha^2}{m^2}|\psi_S(0)|^2 = \frac{4\alpha^2}{m^2}|R_S(0)|^2$$

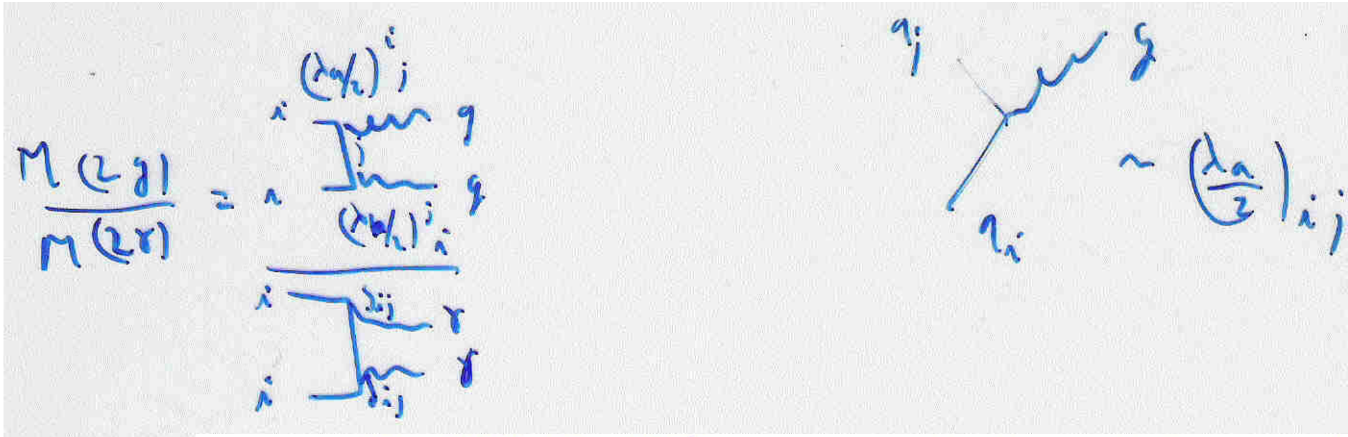
$$\Gamma(^3P_0 \to 2\gamma) = \frac{256}{3}\frac{\alpha^2}{m^4}|R'_P(0)|^2$$

$$\Gamma(^3P_2 \to 2\gamma) = \frac{4}{15}\Gamma(^3P_0 \to \gamma\gamma)\left(\frac{M_0}{M_2}\right)$$

$$\Gamma(^3S_1 \to 3\gamma) = \frac{16}{9\pi}(\pi^2 - 9)\frac{\alpha^3}{m^2}|R_S(0)|^2$$

To relate to hadron decays include quark charges

For decays to gluons must include α_S and λ's for each gluon



$$= \frac{\alpha_s}{e_q^2 \alpha} \frac{(\lambda_a/2)^i_j \ (\lambda_b/2)^j_i}{\delta^i_j \ \delta^j_i} = \frac{\alpha_s}{\alpha e_q^2} \frac{\mathrm{Tr}(\lambda_a/2 \ \lambda_b/2)}{3} = \frac{\alpha_s}{\alpha} \frac{\frac{1}{2}\delta_{ab}}{e_q^2 3}$$

where $\mathrm{Tr}(\lambda_a/2 \ \lambda_b/2) = \frac{1}{2}\delta_{ab}$

$$\Rightarrow \frac{\Gamma(2g)}{\Gamma(2\gamma)} = \frac{2}{9} \frac{\alpha_s^2}{\alpha^2 e_q^4}$$

For 3 gluons/photons:

$$\frac{M(3g)}{M(3\gamma)} = \frac{\alpha_s^{3/2}}{e_q^3 \alpha^{3/2}} \frac{(\lambda_a/2)^i_j \ (\lambda_b/2)^j_k \ (\lambda_c/2)^k_i}{\delta^i_j \ \delta^j_k ; \delta^k_i} = \frac{\alpha_s^{3/2}}{e_q^3 \alpha^{3/2}} \frac{1}{2} \frac{\mathrm{Tr}(\{\lambda_a/2, \ \lambda_b/2)\}\lambda_c/2)}{\delta^i_j \ \delta^j_k \ \delta^k_i}$$

$$\Rightarrow \frac{\Gamma(2g)}{\Gamma(2\gamma)} = \frac{5}{54} \frac{\alpha_s^3}{\alpha^3 e_q^6} \text{ where } \sum_{a,b,c} (d_{abc})^2 = 40/3$$

$$\Gamma(\eta_c \to 2\gamma) = 12\alpha^2 e_q^4 \frac{|R_S(0)|^2}{M^2}$$

$$\Gamma(\eta_c \to 2g) = \frac{8}{3}\alpha_S \frac{|R_S(0)|^2}{M^2}$$

$$\Gamma(J/\psi \to 3\gamma) = \frac{16(\pi^2 - 9)\alpha^3}{3} e_q^6 \frac{|R_S(0)|^2}{M^2}$$

$$\Gamma(J/\psi \to 3g) = \frac{40}{81\pi}(\pi^2 - 9)\alpha_s^3 \frac{|R_S(0)|^2}{M^2}$$

$$\Gamma(J/\psi \to 2g\gamma) = \frac{32}{9\pi}(\pi^2 - 9)\alpha_s^2 \alpha e_1^2 \frac{|R_S(0)|^2}{M^2}$$

For Completeness:

$$\Gamma(\chi_0 \to 2g) = 96\alpha_s^2 \frac{|R'_{\chi_o}(0)|^2}{M_{\chi_0}^4}$$

$$\Gamma(\chi_1 \to q\bar{q}g) = \frac{n_f}{3}\frac{128}{3\pi}\alpha_s^3 \frac{|R'_{\chi_1}(0)|^2}{M_{\chi_0}^4} \ln\left(\frac{4m_c^2}{4m_c^2 - M_\chi^2}\right)$$

$$\Gamma(\chi_2 \to 2g) = \frac{128}{5}\alpha_s^2 \frac{|R'_{\chi_2}(0)|^2}{M_{\chi_0}^4}$$

$$\Gamma(h_c \to q\bar{q}g) = \frac{320}{9\pi}\alpha_s^3 \frac{|R'_{h_c}(0)|^2}{M_{h_c}^4} \ln\left(\frac{4m_c^2}{4m_c^2 - M_{h_c}^2}\right)$$

In the last decade or so these calculations have been studied in greater detail.

It was recognized that soft gluon effects could be important

This leads to the annihilation matrix element having colour octet contributions

All this falls into the realm of NRQCD

Production of the singlet P-wave states

S.G + J. Rosner, PR D66,1014102 (2002)

Two interesting cascades:

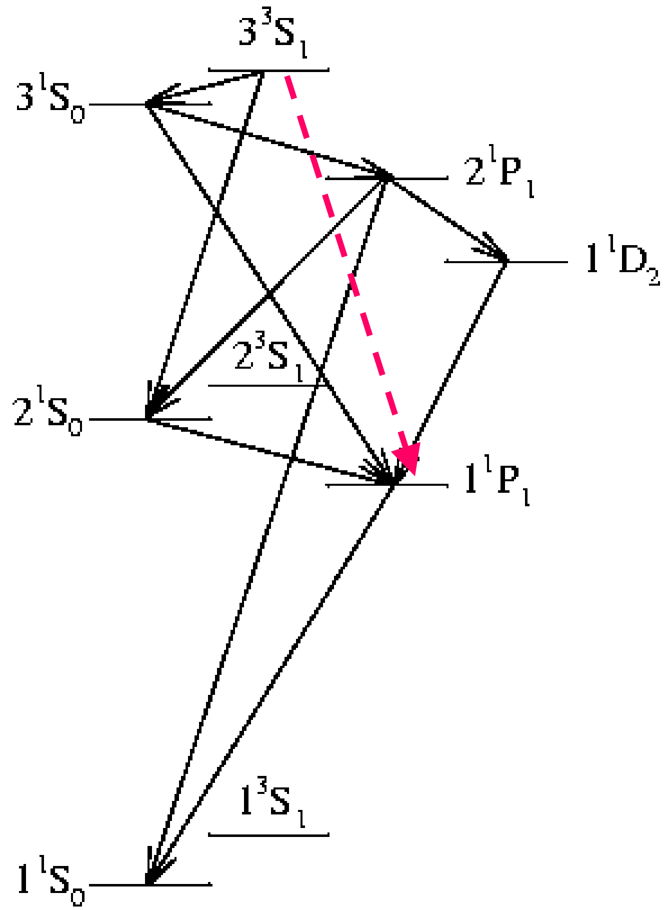
M1 E1 E1

$\Upsilon(3S) \to \eta_b(2S) + \gamma \to h_b + \gamma\gamma \to \eta_b + \gamma\gamma\gamma$

$\psi(2S) \to \eta_c(2S) + \gamma \to h_c + \gamma\gamma \to \eta_c + \gamma\gamma\gamma$

$\Upsilon(3S) \to h_b + \pi \to \eta_b + \gamma + \pi$

$\psi(2S) \to h_c + \pi \to \eta_c + \gamma + \pi$



Need branching ratios and hence partial widths

$$\Gamma[\eta(2^1S_0)\to h_b(1^1P_1)+\gamma]=\frac{4}{3}\alpha e_Q^2\left|\left\langle {}^1P_1|r|{}^1S_0\right\rangle\right|^2\omega^3=2.3\ \mathrm{keV}$$

$$\Gamma[h_b(1^1P_1)\to \eta_b(1^1S_0)+\gamma]=\frac{4}{9}\alpha e_Q^2\left|\left\langle {}^1S_0|r|{}^1P_1\right\rangle\right|^2\omega^3=37\ \mathrm{keV}$$

$$\Gamma[\eta_b(2^1S_0)\to gg]=\frac{27\pi}{5(\pi^2-9)\alpha_S}\times\Gamma[\Upsilon(2^3S_1)\to ggg]=4.1\pm0.7\ \mathrm{MeV}$$

BR($3^3S_1\ \gamma \to 2^1S_0\ \gamma$)=4.7 x 10^{-4} and BR($2^1S_0\ \gamma \to 1^1P_1\ \gamma$)=5.7x 10^{-5}

BR[$\Upsilon(3S) \to 2^1S_0\ \gamma \to 1^1P_1\ \gamma$] = 2.6 x 10^{-7} $\Rightarrow$ 0.3 events/10^6 $\Upsilon(3S)$'s

Similarly

BR[$\psi(2S) \to 2^1S_0\ \gamma \to 1^1P_1\ \gamma$] = 10^{-6} $\Rightarrow$ 1 event /10^6 $\Upsilon(3S)$'s

(A challenge for the experimentalists!)

A more promising approach:

$\Upsilon(3S) \to h_b + \pi \to \eta_b + \gamma + \pi$

$\psi(2S) \to h_c + \pi \to \eta_c + \gamma + \pi$

Utilizes: $BR[\Upsilon(3S) \to \pi\ 1^1P_1] = 0.1\%$

$$\Gamma[h_b(1^1P_1) \to \eta_b(1^1S_0) + \gamma] = \frac{4}{9}\alpha e_Q^2 \left|\langle {}^1S_0 | r | {}^1P_1 \rangle\right|^2 \omega^3 = 37 \text{ keV}$$

$$\Gamma[h_b(1^1P_1) \to ggg] = \frac{5}{2n_f}\Gamma[\chi_{b1}(1^3P_1) \to q\bar{q}g] = 50.8 \text{ keV}$$

$BR[\Upsilon(3S) \to \pi\ 1^1P_1 \to 1^1S_0\,\gamma] = 4 \times 10^{-4}$

$\Rightarrow$ 400 events/10^6 $\Upsilon(3S)$'s

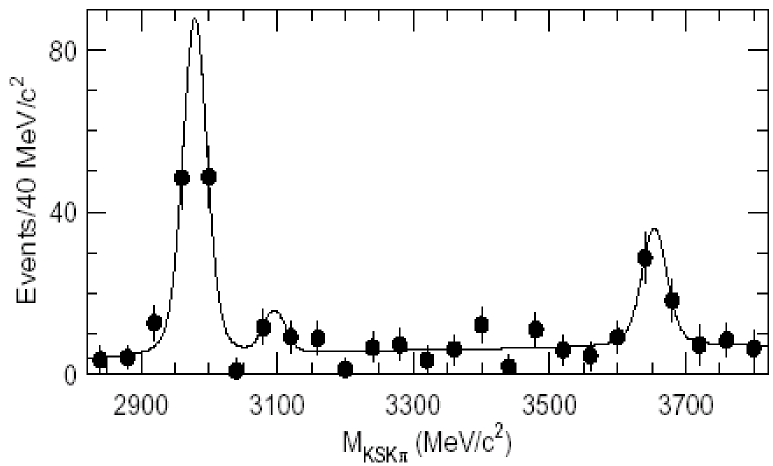
$BR[\psi(2S) \to \pi\ 1^1P_1 \to 1^1S_0\,\gamma] = 3.8 \times 10^{-4}$

$\Rightarrow$ ~400 event /10^6 $\psi(2S)$'s

Charmonium in B decays

Recent observation by Belle of $\eta_c(2S)$ in: $B \rightarrow \eta_c(2S)\,K \rightarrow KK_SK^-\pi^+$

$M=3654 \pm 6$ (stat) $\pm$ 8 (sys) MeV
$\Gamma<55$ MeV (90% C.L.)



Belle had previously reported the observation of

$$B^+ \rightarrow \chi_{c0} K^+$$
$$B \rightarrow \chi_{c2} X$$

And $B \rightarrow \chi_{c1} K$ has been observed by both BaBar and Belle

Search for the h_c in

$$B \rightarrow h_c\, X$$

News Front Page
Africa
Americas
Asia-Pacific
Europe
Middle East
South Asia
UK
Business
Health
Science/Nature
Technology
Entertainment
Have Your Say
Country Profiles
In Depth
Programmes
RELATED SITES
BBC SPORT
BBC WEATHER
BBC ON THIS DAY

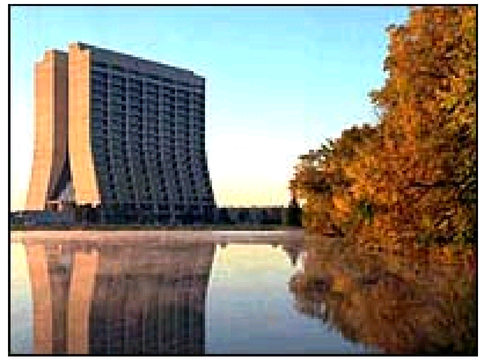
Last Updated: Tuesday, 18 November, 2003, 11:36 GMT

E-mail this to a friend | Printable version

Scientists find mystery particle

By Dr David Whitehouse
BBC News Online science editor

Scientists have found a sub-atomic particle they cannot explain using current theories of energy and matter.



Fermilab confirmed the discovery

The discovery was made by researchers based at the High Energy Accelerator Research Organisation in Tsukuba.

Classified as X(3872), the particle was seen fleetingly in an atom smasher and has been dubbed the "mystery meson".

The Japanese team says understanding its existence may require a change to the Standard Model, the accepted theory of the way the Universe is constructed.

An eternity

X(3872) was found among the decay products of so-called beauty mesons - sub-atomic particles that are produced in large numbers at the Tsukuba "meson factory".

It weighs about the same as a single atom of helium and exists for only about one billionth of a trillionth of a second before it decays into other longer-lived, more familiar

SEE ALSO:
- Behold the pentaquark
  01 Jul 03 | Science/Nature
- Physicists find 'rebel' particle
  30 Apr 03 | Science/Nature
- Did quark matter strike Earth?
  22 Nov 02 | Science/Nature

RELATED INTERNET LINKS:
- KEK
- Fermi National Accelerator
- Quarks

TOP SCIENCE/NATURE STORIES NOW
- Oblivion threat to 12,000 species
- Scientists find mystery particle
- Centre opens for X-rated science
- Huge rock-ice body circles Sun

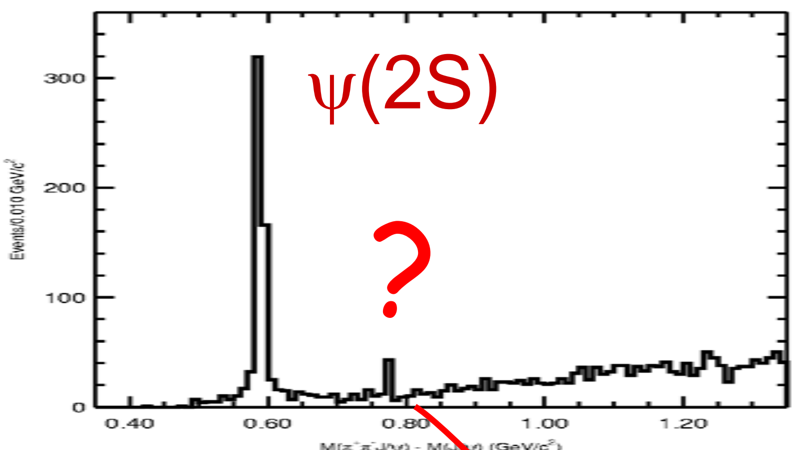
New state observed by Belle: X(3871)

hep-ex/0309032

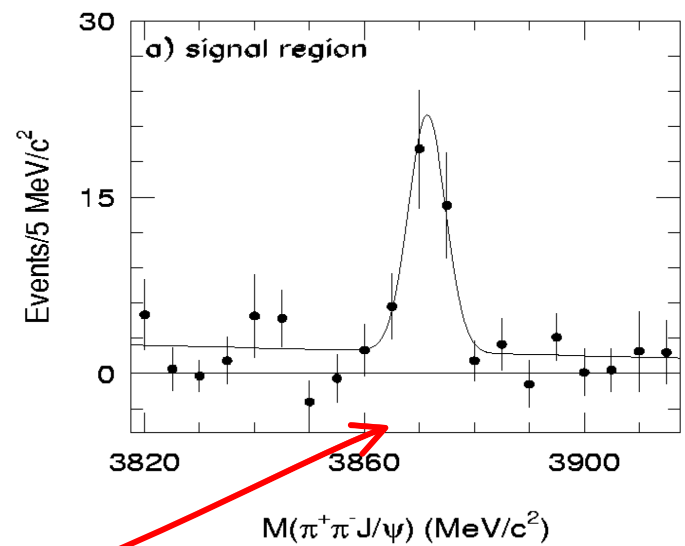
BELLE-CONF-0352

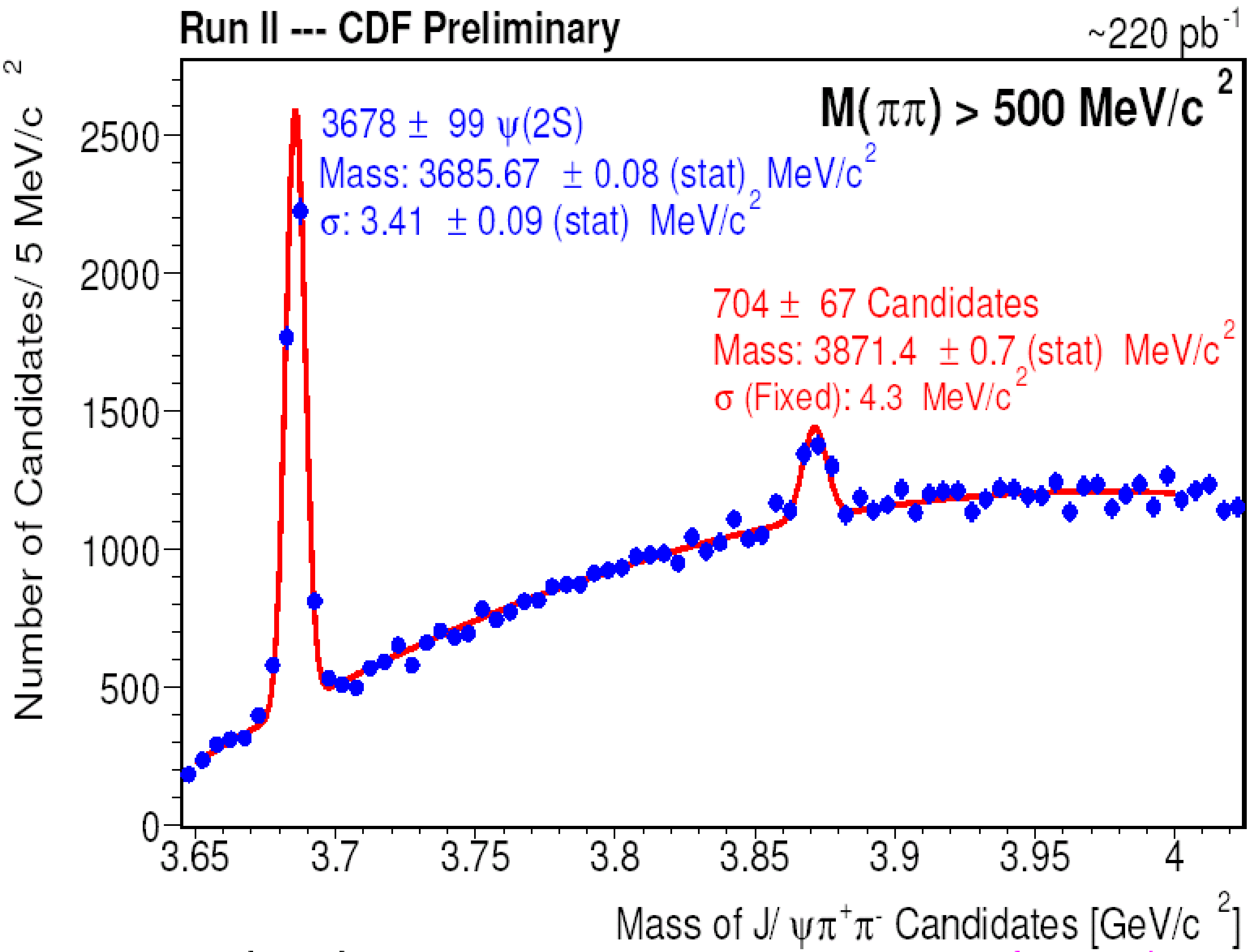
Observation of a new narrow charmonium state in exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ decays

We report the first observation of a narrow charmonium state produced in the exclusive decay process $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$. This state, which decays into $\pi^{+}\pi^{-}J/\psi$, has a mass of $3871.8 \pm 0.7(\text{stat}) \pm 0.4(\text{syst})$ MeV, which is very near the $M_D + M_{D^*}$ mass threshold. The results are based on an analysis of 152M $B\bar{B}$ events collected at the $\Upsilon(4S)$ resonance in the Belle detector at the KEKB collider.



$M(J/\psi\ \pi^{+}\pi^{-}) - M(J/\psi)$ GeV




Run II --- CDF Preliminary
~220 pb-1
M(ππ) > 500 MeV/c2
3678 ± 99 ψ(2S)
Mass: 3685.67 ± 0.08 (stat) MeV/c2
σ: 3.41 ± 0.09 (stat) MeV/c2
704 ± 67 Candidates
Mass: 3871.4 ± 0.7 (stat) MeV/c2
σ (Fixed): 4.3 MeV/c2
Number of Candidates/ 5 MeV/c2
Mass of J/ψπ+π- Candidates [GeV/c2]

Charmonium Options for the X(3872)

T.Barnes,S.Godfrey, Phys Rev D69, 050400 (2004) [hep-ph/0311162]
Eichten, Lane & Quigg, Phys Rev D69, 094019 (2004) [hep-ph/0401210]
Barnes, Godfrey & Swanson, in preparation

New state 1st observed by Belle: X(3871) hep-ex/0309032

Observation of a new narrow charmonium state in exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ decays

- M=3872.0 ± 0.6 ± 0.5 MeV $\Gamma < 2.3$ MeV at 90% C.L.

  width consistent with detector resolution.

1. D^0D^{*0} molecule
2. A charmonium hybrid
3. 1^3D_2 state?

D^0D^{*0} molecule

Quantity	MeV	$M_X - M_{threshold}$
M_X	3871.8±0.7±0.4	
$M_{D^0} + M_{D^{*0}}$	3871.5±0.7	+0.3±1.1
$M_{D^+} + M_{D^{*+}}$	3879.5±0.7	−7.7±1.1

. The mass of the state is right at the D^0D^{*0} threshold!

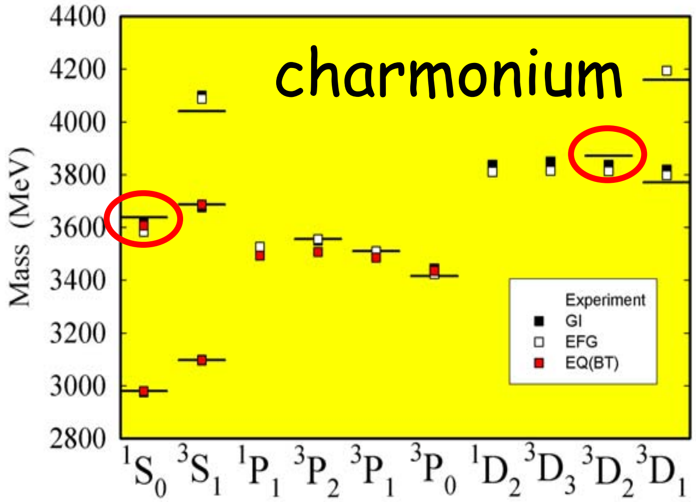
This suggests a loosely bound D^0D^{*0} molecule, right below the dissociation energy

"Molecular Charmonium" discussed in literature since 1975

1^3D_2 state?

Because D-states have negative parity, spin-2 states cannot decay to DD

- They are narrow as long as below the DD* threshold



- Predict: $$\frac{BR(\psi(1^3D_2) \to \gamma\gamma J/\psi)}{BR(\psi(1^3D_2) \to \pi^+\pi^- J/\psi)} \sim 3$$

Should easily see $\psi(1^3D_2) \longrightarrow \gamma\gamma J/\psi$

BUT: $$\frac{BR(X(3872) \to \gamma\chi_{c1})}{BR(X(3872) \to \pi^+\pi^- J/\psi)} < 0.89 \quad (90\% \text{ CL})$$

Belle hep-ex/0309032

- Most models predict $\psi(1^3D_2)$ mass to be ~70 MeV lower than the measured X(3872) mass.
- At the same time they reproduce the $\Upsilon(1^3D_2)$ mass very well.

No models appear to accommodate $\psi(3770)$ and X(3872) in same 1^3D_J triplet!

Can coupled channel effects and $\psi(1^3D_1)$- $\psi(2^3S_1)$ mixing change this?

Charmonium Options for the X(3872)

- Consider all 1D and 2P cc possibilities
- Assume M=3872 MeV
    - calculate radiative widths and
    - strong decay widths

Strong Decays:

1. Zweig-allowed open-charm decays (DD)

   expect 1^3D_2 and 1^1D_2 but 1^3D_3 also narrow because of angular momentum barrier

2. Annihilation type decays

summarized in Ref.[50]. Expressions for decay widths relevant to the 1D and 2P $c\bar{c}$ states in particular are:

$$\Gamma(^3\mathrm{D_J} \to \mathrm{ggg}) = \frac{10\alpha_s^3}{9\pi} C_J \frac{|R''_\mathrm{D}(0)|^2}{m_Q^6} \ln(4m_Q\langle r\rangle) \quad (7)$$

$$\Gamma(^1\mathrm{D_2} \to \mathrm{gg}) = \frac{2\alpha_s^2}{3} \frac{|R''_\mathrm{D}(0)|^2}{m_Q^6} \quad (8)$$

$$\Gamma(^3\mathrm{P_2} \to \mathrm{gg}) = \frac{8\alpha_s^2}{5} \frac{|R'_\mathrm{P}(0)|^2}{m_Q^4} \quad (9)$$

$$\Gamma(^3\mathrm{P_1} \to \mathrm{q\bar{q}g}) = \frac{8n_f\alpha_s^3}{9\pi} \frac{|R'_\mathrm{P}(0)|^2}{m_Q^4} \ln(m_Q\langle r\rangle) \quad (10)$$

$$\Gamma(^1\mathrm{P_1} \to \mathrm{ggg}) = \frac{20\alpha_s^3}{9\pi} \frac{|R'_\mathrm{P}(0)|^2}{m_Q^4} \ln(m_Q\langle r\rangle) \quad (11)$$

$$\Gamma(^1\mathrm{P_1} \to \mathrm{gg\gamma}) = \frac{36}{5} e_q^2 \frac{\alpha}{\alpha_s} \Gamma(^1\mathrm{P_1} \to \mathrm{ggg}) \quad (12)$$

$$\Gamma(^3\mathrm{P_0} \to \mathrm{gg}) = 6\alpha_s^2 \frac{|R'_\mathrm{P}(0)|^2}{m_Q^4} \quad (13)$$

3. Hadronic transitions

Radiative transitions:

$$\Gamma(\mathrm{n}\,^{2\mathrm{S}+1}\mathrm{L_J} \to \mathrm{n}'\,^{2\mathrm{S}'+1}\mathrm{L}'_{\mathrm{J}'} + \gamma) = \frac{4}{3}\, e_c^2\, \alpha\, \omega^3\, C_{fi}\, \delta_{\mathrm{SS}'}\, |\, \langle \mathrm{n}'\,^{2\mathrm{S}'+1}\mathrm{L}'_{\mathrm{J}'} |\, r\, |\, \mathrm{n}\,^{2\mathrm{S}+1}\mathrm{L_J}\rangle\, |^2 \ ,$$

$$C_{fi} = \max(\mathrm{L},\ \mathrm{L}')(2\mathrm{J}'+1) \begin{Bmatrix} \mathrm{L}' & \mathrm{J}' & \mathrm{S} \\ \mathrm{J} & \mathrm{L} & 1 \end{Bmatrix}^2 .$$

TABLE II: Radiative transitions in scenario 1: Predictions for the E1 transitions 1D→1P, 2P→2S, 2P→1S and 2P→1D, assuming in all cases that the initial $c\bar{c}$ state has a mass of 3872 MeV. The matrix elements were obtained using the wavefunctions of the Godfrey-Isgur model, Ref.[17]. Unless otherwise stated, the widths are given in keV and the final $c\bar{c}$ masses are PDG values [38].

Initial state X(3872)	Final state	M_f (MeV)	ω (MeV)	$\langle f\|r\|i\rangle$ (GeV^{-1})	C_{fi}	Width (keV)
$1^3\mathrm{D}_3$	$\chi_{c2}(1^3\mathrm{P}_2)\,\gamma$	3556.2	303	2.762	$\frac{2}{5}$	367
$1^3\mathrm{D}_2$	$\chi_{c2}(1^3\mathrm{P}_2)\,\gamma$	3556.2	303	2.769	$\frac{1}{10}$	92
	$\chi_{c1}(1^3\mathrm{P}_1)\,\gamma$	3510.5	345	2.588	$\frac{3}{10}$	356
$1^3\mathrm{D}_1$	$\chi_{c2}(1^3\mathrm{P}_2)\,\gamma$	3556.2	303	2.769	$\frac{1}{90}$	10.2
	$\chi_{c1}(1^3\mathrm{P}_1)\,\gamma$	3510.5	345	2.598	$\frac{1}{6}$	199
	$\chi_{c0}(1^3\mathrm{P}_0)\,\gamma$	3415	430	2.390	$\frac{2}{9}$	437
$1^1\mathrm{D}_2$	$h_c(1^1\mathrm{P}_1)\,\gamma$	3517[a]	339	2.627	$\frac{2}{5}$	464

TABLE IV: Partial widths and branching fractions for strong and electromagnetic transitions in scenario 1: We assume in all cases that the initial $c\bar{c}$ state has a mass of 3872 MeV. Details of the calculations are given in the text.

Initial state	Final state	Width (MeV)	B.F. (%)
1^3D_3	DD	4.04	84.2
	ggg	0.18	3.8
	$J/\psi\pi\pi$	0.21 ± 0.11	4.4
	$\chi_{c2}(1^3P_2)\gamma$	0.37	7.7
	Total	4.80	100
1^3D_2	ggg	0.08	10.8
	$J/\psi\pi\pi$	0.21 ± 0.11	28.4
	$\chi_{c2}(1^3P_2)\gamma$	0.09	12.2
	$\chi_{c1}(1^3P_1)\gamma$	0.36	48.6
	Total	0.74	100
1^3D_1	DD	184	98.9
	ggg	1.15	0.6
	$J/\psi\pi\pi$	0.21 ± 0.11	0.1
	$\chi_{c1}(1^3P_1)\gamma$	0.20	0.1
	$\chi_{c0}(1^3P_0)\gamma$	0.44	0.2
	Total	186	100
1^1D_2	gg	0.19	22.1
	$\eta_c\pi\pi$	0.21 ± 0.11	24.4
	$h_c(1^1P_1)\gamma$	0.46	53.5
	Total	0.86	100
2^3P_2	DD	21.1	82.4
	gg	4.4	17.2
	$\psi'(2^3S_1)\gamma$	0.06	0.2
	$J/\psi(1^3S_1)\gamma$	0.04	0.2
	Total	25.6	100
2^3P_1	$q\bar{q}g$	1.65	95.9
	$\psi'(2^3S_1)\gamma$	0.06	3.5
	$J/\psi(1^3S_1)\gamma$	0.01	0.6
	Total	1.72	100
2^3P_0	DD	13.7 (see text)	24.6
	gg	42.	75.3
	$\psi'(2^3S_1)\gamma$	0.07	0.1
	$\psi''(1^3D_1)\gamma$	0.02	4×10^{-2}
	Total	55.8	100
2^1P_1	ggg	1.29	81.6
	$gg\gamma$	0.13	8.2
	$\eta_c'(2^1S_0)\gamma$	0.09	5.7
	$\eta_c(1^1S_0)\gamma$	0.07	4.4
	Total	1.58	100

too wide

too wide

too wide

1^3D_2 and 1^1D_2 and 1^3D_3

1^3D_2	ggg	0.08	10.8
	$J/\psi\pi\pi$	0.21 ± 0.11	28.4
	$\chi_{c2}(1^3P_2)\gamma$	0.09	12.2
	$\chi_{c1}(1^3P_1)\gamma$	0.36	48.6
	Total	0.74	100

1^1D_2	gg	0.19	22.1
	$\eta_c\pi\pi$	0.21 ± 0.11	24.4
	$h_c(1^1P_1)\gamma$	0.46	53.5
	Total	0.86	100

1^3D_3	DD	4.04	84.2
	ggg	0.18	3.8
	$J/\psi\pi\pi$	0.21 ± 0.11	4.4
	$\chi_{c2}(1^3P_2)\gamma$	0.37	7.7
	Total	4.80	100

2^3P_1 and 2^1P_1

2^3P_1	$q\bar{q}g$	1.65	95.9
	$\psi'(2^3S_1)\gamma$	0.06	3.5
	$J/\psi(1^3S_1)\gamma$	0.01	0.6
	Total	1.72	100
2^1P_1	ggg	1.29	81.6
	$gg\gamma$	0.13	8.2
	$\eta_c'(2^1S_0)\gamma$	0.09	5.7
	$\eta_c(1^1S_0)\gamma$	0.07	4.4
	Total	1.58	100

The problem here is that the BR to γ and $\pi\pi$ is quite small and not the final states being looked for

So far haven't distinguished between C=+ or C=-

- $J/\psi\ \pi\pi$ implies C=- so expect $\pi^0\pi^0$ final state in ratio of 1/2
- $J/\psi\ \rho$ implies C=+ but only $\pi^+\pi^-$ final state

Therefore observation or non observation of $\pi^0\pi^0$ distinguishes C

ie. C=- gives 1^3D_2 1^3D_3 or 2^1P_1

While C=+ gives 1^1D_2 or 2^3P_1

Radiative decays can then distinguish between the remaining possibilities

NOTE: Belle $\dfrac{BR(X \to \gamma\chi_{c2})}{BR(X \to \pi^+\pi^- J/\psi)} < 1.1\ (90\%\ \mathrm{CL})$ tests 1^3D_3

angular distribution analysis rules out 2^1P_1

Differences of $\pi^0\pi^0$ / $\pi^+\pi^-$ from 1/2 suggests DD^* admixtures

Probably the most useful result is that all 4 D-wave states Should be observable in B-decay!

Coupled Channel effects

Eichten et al, Phys Rev D17, 3090 (1978); D21, 203 (1980).

Interaction Hamiltonian:

$$H_I = \frac{1}{2}\sum_{a=1}^{8}\int :\rho_a(\vec{r})\,V_0(\vec{r}-\vec{r}\,')\rho_a(\vec{r}\,'):\,d^3r\,d^3r'$$

$$\rho_a(\vec{r}) = \psi^\dagger(\vec{r})\,\tfrac{1}{2}\lambda_a\psi(\vec{r})$$

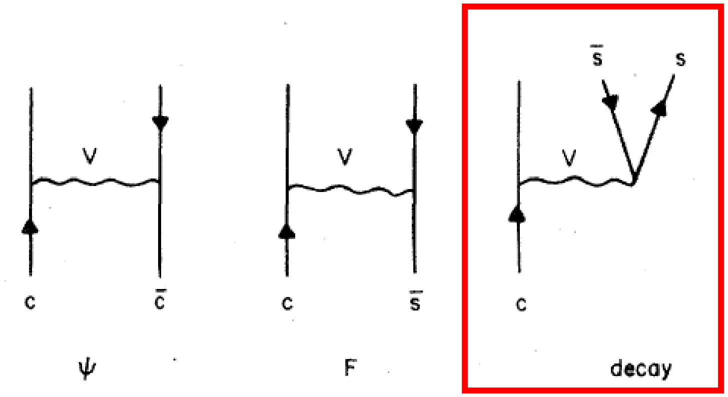
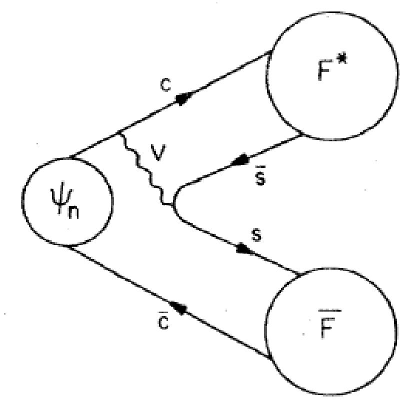


FIG. 6. Some interactions contained in Eq. (3.1).

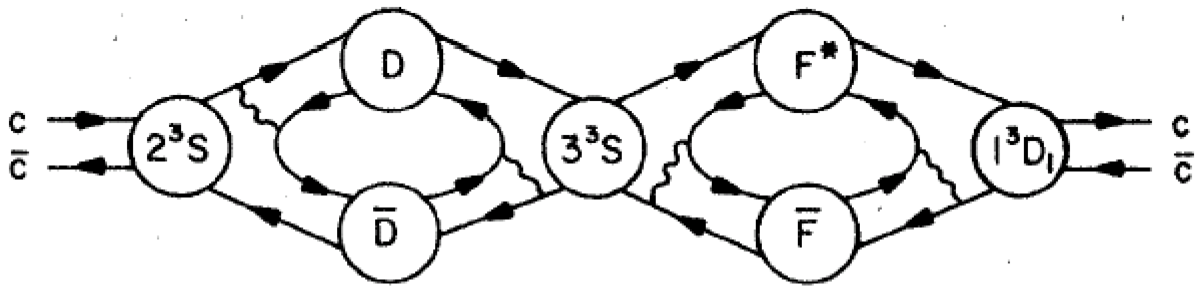


$$\langle C_1(\vec{P}\lambda_1)\bar{C}_2(\vec{P}'\lambda_2)|H_I|\psi_n\rangle = -i(2\pi)^{-3/2}\delta^3(\vec{p}+\vec{p}\,')3^{-1/2}A_{12}(\vec{P}\lambda_1\lambda_2;n)\,,$$

where

$$A_{12}(\vec{P}\lambda_1\lambda_2;n) = \frac{1}{m_q}\sum_{\{s\}}\int d^3x\,d^3y\,[\chi^\dagger(s_2')\vec{\sigma}\cdot\hat{x}\chi(-s_1')]\frac{dV(|\vec{x}|)}{d|\vec{x}|}\phi_1^*(\vec{x}s_1s_1')\phi_2^*(\vec{x}-\vec{y},\,s_2s_2')\psi_n(\vec{y}s_1s_2)e^{-i\mu_c\vec{p}\cdot\vec{y}}$$

Pair produced in pseudoscalar static potential produces ^1S state



$$|\psi'\rangle = \sum_n a_n |n^3S(c\bar{c})\rangle + \sum_n b_n |n^3D_1(c\bar{c})\rangle$$

$$+\ \alpha |D\bar{D}; p\text{-wave}\rangle + \beta |D^*\bar{D}^*; f\text{-wave}\rangle + \cdots ,$$

Expected to be most important for states near threshold

- Induces splittings of states of different J with same L
- Mechanism induces strong 2^3S_1 -1^3D_1 mixing in charmonium:
  - Shifts $\Delta M(2^3S_1)$=mass –118 MeV vs $\Delta M(1^3S_1)$=-48 MeV
  - explains large 1^3D_1 leptonic width
- predicts 3^3S_1 -2^3D_1 mixing in bottomonium and possibly also 4^3S_1 -3^3D_1

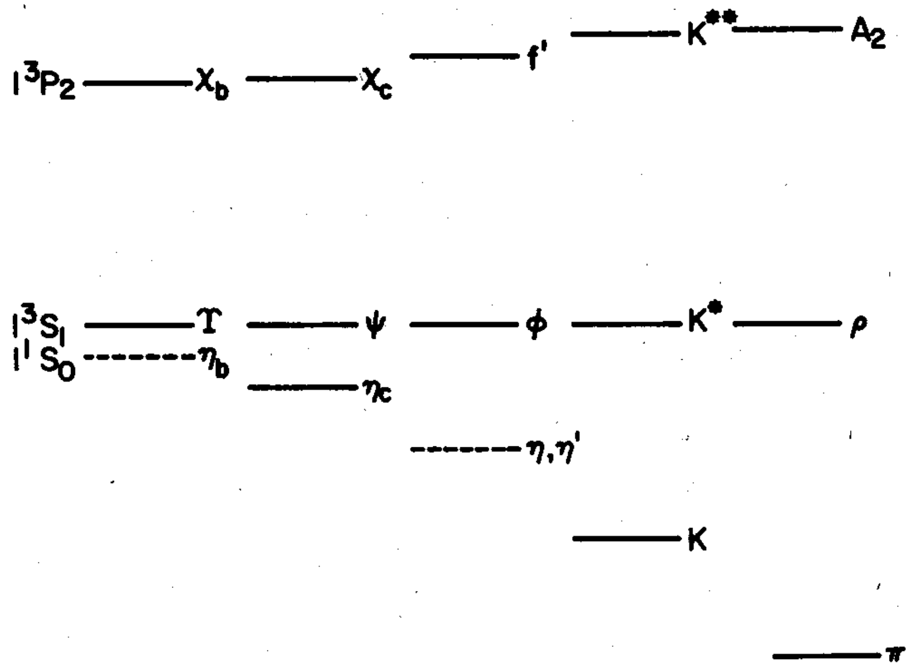
No work on this important subject since!

Summary

- In the last decade there has been much theoretical progress especially in lattice QCD.
- Need comparable experimental results to compare to theoretical results and to understand the nature of confinement in QCD.
- Theory and experiment go hand in hand to fully understand ***Soft QCD***
  - First narrow bb state observed in 19 years!
  - Only long lived L=2 meson
- Expect great progress in heavy quarkonium spectroscopy!

4. What about mesons with light quarks?

Historically, it was the successes of the quark model that led many physicists to believe that the quark model has something to do with reality



Essential features are the same, except:

- Relative importance of relativistic effects
- Hyperfine splittings are comparable in size to orbital splittings

Conclude

- potential models approximately valid

PHYSICAL REVIEW D VOLUME 12, NUMBER 1 1 JULY 1975

Hadron masses in a gauge theory*

A. De Rújula, Howard Georgi,† and S. L. Glashow
Lyman Laboratory, Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 24 February 1975)

We explore the implications for hadron spectroscopy of the "standard" gauge model of weak, electromagnetic, and strong interactions. The model involves four types of fractionally charged quarks, each in three colors, coupling to massless gauge gluons. The quarks are confined within colorless hadrons by a long-range spin-independent force realizing infrared slavery. We use the asymptotic freedom of the model to argue that for the calculation of hadron masses, the short-range quark-quark interaction may be taken to be Coulomb-like. We rederive many successful quark-model mass relations for the low-lying hadrons. Because a specific interaction and symmetry-breaking mechanism are forced on us by the underlying renormalizable gauge field theory, we also obtain new mass relations. They are well satisfied. We develop a qualitative understanding of many features of the hadron mass spectrum, such as the origin and sign of the Σ-Λ mass splitting. Interpreting the newly discovered narrow boson resonances as states of charmonium, we use the model to predict the masses of charmed mesons and baryons.

Flavour content:

$$|\rho^+\rangle, |\pi^+\rangle = -|u\bar{d}\rangle$$

$$|\rho^0\rangle, |\pi^0\rangle = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle$$

$$|\omega\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$$

$$|\eta\rangle = \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle$$

$$|\eta'\rangle = \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle$$

$$|\phi\rangle = |s\bar{s}\rangle$$

$$|K^+\rangle = |u\bar{s}\rangle$$

$$|K^0\rangle = |d\bar{s}\rangle$$

$$|\bar{K}^0\rangle = -|s\bar{d}\rangle$$

$$|K^-\rangle = |s\bar{u}\rangle$$

In heavy quarkonium we used:

$$M = m_1 + m_2 + E_{nl}$$

$$\left[\frac{p^2}{2\mu} + V(r)\right]\psi = E_{nl}\psi$$

$$H_{ij}^{conf} = -\frac{4}{3}\frac{\alpha_s(r)}{r} + br$$

This is a non-relativistic formula (v/c)=

$b\bar{b}$	0.26
$c\bar{c}$	0.45
$s\bar{s}$	0.78
$u\bar{u}$	0.9

What do we do?

- Use it anyway and see what happens. Taking this approach the general features are OK
- Try to relativize it.

Spin dependent interactions:

$$\Delta[M(^3S_1) - M(^1S_0)] = \frac{3\pi\alpha_s}{9m_1m_2}|\psi(0)|^2$$

Approximate $^3S_1$ and $^1S_0$ masses by:

$$M(^3S_1) = M(S) + \frac{1}{4}\frac{a}{m_q m_{\bar{q}}}$$

$$M(^1S_0) = M(S) - \frac{3}{4}\frac{a}{m_q m_{\bar{q}}}$$

If a is approximately constant:

$$\frac{M(\rho) - M(\pi)}{M(K^*) - M(K)} \approx \frac{m_u m_s}{m_u m_u} \approx \frac{m_s}{m_u} \approx \frac{500}{300} \approx 1.7$$

$$\frac{770 - 140}{892 - 495} \approx \frac{630}{400} \approx 1.7$$

Similarly:

$$\frac{M(K^*) - M(K)}{M(D^*) - M(D)} \approx \frac{m_u m_c}{m_u m_s} \approx \frac{m_c}{m_s} \approx \frac{1.6}{0.55} \approx 2.9$$

$$\frac{892 - 494}{2010 - 1870} \approx \frac{400}{140} \approx 2.9$$

So splittings reasonably well described

Because $^3P_{cog}$-$^1P_1$ splitting is small supports short range contact interaction

Electromagnetic transitions:

As before: $$\Gamma_{M1} = \frac{k_\gamma^3}{3\pi} \left| \langle f | i \rangle \right|^2 \left| \sum \mu_i \sigma_{zi} \right|^2$$

For example:

$$K^{*+} \to K^+ \gamma$$

$$\left\langle u\bar{s} \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right| \frac{e_i}{2m_i} \sigma_z \left| u\bar{s} \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right\rangle$$

$$= \tfrac{1}{2} \left\langle u\bar{s} \right| \frac{e_q}{2m_q} + \frac{e_q}{2m_q} - \frac{e_{\bar{q}}}{2m_{\bar{q}}} - \frac{e_{\bar{q}}}{2m_{\bar{q}}} \left| u\bar{s} \right\rangle$$

$$= \frac{1}{2}\left[\frac{e_u}{m_u} - \frac{e_s}{m_s} \right] = \frac{1}{2}\left[\frac{2}{3}\frac{1}{m_u} - \frac{1}{3}\frac{1}{m_s} \right]$$

Strong (Zweig allowed) Decays:

A number of models to calculate strong decays.

Give good qualitative agreement with experiment
with only 1 free parameter (using QM wavefunctions)

Important input to disentangle hadron spectrum

Relativistic effects:

Clearly light quark hadrons are relativistic

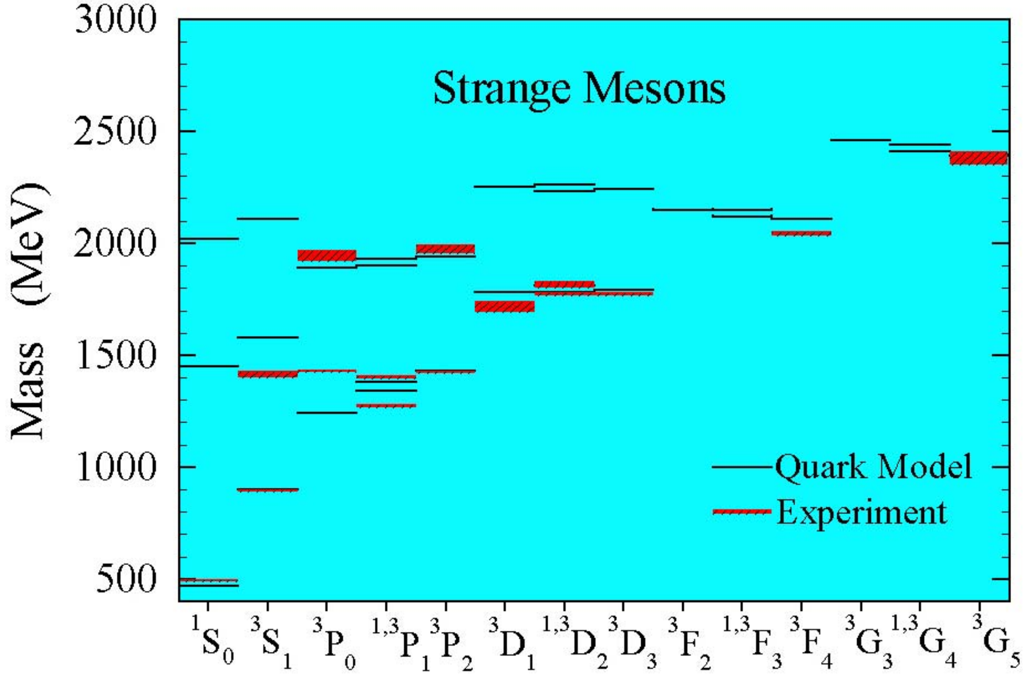
Various attempts to "relativize" QM

Generally improves agreement

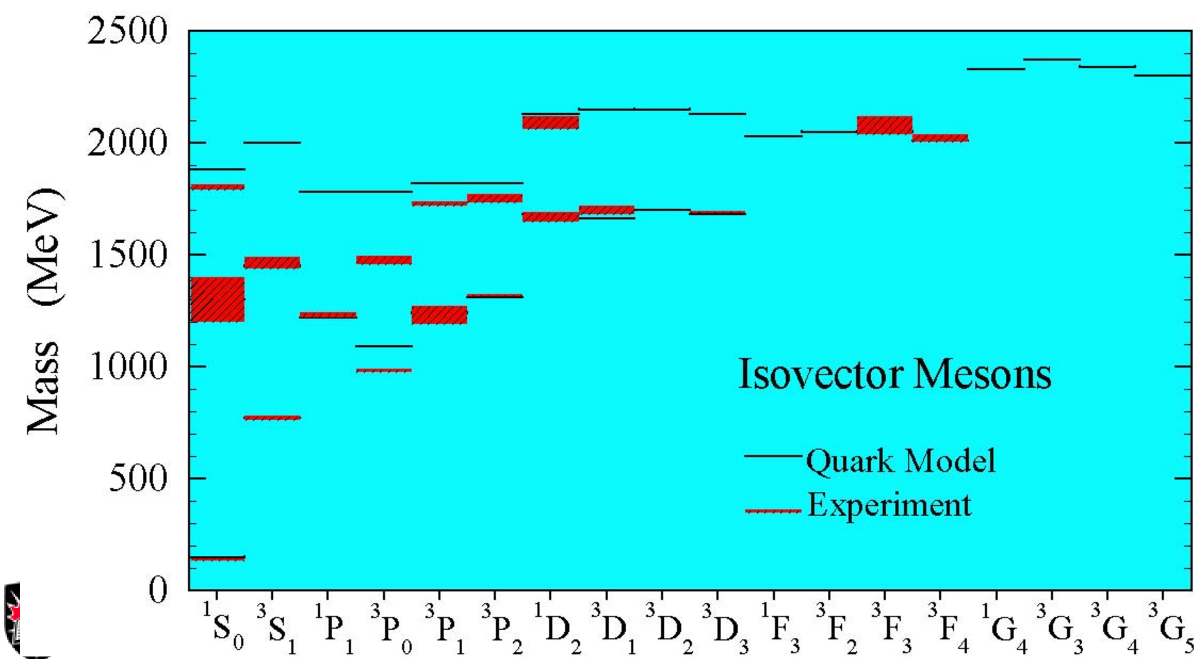
But much is missing. Major battles about what is correct approach.

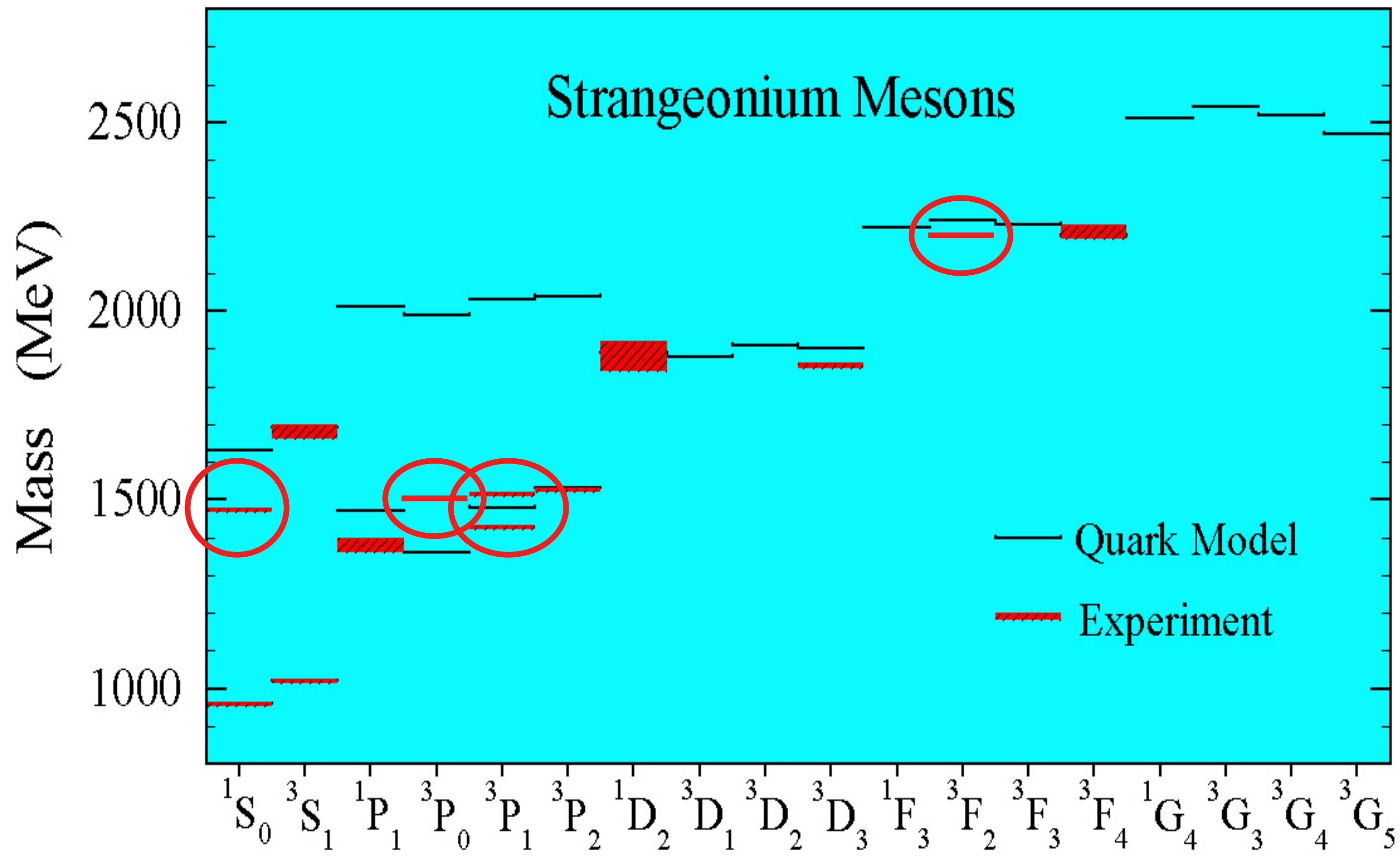
BUT QM seems to get the physics right.

"Better to get the right degrees of freedom"



Generally, good agreement for confirmed states





- Many unconfirmed states:

  $f_1(1530)$, $h_1(1380)$

- Many puzzles:

  $\eta(1440)$, $f_1(1420)$, $f_0(1500)$ $f_J(1710)$, $f_J(2200)$

Baryon Spectroscopy:

Can also describe baryons using QM

But more degrees of freedom so much more complicated to deal with.

Simple exercise to calculate ground state Baryon magnetic moments using M1 operator

Leave this for another time.

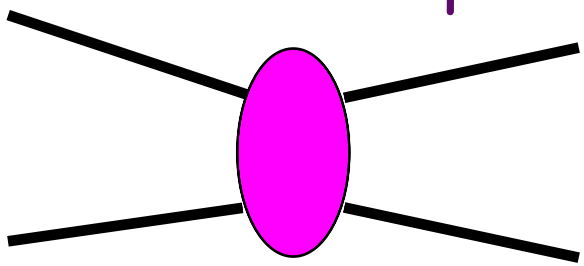
Spin-dependent potentials:

Spin-dependent interactions are $(v/c)^2$ corrections

Lorentz structure of confining potential:

scalar? vector? pseudoscalar? …

M_{if} =



$$= \left[\bar{u}\Gamma_\mu u\right] V(Q^2) \left[\bar{v}\Gamma^\mu v\right]$$

1. Lorentz vector 1-gluon exchange + scalar confinement

2. If the confining interaction couples to the colour charge density so interaction is $\gamma_0 \otimes \gamma_0$

Gives rise to spin-dependent interactions

$$H^{spin-spin}_{vector\ \ conf t.} = +\frac{4b_v}{3m_c^2 r}\,\vec{S}_q \cdot \vec{S}_{\bar{q}}$$

Radiative Transitions:

$$\Gamma = \frac{1}{8\pi M_i^2}\left|M_{if}\right|^2 p$$

E1 transitions:

$$\langle f|H_I|i\rangle = -\frac{ie\omega}{2}\langle f|\vec{r}|i\rangle\cdot\vec{\varepsilon}$$

M1 transitions:

$$M_{if} = i\mu\langle f|\vec{\sigma}|i\rangle\cdot\vec{k}\times\vec{\varepsilon}^{*} = \frac{ie_q}{2m_q}k_\gamma\langle f|\sigma_z|i\rangle$$

where $\mu = \dfrac{e_q}{2m_q}$

(subtleties about how we define wavefunction)

Leptonic Decays:

$$\Gamma = \frac{16\pi\alpha^2 e_q^2}{M_i^2}\left|\psi(0)\right|^2$$

Also have decays via annihilation to photons and gluons