

# 3. Heavy Quarkonia

1. Spectroscopy
2. em decays
3. decays



# 2. The November Revolution:

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PHYSICAL REVIEW LETTERS

2 DECEMBER 1974

## Experimental Observation of a Heavy Particle $J^\dagger$

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorrison, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu  
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and

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 (Received 12 November 1974)

We report the observation of a heavy particle  $J$ , with mass  $m = 3.1$  GeV and width approximately zero. The observation was made from the reaction  $p + Be \rightarrow e^+ + e^- + x$  by measuring the  $e^+e^-$  mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

## Discovery of a Narrow Resonance in $e^+e^-$ Annihilation\*

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 (Received 13 November 1974)

We have observed a very sharp peak in the cross section for  $e^+e^- \rightarrow$  hadrons,  $e^+e^-$ , and possibly  $\mu^+\mu^-$  at a center-of-mass energy of  $3.105 \pm 0.003$  GeV. The upper limit to the full width at half-maximum is 1.3 MeV.

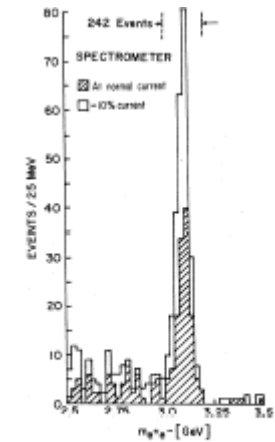


FIG. 2. Mass spectrum showing the existence of  $J$ . Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

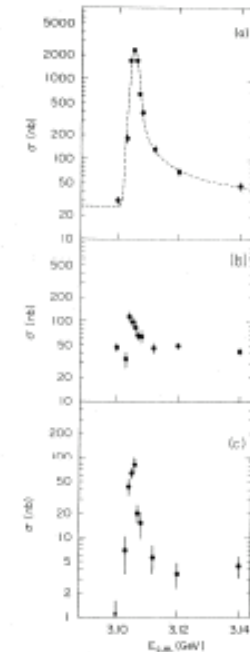


FIG. 1. Cross section versus energy for (a) multi-hadron final states, (b)  $e^+e^-$  final states, and (c)  $\mu^+\mu^-$ ,  $e^+e^-$ , and  $K^+K^-$  final states. The curve in (a) is the expected shape of a  $\delta$ -function resonance folded with the Gaussian energy spread of the beams and including radiative processes. The cross sections shown in (a) and (b) are integrated over the detector acceptance. The total hadron cross section, (c), has been corrected for detection efficiency.

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## Is Bound Charm Found?\*

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and

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(Received 27 November 1974)

We argue that the newly discovered narrow resonance at 3.1 GeV is a  ${}^3S_1$  bound state of charmed quarks and we show the consistency of this interpretation with known meson systematics. The crucial test of this notion is the existence of charmed hadrons near 2 GeV.



## Spectroscopy of the New Mesons\*

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(Received 11 December 1974)

The interpretation of the narrow boson resonances at 3.1 and 3.7 GeV as charmed quark-antiquark bound states implies the existence of other states. Some of these should be copiously produced in the radiative decays of the 3.7-GeV resonance. We estimate the masses and decay rates of these states and emphasize the importance of  $\gamma$ -ray spectroscopy.

Two earlier papers<sup>1,2</sup> present our case that the recently discovered<sup>3,4</sup> and confirmed<sup>5</sup> resonance at 3.106 GeV is the ground state of a charmed quark bound to its antiquark, by colored gauge gluons: orthocharmonium I. More recently, a second state at 3.695 GeV has been reported<sup>6</sup> with an estimated width of 0.5–2.7 MeV and a partial decay rate  $\sim 2$  keV into  $e^+e^-$ . We interpret this state as an S-wave radial excitation, orthocharmonium II, with  $J^P = 1^-$  and  $I^G = 0^-$ . Here are three indications of the correctness of our interpretation: (1) Much of the time, orthocharmonium II decays into orthocharmonium I and two pions. This behavior suggests that orthocharmonium II is an excited state of orthocharmonium I.<sup>7</sup> (2) The leptonic width of orthocharmonium II is about half that of orthocharmonium I, not unexpected for an excited state whose wave function at the origin is smaller. (3) Orthocharmonium II is not seen in the Brookhaven National Laboratory–Massachusetts Institute of Technology experiment.<sup>8</sup> In a thermodynamic model,<sup>9</sup> the production cross section of a hadron of 3.7 GeV is suppressed by  $\sim 10^{-2}$  relative to that of a hadron of 3.1 GeV. Moreover, the leptonic branching ratio of orthocharmonium II is smaller than that of orthocharmonium I by a factor of 10.

We predict the existence of other states of charmonium with masses less than 3.7 GeV, a

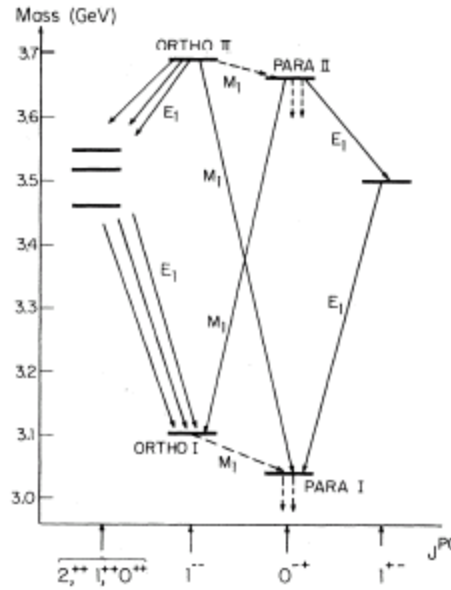


FIG. 1. Masses and radiative transitions of charmonium.

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## Spectrum of Charmed Quark-Antiquark Bound States\*

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(Received 17 December 1974)

The discovery of narrow resonances at 3.1 and 3.7 GeV and their interpretation as charmed quark-antiquark bound states suggest additional narrow states between 3.0 and 4.3 GeV. A model which incorporates quark confinement is used to determine the quantum numbers and estimate masses and decay widths of these states. Their existence should be revealed by  $\gamma$ -ray transitions among them.

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TABLE I.  $\gamma$  ray widths.<sup>a</sup>

Transition	$\Gamma_\gamma$	$\Gamma_\gamma$ (keV)
$2^3S \rightarrow 3P_2$	$5I_4\alpha k^3$	120
$\rightarrow 3P_1$	$3I_4\alpha k^3$	70
$\rightarrow 3P_0$	$I_4\alpha k^3$	25
$3P_2 \rightarrow 1^3S$	$I_4\alpha k^3$	240
$3P_1 \rightarrow 1^3S$	$I_4\alpha k^3$	240
$3P_0 \rightarrow 1^3S$	$I_4\alpha k^3$	240
$3D_1 \rightarrow 3P_2$	$1I_4\alpha k^3$	7
$\rightarrow 3P_1$	$15I_4\alpha k^3$	110
$\rightarrow 3P_0$	$20I_4\alpha k^3$	150
$2^3S \rightarrow 1^1S$	$I_4\alpha k^4$	$\sim 1$

<sup>a</sup>In the second column  $1/\alpha = 137$ ,  $k$  is the energy of the transition, and  $I_4$  is a radial integral. The last column is based on our wave functions and energy differences, with fine-structure splittings and S-D mixing ignored.

P multiplet lies about 230 MeV below that of the 2S levels. This energy difference is not very sensitive to our choice of parameters: It decreases to 160 MeV if  $\alpha_s$  and  $m_c$  assume the unreasonable values of 0.8 and 0.9 GeV, respectively.

(b) The c.o.g. of the lowest D multiplet is 70 MeV above that of the 2S levels. These D levels may therefore lie below the threshold  $M_c$ .

(c) The 3S level lies at  $\sim 4.2$  GeV. As no sharp resonance has been found in this region,<sup>3</sup> this implies that  $M_c < 4.2$  GeV.

(d) The almost inevitable presence of tensor forces has an intriguing consequence: for it enar-

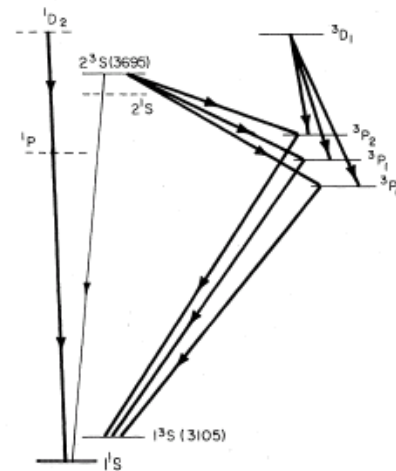


FIG. 1. The spectrum of charmonium. The vertical scale is schematic; our predictions of masses for the P and D levels are given in the text. The  $3D_2$  and  $3D_1$  levels are not shown as their position relative to  $3D_1$  is sensitive to  $2^3S_1$ - $3D_1$  mixing. Heavy lines are allowed E1  $\gamma$  transitions; the  $2^3S_1 \rightarrow 1^3S$  decay is a highly suppressed M1 transition. Dashed levels are unlikely to be produced or fed from above at an  $e^+e^-$  storage ring. Transitions among levels of an LS multiplet are probably unobservable, while  $\gamma$  transitions between states having the same value of  $C = (-1)^{L+S}$  are rigorously forbidden.



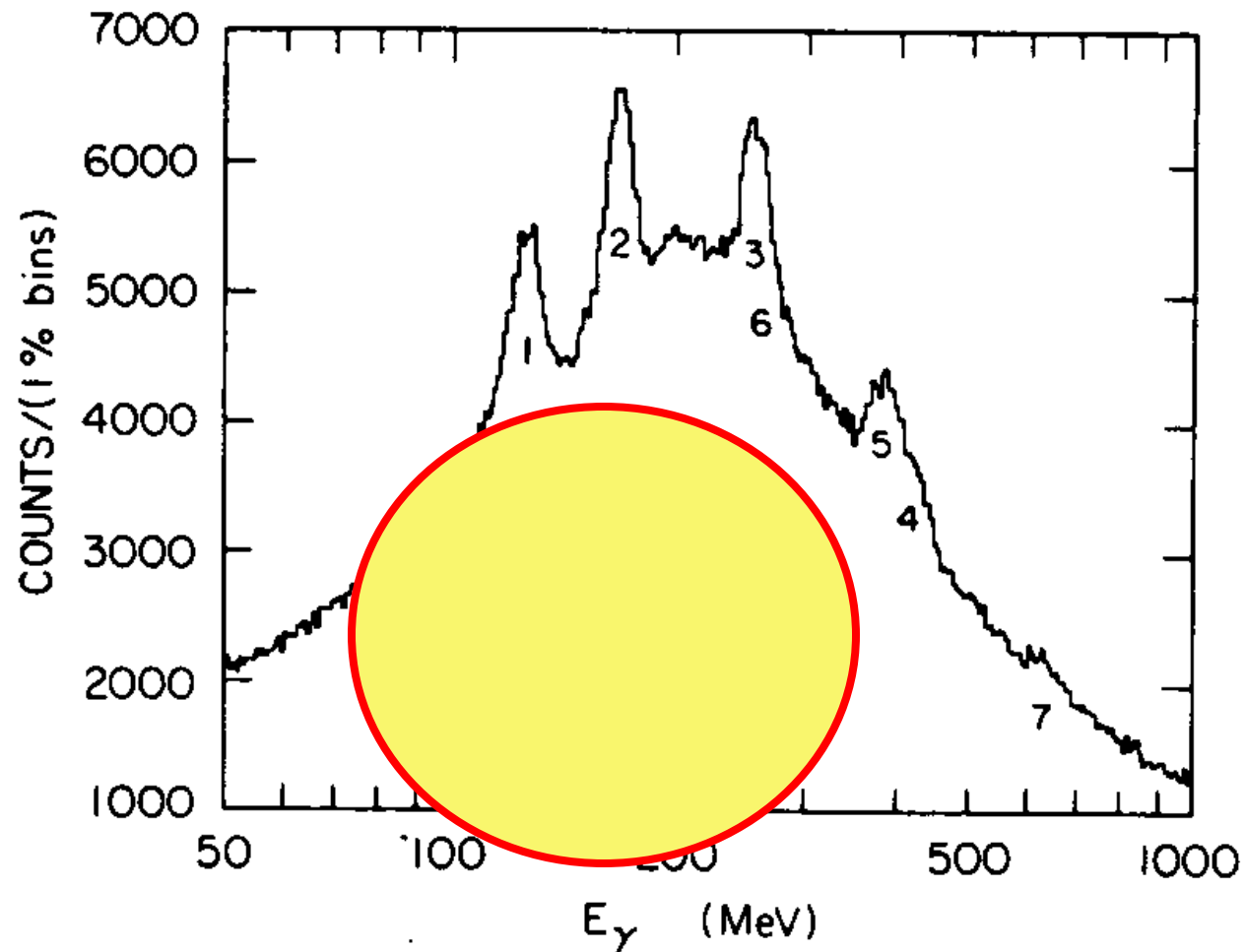
# The Charmonium Spectrum

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PHYSICAL REVIEW LETTERS

6 OCTOBER 1980

Observation of an  $\eta_c$  Candidate State with Mass  $2978 \pm 9$  MeV



S. Godfr

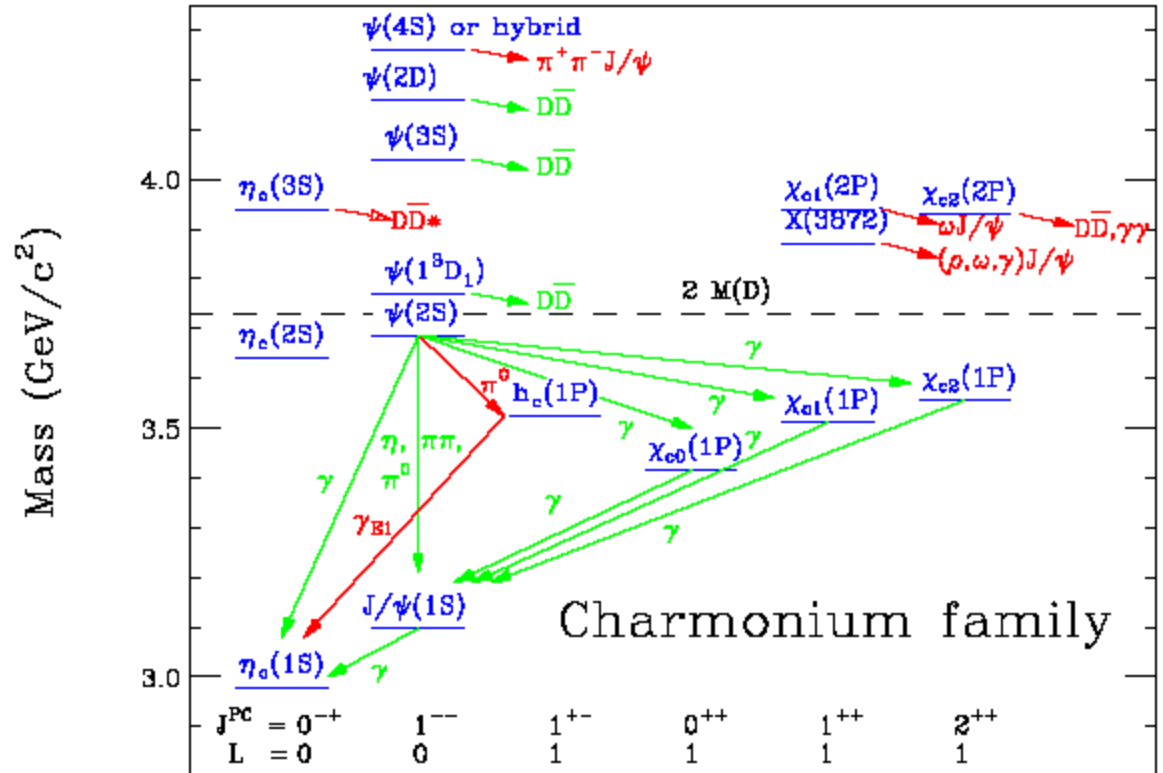


Richter

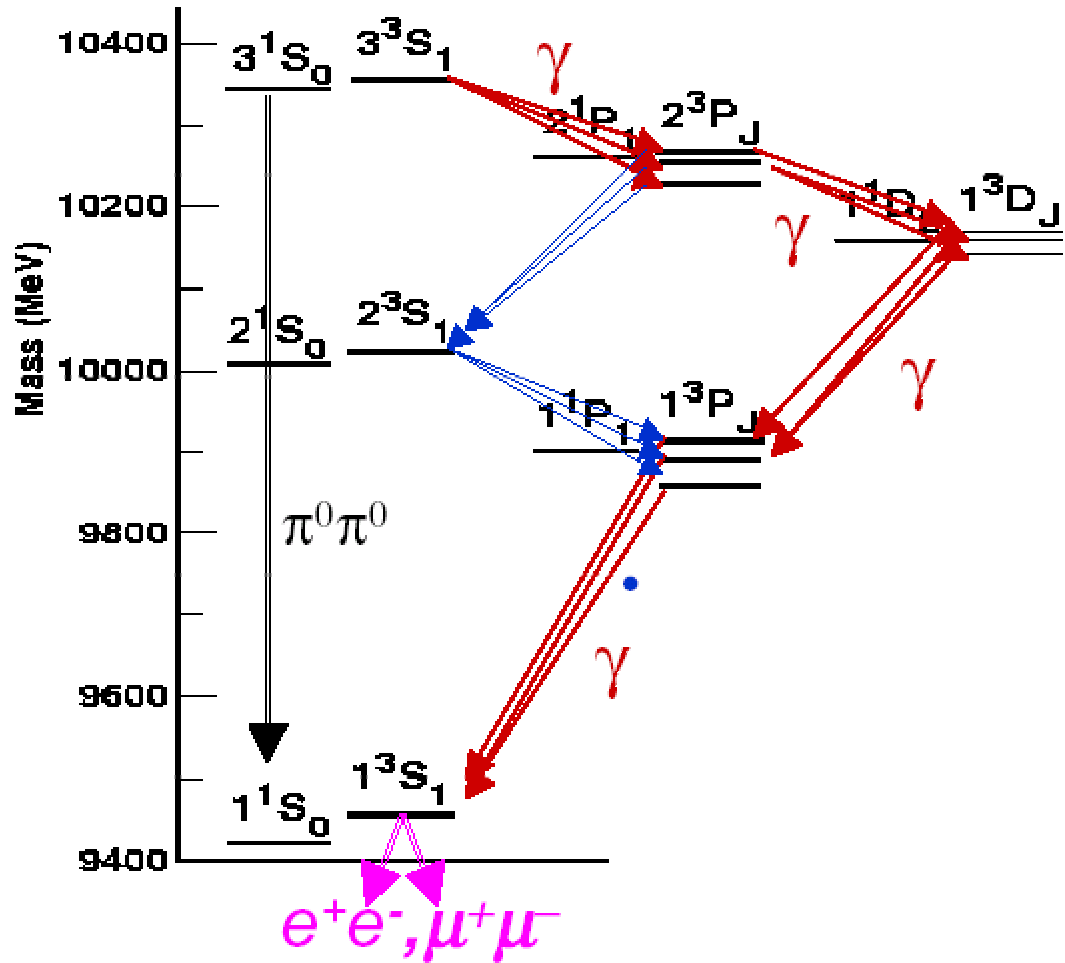
Ting

Spectroscopy convinced us that quarks were real

# "New" Spectroscopy of Mesons



# "New" Spectroscopy of Mesons



# 1. Potential Models:

- Spin independent potentials
- Relativistic corrections
- Spin dependent effects
- Coupled channel effects

## Reviews:

Kwong and Rosner, *Ann. Rev. Nucl. Part. Sci.* 37, 325 (1987)

Buchmuller and Cooper, *Adv. Ser. Direct. High Energy Phys.* 1, 412 (1988)

Konigsmann, *Phys. Rept.* 139, 243 (1986).

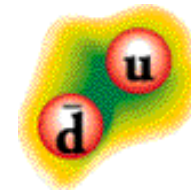
Thomas as has recent review and maybe quigg?





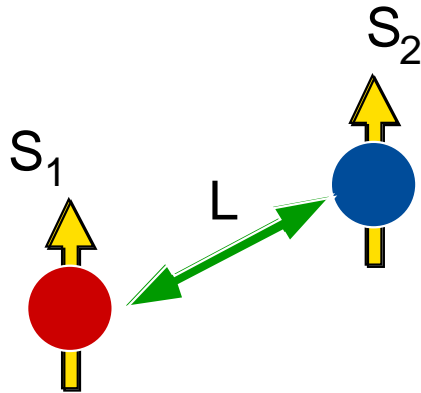
# Mesons are composed of a quark-antiquark pair

Combine u, d, s, c, b quark and antiquark to form various mesons:



$\pi$  meson

Meson quantum numbers characterized by given  $J^{PC}$



$$S = S_1 + S_2$$

$$J = L + S$$

$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

Allowed:

$$J^{PC} = 0^{-+} \quad 1^{--} \quad 1^{+-} \quad 0^{++} \quad 1^{++} \quad 2^{++} \dots$$

Not allowed: exotic combinations:

$$J^{PC} = 0^{--} \quad 0^{+-} \quad 1^{-+} \quad 2^{+-} \dots$$



## 4.1 The Spin-Independent Potential

Previously gave qualitative arguments why the spin-independent potential is linear + Coulomb

$$V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + br \quad b \simeq 0.18 \text{ GeV}^2$$

We also saw how this potential is consistent with results from Lattice QCD

**However**, Historically this form was arrived at through trial and error (Although Appelquist and Politzer got it right in an early paper ~ 1975)

Emperically, the Schrodinger eqn was solved for a given potential which was modified until agreement was achieved between theory and experiment.



$$M = m_1 + m_2 + E_{nl}$$

$$\left[ \frac{p^2}{2\mu} + V(r) \right] \psi = E_{nl} \psi \quad \left( \mu = \frac{m_1 m_2}{m_1 + m_2} \right)$$

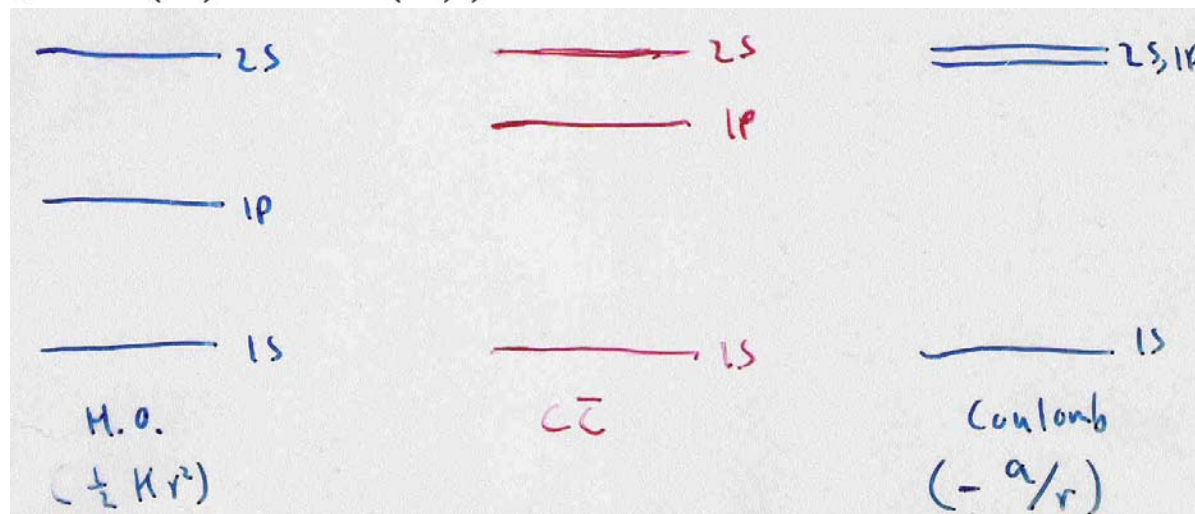
$$\left[ \frac{\hbar^2}{2\mu} \nabla^2 + V(r) \right] \psi = E_{nl} \psi$$

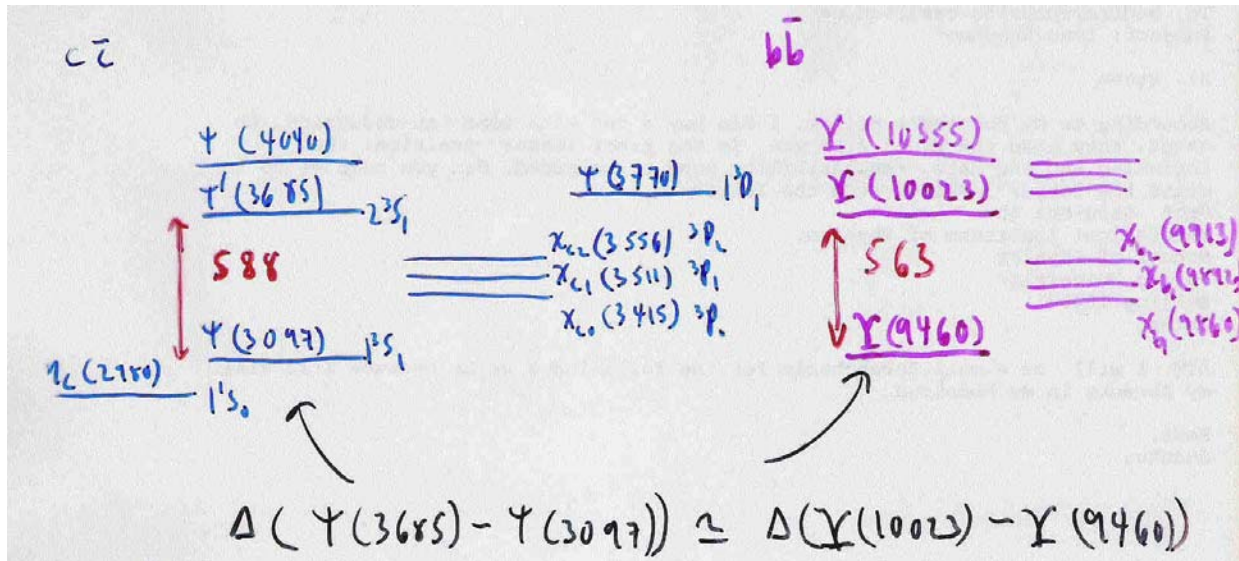
$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

$$\psi(r, \theta, \phi) = R(r) Y_{\ell m}(\theta, \phi) \quad U(r) \equiv r R(r)$$

$$\frac{\hbar^2}{2\mu} \frac{d^2 U}{dr^2} + \left[ V(r) + \frac{\hbar^2}{2\mu} \frac{\ell(\ell+1)}{r^2} \right] U = E_{nl} U$$

$$(U(0) = 0, U'(0) = R(0))$$



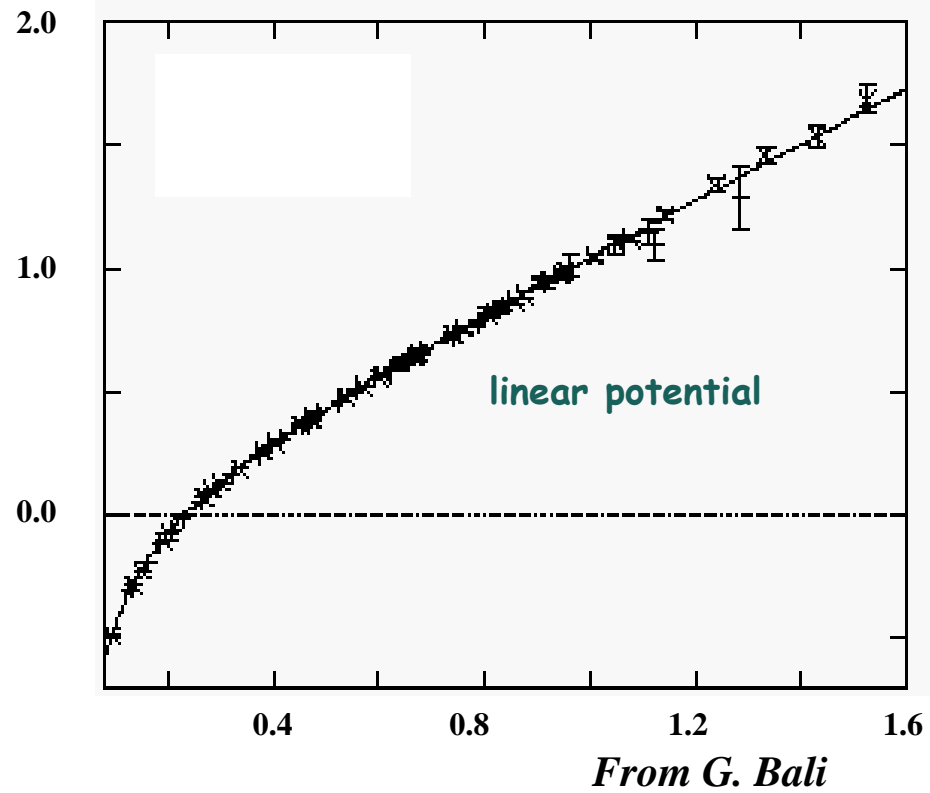


$$\Rightarrow \begin{cases} V(r) = \lambda r^\nu & r \simeq 0.1 \\ V(r) = c \ln(r/r_0) & c \simeq 0.73 \end{cases}$$

Also  $V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + br$  for suitable  $\alpha_s, b$



# Lattice QCD gives qq potential:



# Quark-antiquark Potential

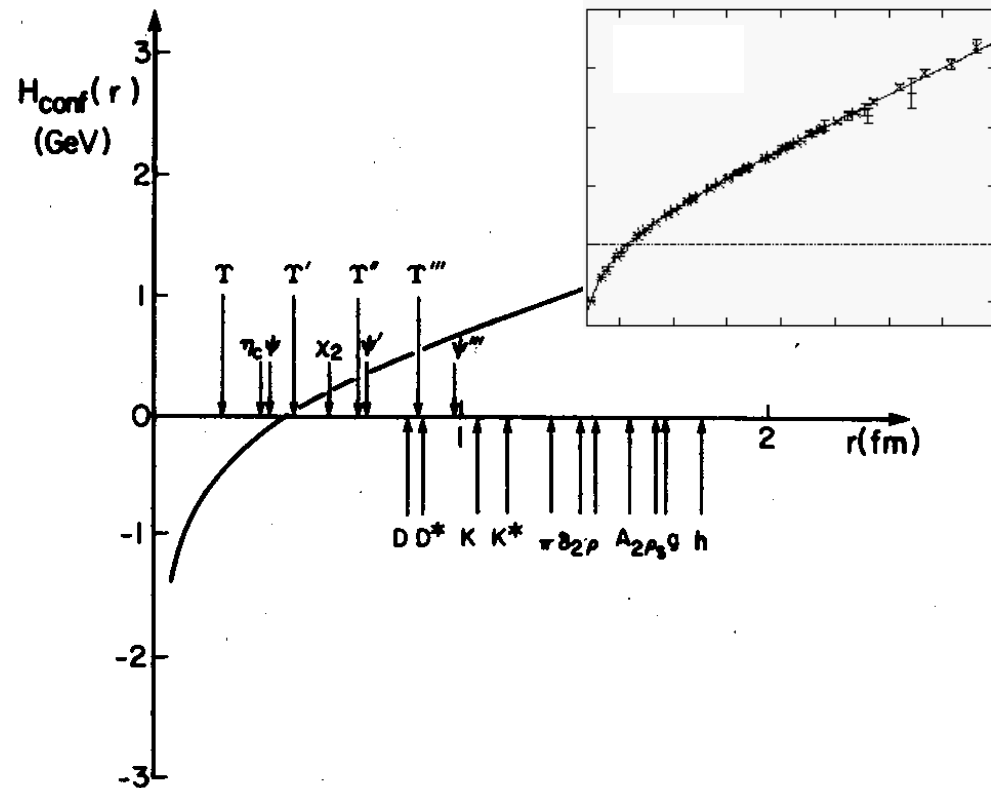
For given spin and orbital angular momentum configurations & radial excitations generate our known spectrum of light quark mesons

$$H_{ij}^{\text{conf}} = -\frac{4}{3} \frac{\alpha_s(r)}{r} + br$$

$$M = m_1 + m_2 + E_{nl}$$

$$\left[ \frac{p^2}{2\mu} + V(r) \right] \psi = E_{nl} \psi$$

Solve Schrodinger eqn  
for meson masses



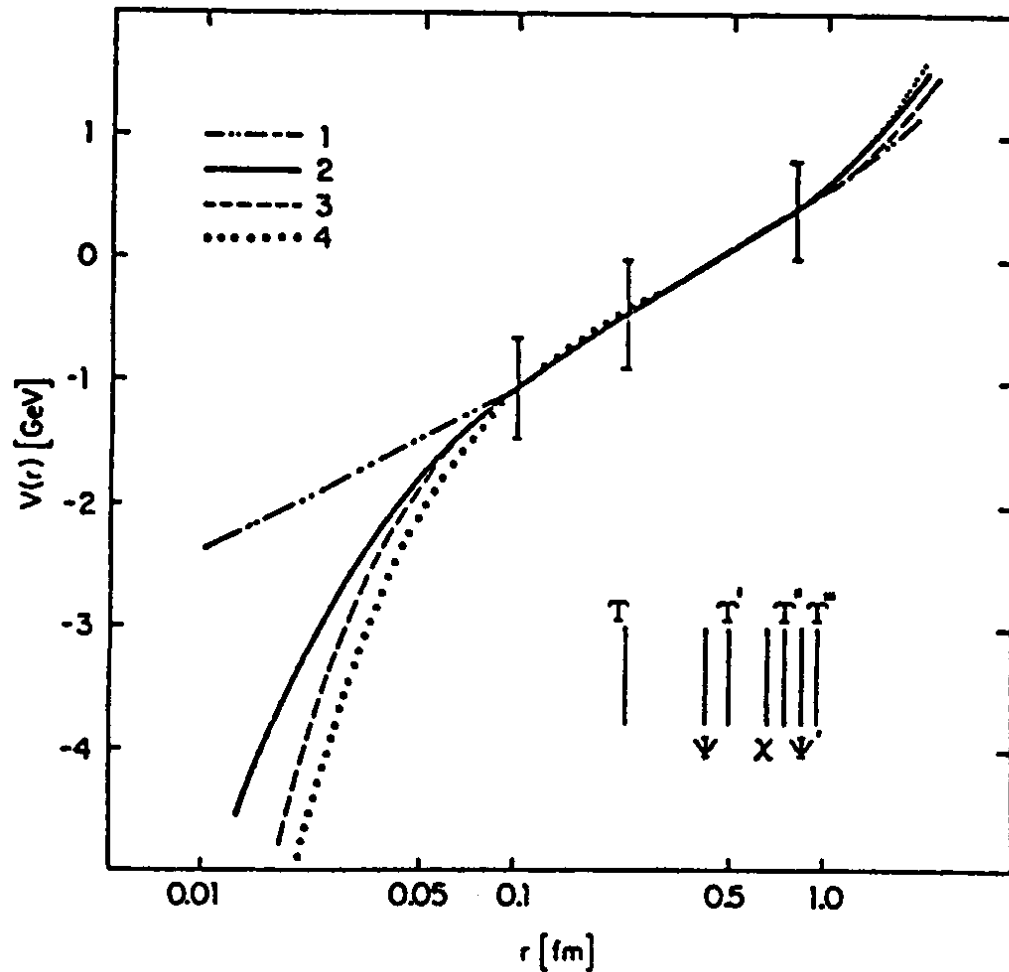


Figure 21: Various  $Q\bar{Q}$  potentials. The potentials have been shifted to agree at  $r=0.5$  fm. The numbers refer to the following references: 1: Martin [101], 2: Buchmüller, Grunberg and Tye [99], 3: Bhanot and Rudaz [102], 4: Cornell group [97].

From Buchmuller & Tye  
PR D24, 132 (1981)

Quark potential models are strongly supported by emperical agreement with quarkonium spectroscopy and with lattice QCD



Could also use position of P-waves

Spin averaged  ${}^3P_J$  gives

$$\bar{M} = (5M_{3P_2} + 3M_{3P_1} + M_{3P_0})/9$$

For  $c\bar{c}$   $\bar{M} = 3522$  MeV

$$\frac{M(2S) - M(1P)}{M(2S) - M(1S)} \begin{cases} 1/2 & \text{H.O.}(\nu = 2) \\ \simeq 1/4 & \text{for } \nu = 0 \\ 0 & \text{Coulomb}(\nu = -1) \end{cases}$$

$$c\bar{c} \Rightarrow \nu \simeq 0.15$$





# Spin-dependent potentials:

Generally expect spin-dependent Interactions:

$$\vec{S}_1 \cdot \vec{S}_2 \quad \vec{L} \cdot \vec{S} \quad S_{12}$$

Start by looking at spin-dependent interactions of QED in hydrogen atom

**Spin-Orbit:** electron sees the proton circling around

• The orbital motion creates a magnetic field at the centre:

$$B = \frac{ev}{cr^2}$$

• In terms of  $L = mvr$

$$\vec{B} = \frac{e}{mcr^3} \vec{L}$$

• The spinning electron constitutes a magnetic dipole

$$\vec{\mu} = -\frac{e}{mc} \vec{S}$$

• The interaction energy is

$$W = -\vec{\mu} \cdot \vec{B}$$



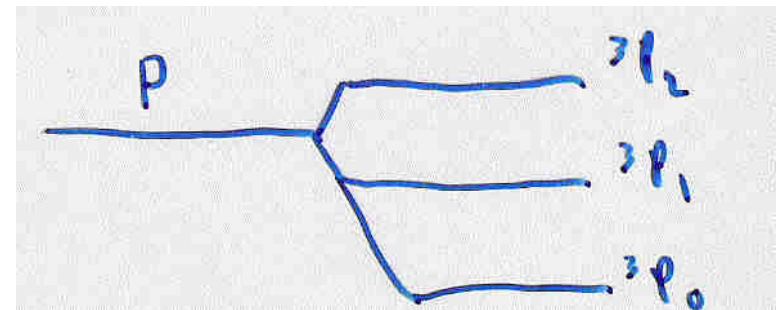
More rigorously (derived as a succession of infinitesimal Lorentz transformations) leads to the Thomas precession with a factor of 1/2

$$\Delta H_{S.O.} = \frac{e^2}{2m^2 c^2 r^3} \vec{L} \cdot \vec{S}$$

$$\vec{J}^2 = \vec{L}^2 + \vec{S}^2 + 2\vec{L} \cdot \vec{S}$$

$$\begin{aligned} \Rightarrow \vec{L} \cdot \vec{S} &= \frac{1}{2}[\vec{J}^2 - \vec{L}^2 - \vec{S}^2] \\ &= \frac{1}{2}[J(J+1) - L(L+1) - S(S+1)] \end{aligned}$$

$$\text{For } \begin{cases} {}^3P_2 & \vec{L} \cdot \vec{S} = 1 \\ {}^3P_1 & \vec{L} \cdot \vec{S} = -1 \\ {}^3P_0 & \vec{L} \cdot \vec{S} = -2 \end{cases}$$

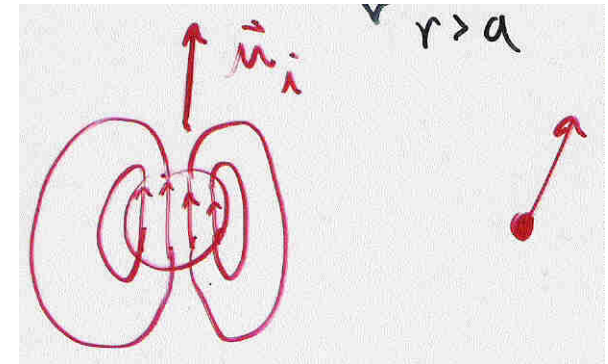


**Hyperfine:** Again in hydrogen, the proton has dipole moment:

$$\vec{\mu}_P = \gamma_P \frac{e}{m_P c} \vec{S}_P \quad (\gamma_P = 2.73)$$

The magnetic dipole has a field:

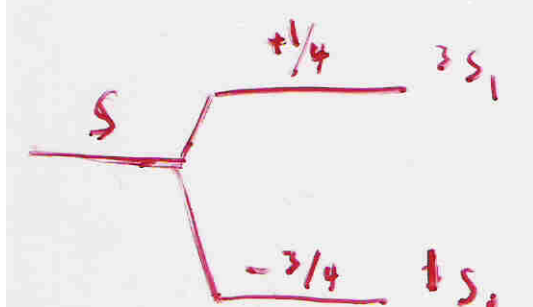
$$\vec{B}(\vec{r}) = \underbrace{\frac{1}{r^3} \left[ \frac{3(\vec{\mu} \cdot \vec{r})\vec{r}}{r^2} - \vec{\mu} \right]}_{r > a} + \underbrace{\frac{8\pi}{3} \vec{\mu}}_{r < a}$$



The energy of the electron in the presence of  $\mu_i$

$$\Delta H_{SS} = \frac{\gamma_P e^2}{m m_P c^2} \left\{ \frac{1}{r^3} [3(\vec{S}_P \cdot \hat{r})(\vec{S}_e \cdot \hat{r}) - \vec{S}_P \cdot \vec{S}_e] + \frac{8\pi}{3} \vec{S}_P \cdot \vec{S}_e \delta^3(\vec{r}) \right\}$$

Gives rise to the hyperfine structure of hydrogen



$$\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} [\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2] = \frac{1}{2} [s(s+1) - \frac{3}{2}]$$

21 cm line in hydrogen



One can take this over to 1-gluon interaction of QCD:

$$\Delta H_{ij}^{hyp} = -\frac{\alpha_s(r)}{m_i m_j} \left\{ \frac{8\pi}{3} \vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} [3(\vec{S}_i \cdot \hat{r}_{ij})(\vec{S}_j \cdot \hat{r}_{ij}) - \vec{S}_i \cdot \vec{S}_j] \right\} \vec{F}_i \cdot \vec{F}_j$$

$$\Delta H_{ij}^{S.O.(c.m.)} = -\frac{\alpha_s(r)}{r_{ij}^3} \left( \frac{1}{m_i} + \frac{1}{m_j} \right) \left( \frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right) \cdot \vec{L} \vec{F}_i \cdot \vec{F}_j$$

$$\Delta H_{ij}^{S.O.(TP)} = -\frac{1}{2r_{ij}} \frac{\partial V(r)}{\partial r_{ij}} \left( \frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2} \right) \cdot \vec{L} \vec{F}_i \cdot \vec{F}_j$$

For mesons  $\langle \vec{F}_i \cdot \vec{F}_j \rangle = -\frac{4}{3}$



## Systematic treatment starts with Wilson loop

Eichten and Feinberg, PR D23, 2724 (1981)

Gromes, Yukon Advanced Study Inst.

- Expanding in  $1/m_Q$  write spin-dependent Hamiltonian in terms of static potential and correlation functions of colour electric and magnetic fields
- With some assumptions one obtains:

$$V_{spin}(r) = \frac{1}{m^2} \left( \frac{-k}{2r} + \frac{2\alpha_s}{3r^3} \right) \vec{L} \cdot \vec{S} \\ + \frac{1}{m^2} \frac{4\alpha_s}{3r^3} S_{12} + \frac{1}{m^2} \frac{32\pi\alpha_s}{9} \delta^3(\vec{r}) \vec{S}_1 \cdot \vec{S}_2$$

Which corresponds to short range vector and long range scalar exchange

Observation of  $^1P_1$  states is important test



## Spin-dependent potentials:

- Need some sort of reduction to find spin dependent terms
- Depends on Lorentz nature of potential

we find phenomenologically

short range Lorentz Vector 1-gluon exchange

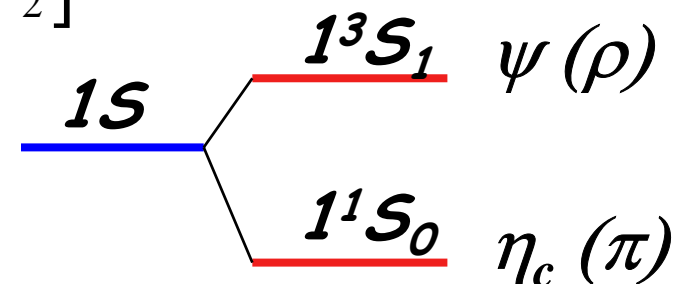
+ long range Lorentz scalar confining potential

- Use Breit-Fermi Hamiltonian
- Spin-dependent interactions are  $(v/c)^2$  corrections

### Spin-spin interactions:

$$H_{ij}^{hyp} = \frac{4\alpha_s(r)}{3m_i m_j} \left\{ \frac{8\pi}{3} \vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} \left[ \frac{3\vec{S}_i \cdot \vec{r}_{ij} \vec{S}_j \cdot \vec{r}_{ij}}{r_{ij}^2} - \vec{S}_i \cdot \vec{S}_j \right] \right\}$$

$$\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} [S^2 - S_1^2 - S_2^2] = \frac{1}{2} [s(s+1) - \frac{3}{2}]$$



## Spin-orbit interactions:

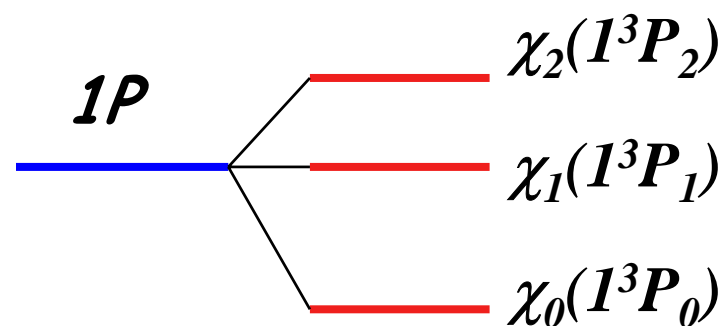
$$H_{ij}^{s.o.(cm)} = \frac{4\alpha_s(r)}{3r_{ij}^3} \left( \frac{1}{m_i} + \frac{1}{m_j} \right) \left( \frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right) \cdot \vec{L}$$

$$H_{ij}^{s.o.(tp)} = \frac{-1}{2r_{ij}} \frac{\partial V(r)}{\partial r_{ij}} \left( \frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2} \right) \cdot \vec{L}$$

$${}^3P_2: \vec{L} \cdot \vec{S} = 1$$

$${}^3P_1: \vec{L} \cdot \vec{S} = -1$$

$${}^3P_0: \vec{L} \cdot \vec{S} = -2$$



Let us examine the spin-dependent splittings in charmonium

- Using H.O. wavefunctions simplifies the calculations
- Fitting the oscillator parameter to the r.m.s. radii of exact solutions is a good approximation:

$$\psi_{1S} = \frac{2}{\pi^{1/4}} \beta^{3/2} e^{-\beta^2 r^2/2} Y_{00} \quad \langle r^2 \rangle_{1S} = \frac{3}{2} \frac{1}{\beta^2} = 2.5 \Rightarrow \beta = 0.77$$

$$\psi_{2S} = \sqrt{\frac{8}{3}} \frac{\beta^{3/2}}{\pi^{1/4}} \left( \frac{3}{2} - \beta^2 r^2 \right) e^{-\beta^2 r^2/2} Y_{00} \quad \langle r^2 \rangle_{2S} = \frac{7}{2} \frac{1}{\beta^2} = 11 \Rightarrow \beta = 0.564$$

$$\psi_{1P} = \sqrt{\frac{8}{3}} \frac{\beta^{5/2} r}{\pi^{1/4}} e^{-\beta^2 r^2/2} Y_{1m} \quad \langle r^2 \rangle_{1P} = \frac{5}{2} \frac{1}{\beta^2} \simeq 7 \Rightarrow \beta = 0.598$$

$$\langle 1/r \rangle_{1P} = \frac{4}{3} \frac{\beta}{\pi^{1/2}} = 0.45$$

$$\langle 1/r^3 \rangle_{1P} = \frac{4}{3} \frac{\beta^3}{\pi^{1/2}} = 0.16$$





# Hyperfine Effects:

$$H_{ij}^{hyp} = \frac{32\pi}{9} \frac{\alpha_s}{m^2} \vec{S}_1 \cdot \vec{S}_2 \delta^3(r_{ij})$$
$$\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}[s(s+1) - 3/2]$$
$$\Rightarrow \begin{cases} \langle {}^3S_1 | \vec{S}_1 \cdot \vec{S}_2 | {}^3S_1 \rangle = +1/4 \\ \langle {}^1S_0 | \vec{S}_1 \cdot \vec{S}_2 | {}^1S_0 \rangle = -3/4 \end{cases}$$

$$\begin{aligned} \therefore M({}^3S_1) - M({}^1S_0) &= \frac{32\pi}{9} \frac{\alpha_s}{m^2} \langle \delta^3(r_{ij}) \rangle \\ &= \frac{32\pi}{9} \frac{\alpha_s}{m^2} |\psi(0)|^2 \\ &= \frac{32\pi}{9} \frac{\alpha_s}{m^2} \frac{\beta^3}{\pi^{3/2}} \\ &= 115 \text{ MeV (where } \beta = 0.77 \text{ GeV, } \alpha_s = 0.32, m_c = 1.6 \text{ GeV)} \\ &\text{vs } 115 \text{ MeV from experiment} \end{aligned} \tag{1}$$

$$M(2^3S_1) - M(2^1S_0) = 67 \text{ MeV}$$



## Fine Structure:

We can write the  ${}^3P_J$  Masses as:

$$M = M(1P) + a\langle \vec{L} \cdot \vec{S} \rangle + b\langle S_{12} \rangle$$

$$M({}^3P_2) = M(1P) + a - \frac{2}{5}b = 3556$$

$$M({}^3P_1) = M(1P) - a - 2b = 3511$$

$$M({}^3P_0) = M(1P) - 2 - 4b = 3415$$

$$M(1P) = 3525$$

Lorentz Vector 1-gluon exchange gives:

$$a = \frac{3}{2m^2} \frac{4}{3} \frac{\alpha_s}{r^3} = 40 \text{ MeV}$$

$$b = \frac{1}{4m^2} \frac{4}{3} \frac{\alpha_s}{r^3} = 7 \text{ MeV}$$



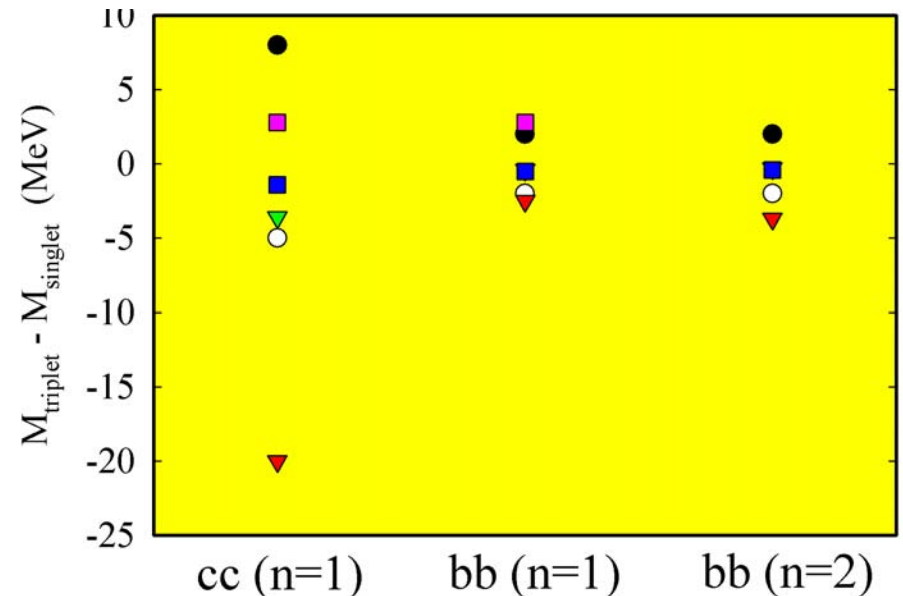
# $^1P_1$ vs $^3P_{\text{cog}}$ mass - distinguish models

- Important to distinguish models

- In QM triplet-singlet splittings test

- the Lorentz nature of the confining potential
- Relativistic effects

- important validation of
  - lattice QCD calculations
  - NRQCD calculations



Observation of  $^1P_1$  states is an important test of theory



# Wide variation of theoretical predictions:

TABLE I. Predictions for hyperfine splittings  $M(n^3P_{\text{cog}}) - M(n^1P_1)$  for  $c\bar{c}$  and  $b\bar{b}$  levels.

Reference	Approach	$n=1$ $c\bar{c}$ (MeV)	$n=1$ $b\bar{b}$ (MeV)	$n=2$ $b\bar{b}$ (MeV)
QM	GI85 [14]	8	2	2
	MR83 [15]	0	0	1
	LPR92 [16]	4	2	1
	OS82 [17]	10	3	3
QM	MB83 [18]	-5	-2	-2
	GRR86 [19]	-2	-1	-1
	IO87 [20]	$24.1 \pm 2.5$	$3.73 \pm 0.1$	$3.51 \pm 0.02$
	GOS84 $\eta_s=1$ [21]	6	3	2
	GOS84 $\eta_s=0$ [21]	17	8	6
QM	PJF92 [22]	$-20.3 \pm 3.7$	$-2.5 \pm 1.6$	$-3.7 \pm 0.8$
	HOOS92 [23]	$-0.7 \pm 0.2$	$-0.18 \pm 0.03$	$-0.15 \pm 0.03$
PQCD	PTN86 [25]	-3.6	-0.4	-0.3
	PT88 [26]	-1.4	-0.5	-0.4
lattice	SESAM98 [31]	-	$\sim -1$	-
	CP-PACS00 [33]	1.7-4.0	1.6-5.0	-
EFG		0	-1	-1



## Quark Potential Models with 1-gluon exchange:

$$H_{q\bar{q}}^{hyp} = \frac{32\pi}{9} \frac{\alpha_s}{m_q m_{\bar{q}}} \vec{S}_q \cdot \vec{S}_{\bar{q}} \delta^3(\vec{r})$$

$\delta$  function is short range but smeared by relativistic effects modeled by a Gaussian.

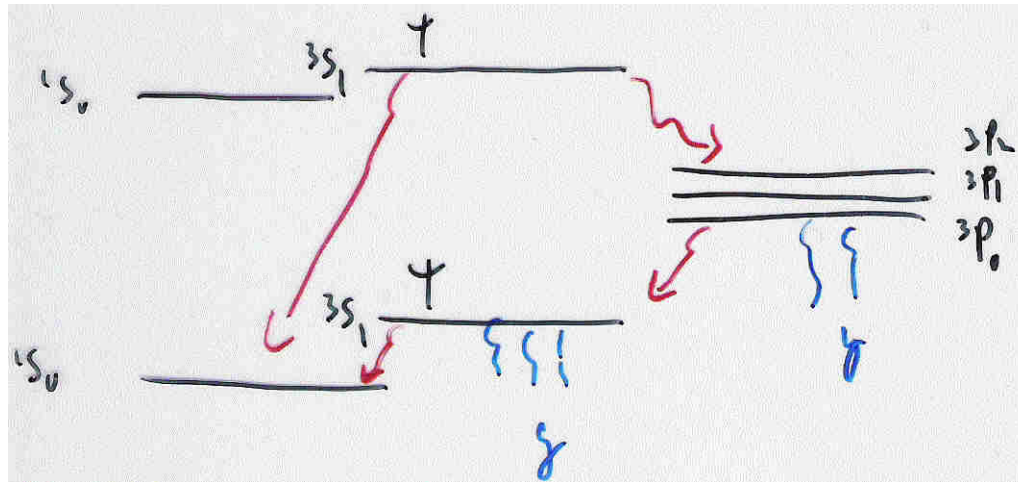
• gives  $M(^3P_{cog}) > M(^1P_1)$

Godfrey & Isgur, PR D32, 189 (1985)

wide variation in predictions indicates need for experimental data



# Decays and Transitions



- To calculate Decays and Transitions we need to calculate hadronic matrix elements.
- Define a "Mock" meson which we equate with the wavefunction of the physical meson

$$|M(\vec{K})\rangle = \sqrt{2E_M} \int d^3p \Phi(\vec{p}) \chi_{s\bar{s}} \phi_{q\bar{q}} \phi_{colour} |q(\frac{m_q}{m_q + m_{\bar{q}}} \vec{K} + p, s) \bar{q}(\frac{m_{\bar{q}}}{m_q + m_{\bar{q}}} \vec{K} - p, \bar{s})\rangle$$



There are two generic types of matrix elements:

$$\langle 0|A|M_i\rangle \text{ like in } J/\psi \rightarrow e^+ e^-$$

$$\langle M_f|A|M_i\rangle \text{ like in } \chi_{c2} \rightarrow J/\psi + \gamma$$

A is some sort of transition operator like:

$$j_{em}^\mu = \bar{q}\gamma^\mu q$$



# Crystal Ball

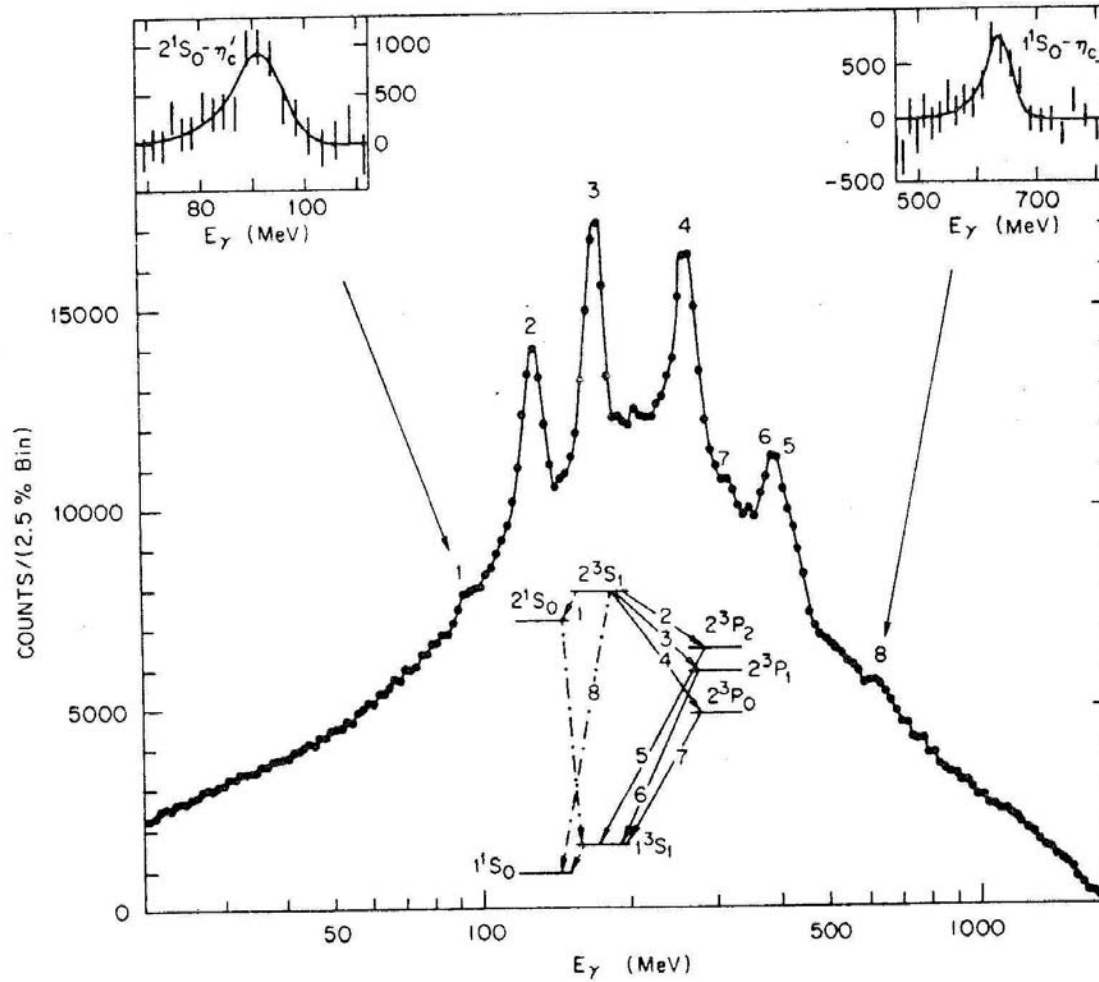


Fig. 5. Inclusive photon spectrum at the  $\psi'$  obtained by the Crystal Ball experiment. Note that the logarithmic energy scale yields bin sizes approximately proportional to photon energy resolution. The numbers over the spectrum correspond to the expected radiative transitions shown in the spectrum inset





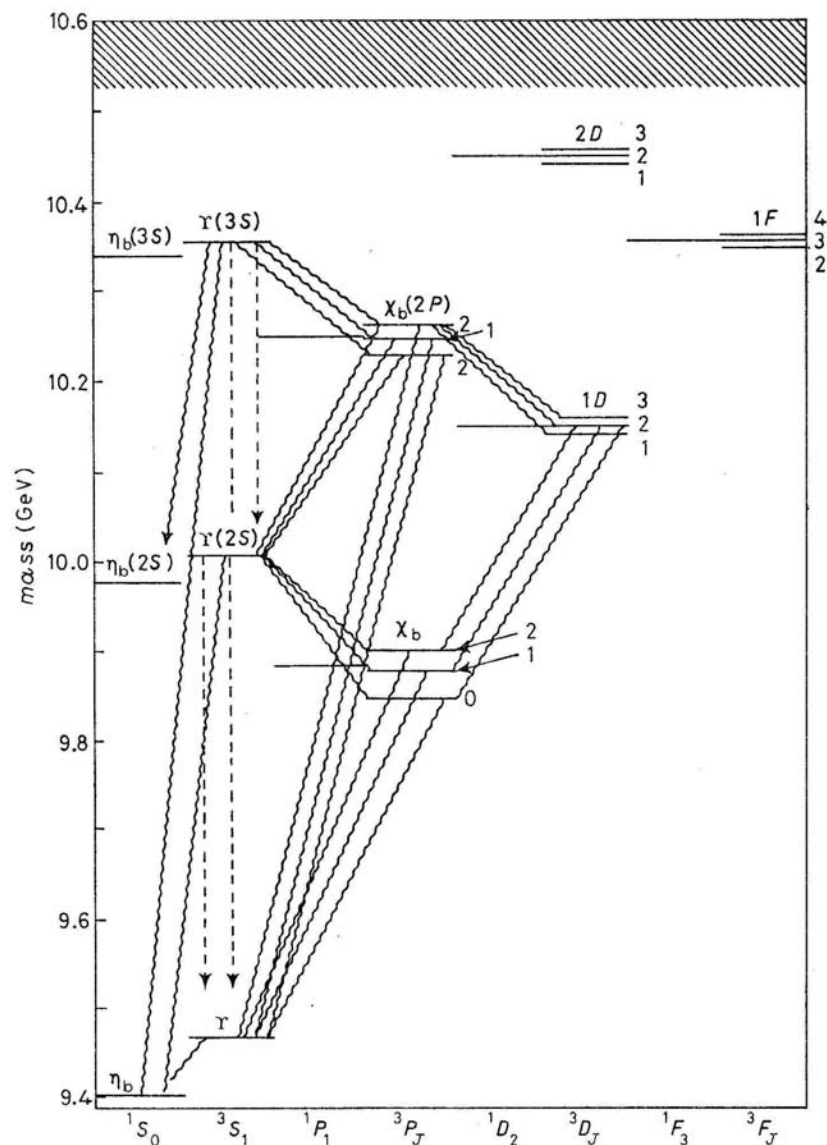


Fig. 4. - The  $b\bar{b}$  level diagram showing transitions between states. We use the familiar spectroscopic notation  $n^{2S+1}L_J$ , where  $n$  is the principal quantum number (with the convention that  $n$  is one plus the number of nodes in the wavefunction), and  $L$ ,  $S$ , and  $J$  are the orbital angular momentum, total spin, and total angular momentum. The parity and  $C$ -parity are given by  $P = (-)^{L+1}$  and  $C = (-)^{L+S}$ . Note that not all states and transitions shown have been observed and not all possible transitions are shown.  $\sim$   $\gamma$ -transitions,  $---$  hadronic transitions.



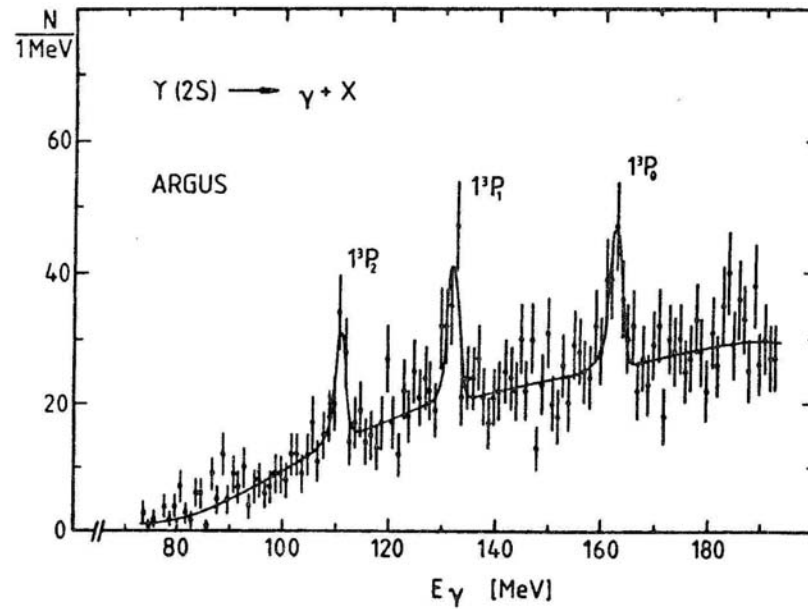


Figure 11: ARGUS [74]  $\Upsilon(2S) \rightarrow \gamma + \text{hadrons}$  with  $\gamma \rightarrow e^+e^-$ .

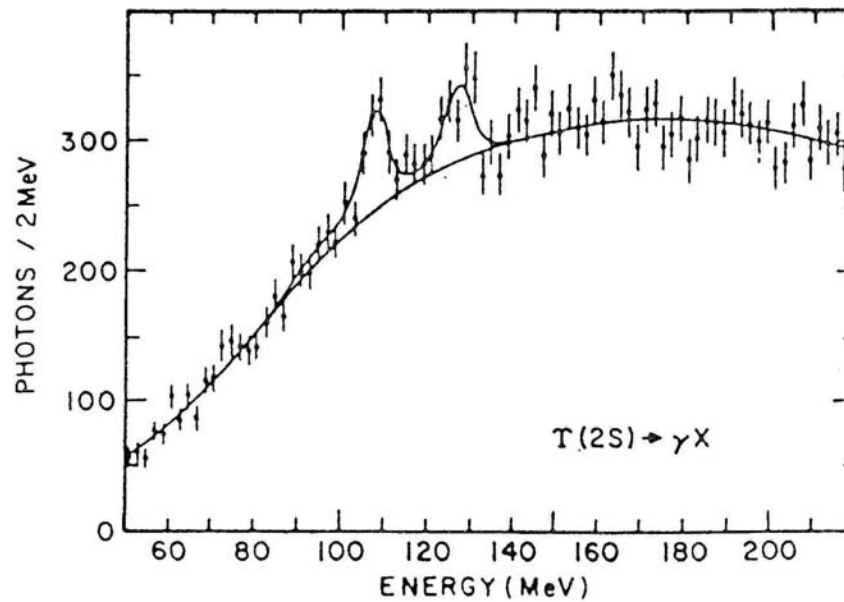


Figure 12: CLEO [75]  $\Upsilon(2S) \rightarrow \gamma + \text{hadrons}$  with  $\gamma \rightarrow e^+e^-$ . The data do not represent the  $\Upsilon(2S) \rightarrow \gamma + \text{hadrons}$  line, and it is not included in the fit shown.



Photon Transitions in  $Y(2S)$  and  $Y(3S)$  Decays

(CLEO Collaboration)

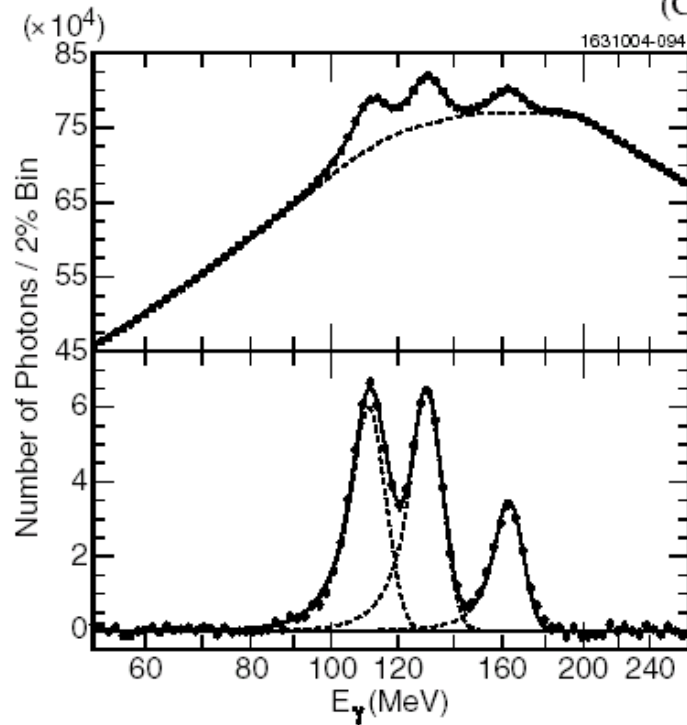


FIG. 2. Fit to the  $Y(2S) \rightarrow \gamma\chi_{bJ}(1P)$  ( $J = 2, 1, 0$ ) photon lines in the data. The points represent the data (top plot). Statistical errors on the data are smaller than the point size. The solid line represents the fit. The dashed line represents total fitted background. The background subtracted data (points with error bars) are shown at the bottom. The solid line represents the fitted photon lines together. The dashed lines show individual photon lines.

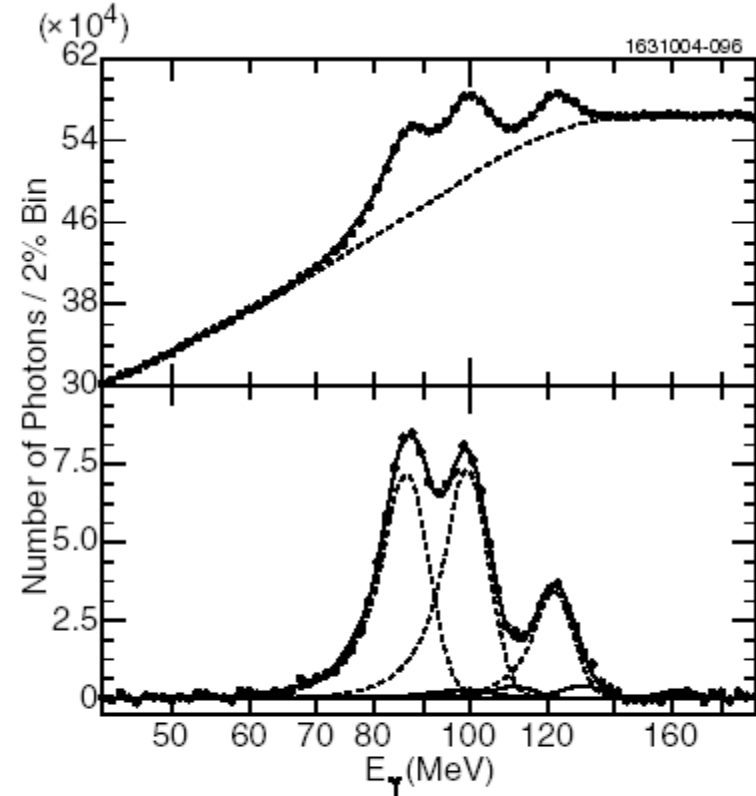
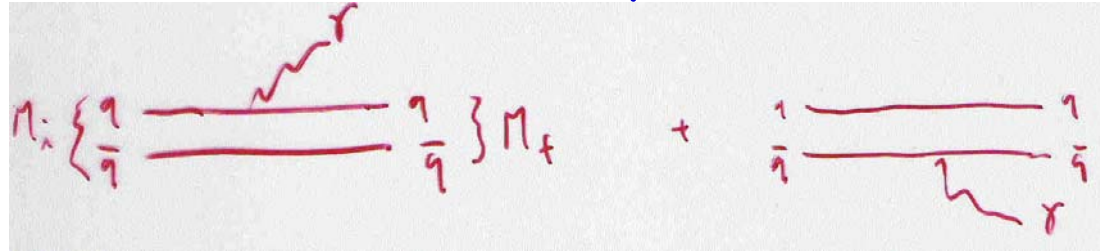


FIG. 3. Fit to the  $Y(3S) \rightarrow \gamma\chi_{bJ}(2P)$  ( $J = 2, 1, 0$ ) photon lines in the data. See caption of Fig. 2 for the description. Small solid line peaks in the bottom plot show the  $\chi_{bJ}(2P) \rightarrow \gamma Y(1D)$  and  $Y(2S) \rightarrow \gamma\chi_{bJ}(1P)$  contributions.



# Radiative (e.m.) Transitions

Same physics as in atomic and nuclear systems  
 An e.m. transition is described by:



For 2 body decay  $M_i \rightarrow M_f \gamma$

$$d\Gamma = \frac{(2\pi)^4 \delta^4(P_f + p_\gamma - p_i)}{2M_i} |M_{fi}|^2 \frac{d^3 p_f}{(2\pi)^3 (2E_f)} \frac{d^3 p_\gamma}{(2\pi)^3 (2E_\gamma)}$$

$$\Gamma = \frac{1}{2\pi M^2} |M_{if}|^2 p$$

$$\text{where } p = \frac{(M_i^2 - M_f^2)}{2M_i}$$

$$= \frac{|M_{if}|^2}{8\pi M} (1 - M_f^2/M_i^2)$$

$$\frac{d\Gamma}{d\cos\theta} = \frac{|M_{if}|^2}{16\pi^2 M_i} (1 - M_f^2/M_i^2) = \frac{|M_{if}|^2}{8\pi^2 M_i^2} k_\gamma$$



## Start with E1 Transitions:

$$\frac{p^2}{2m} \rightarrow \frac{(\vec{p} - e\vec{A})^2}{2m} = \frac{p^2}{2m} - \frac{e\vec{p} \cdot \vec{A}}{2m} - \frac{e\vec{A} \cdot \vec{p}}{2m} + e^2 \frac{\vec{A}^2}{2m}$$

$p^2/2m$  is the original kinetic energy term

drop higher order  $e^2 \vec{A}^2$  terms

Interested in:

$$H_I = -\frac{e}{2m} (\vec{A} \cdot \vec{p} + \vec{p} \cdot \vec{A})$$

$$\vec{A}(x) = \frac{1}{\sqrt{2\omega}} \vec{\epsilon}(\vec{k}) e^{i\vec{k} \cdot \vec{x}}$$

$$e^{i\vec{k} \cdot \vec{x}} \simeq 1 + i\vec{k} \cdot \vec{x} + \dots$$

in the long wavelength limit  $\frac{1}{k} \gg r$

$$\Rightarrow \vec{A}(x) \simeq \frac{1}{\sqrt{2\omega}} \vec{\epsilon}(\vec{k})$$

$$H_I = -\frac{e}{2m} (\vec{\epsilon} \cdot \vec{p} + \vec{p} \cdot \vec{\epsilon})$$



To evaluate  $\langle A|\vec{p}|B\rangle \cdot \vec{\epsilon}$

Start with  $[p_i, r_j] = -i\delta_{ij}$

$$\Rightarrow [\vec{p}^2, r_j] = p_i[p_i, r_j] + [p_i, r_j]p_i = -2ip_j$$

$$\langle A|p_i|B\rangle = i\langle A|[\vec{p}^2/2, r_j]|B\rangle$$

$$= i\mu\langle A|[H, r_j]|B\rangle$$

$$(H = p^2/2\mu + V(r) \text{ but } [V(r), r] = 0)$$

$$= i\mu\langle A|Hr_j - r_jH|B\rangle$$

$$= i\mu(E_A - E_B)\langle A|r_j|B\rangle$$

$$= i\frac{m}{2}\omega\langle A|r_j|B\rangle$$

$$\langle A|H_I|B\rangle = -\frac{iem\omega}{2m}\langle A|r_i|B\rangle\epsilon_i$$

$$= -\frac{i\epsilon\omega}{2}\langle A|r_i|B\rangle\epsilon_i$$

$$= -\frac{i\epsilon\omega}{2}\langle A|\vec{r}|B\rangle \cdot \vec{\epsilon}$$



There are two methods for evaluating the matrix element

**Method 1:**

The sum over final polarizations is:

$$\sum_{pol} \vec{\epsilon}_i(k) \vec{\epsilon}^*(k) = \delta_{ij} - k_i k_j / k^2$$

So:

$$\sum_{pol} |\langle B | H_I | A \rangle|^2 = \omega^2 e^2 Q^2 \left\{ |\langle B | \vec{r} | A \rangle|^2 - |\langle B | \vec{r} \cdot \hat{k} | A \rangle|^2 \right\}$$

Averaging over directions:

$$= \omega^2 e^2 Q^2 \frac{2}{3} |\langle B | \vec{r} | A \rangle|^2$$

**Start with  ${}^3P_J \rightarrow {}^3S_1$**

- The orbital angular momentum is zero in the final state
- We may choose any  $J_z$  since we averaged over the photon directions

Convenient to choose  $J_z = J$



Start by writing down the meson wavefunction:  $|M\rangle = \sqrt{2M}\psi(r)$   
 where  $\sqrt{2M}$  is introduced to normalize the wavefunction when  
 integrating over relativistic phase space.

$${}^3P_2(J_z = 2) : |J = J_z = 2\rangle = |L = L_z = 1\rangle \otimes |S = S_z = 1\rangle = |Y_{11} \uparrow\uparrow\rangle$$

Only  $J'_z = S'_z = 1$  contributes since  $H_I$  does not flip spin.

$$\langle f|\vec{r}|i\rangle = \langle f|r|i\rangle \int \langle Y_{00} \uparrow\uparrow | \sqrt{\frac{4\pi}{3}} Y_{1-1} | Y_{11} \uparrow\uparrow \rangle d\Omega = \langle f|r|i\rangle \sqrt{\frac{1}{3}}$$

$$\text{where } \langle f|r|i\rangle = \int r^2 dr R_f(r) r R_i(r) \sqrt{2M_i} \sqrt{2M_f}$$

$$\begin{aligned} \Gamma({}^3P_2 \rightarrow {}^3S_1) &= \frac{1}{8\pi M^2} |M_{if}|^2 \omega \\ &= \frac{\omega}{8\pi M^2} \omega^2 e^2 Q^2 |\langle f|r|i\rangle|^2 (sM_i)(2M_f) \times \frac{2}{3} \times \frac{1}{3} \\ &= \frac{4\pi\alpha\omega^3 e_q^2}{8\pi} \frac{8}{9} |\langle f|r|i\rangle|^2 \left( \frac{M_i M_f}{M_i M_i} \right) \\ &= \frac{4}{9} \alpha\omega^3 e_q^2 |\langle f|r|i\rangle|^2 \left( \frac{M_f}{M_i} \right) \end{aligned}$$





For  ${}^3P_1 \rightarrow {}^3S_1$

$$|J = J_Z = 1\rangle = \frac{1}{\sqrt{2}}|Y_{11} \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) - Y_{10} \uparrow\uparrow\rangle$$

so that

$$\begin{aligned} \langle Y_{00} | \vec{r} | J = J_Z = 1 \rangle &= \frac{1}{\sqrt{2}} \langle Y_{00} | \vec{r} | Y_{11} \rangle - \frac{1}{\sqrt{2}} \langle Y_{00} | \vec{r} | Y_{10} \rangle \\ &= \left[ \frac{1}{\sqrt{2}} \langle Y_{00} | \frac{1}{\sqrt{3}} \left( \frac{-\hat{x} + i\hat{y}}{\sqrt{2}} \right) | Y_{11} \rangle - \frac{1}{\sqrt{2}} \langle Y_{00} | \frac{1}{\sqrt{3}} \hat{z} | Y_{10} \rangle \right] \langle 1S | r | 1P \rangle \\ \Rightarrow |\langle {}^3S_1 | \vec{r} | {}^3P_1 \rangle|^2 &= \left[ \frac{1}{2} \frac{1}{3} + \frac{1}{2} \frac{1}{3} \right] |\langle 1S | r | 1P \rangle|^2 \end{aligned}$$

${}^3P_0 \rightarrow {}^3S_1$

$$|J = J_Z = 0\rangle = \sqrt{\frac{1}{3}}|Y_{11} \downarrow\downarrow - Y_{10} \sqrt{\frac{1}{2}}(\uparrow\downarrow + \downarrow\uparrow) + Y_{1-1} \uparrow\uparrow\rangle$$

resulting in  $|\langle {}^3S_1 | \vec{r} | {}^3P_0 \rangle|^2 = \left[ \frac{1}{3} \frac{1}{3} + \frac{1}{3} \frac{1}{3} + \frac{1}{3} \frac{1}{3} \right] |\langle 1S | r | 1P \rangle|^2$



Summarizing all these results we obtain:

$$\Gamma(^3P_2 \rightarrow ^3S_1\gamma) = \frac{\omega^3 e^2 Q^2}{3\pi} \frac{1}{3} |\langle 1S|r|1P \rangle|^2$$

$$\Gamma(^3P_1 \rightarrow ^3S_1\gamma) = \frac{\omega^3 e^2 Q^2}{3\pi} \left\{ \frac{1}{2} \frac{1}{3} + \frac{1}{2} \frac{1}{3} \right\} \frac{1}{3} |\langle 1S|r|1P \rangle|^2$$

$$\Gamma(^3P_0 \rightarrow ^3S_1\gamma) = \frac{\omega^3 e^2 Q^2}{3\pi} \left\{ \frac{1}{3} \frac{1}{3} + \frac{1}{3} \frac{1}{3} + \frac{1}{3} \frac{1}{3} \right\} |\langle 1S|r|1P \rangle|^2$$

Comparing these expressions we see that in all cases

$$\Gamma(^3P_J \rightarrow ^3S_1\gamma) = \frac{4\alpha\omega^3 Q^2}{9} |\langle 1S|r|1P \rangle|^2$$

Similarly we obtain:

$$\Gamma(^3S_1 \rightarrow ^3P_J\gamma) = \frac{4\alpha\omega^3 Q^2 (2J + 1)}{27} |\langle 1S|r|1P \rangle|^2$$



Let us return to our *effective* wavefunctions:

$$\psi_{1S} = \frac{2}{\pi^{1/4}} \beta^{3/2} e^{-\beta^2 r^2 / 2} Y_{00} \quad \beta = 0.77 \text{ GeV}$$

$$\psi_{1P} = \sqrt{\frac{8}{3}} \frac{\beta^{5/2} r}{\pi^{1/4}} e^{-\beta^2 r^2 / 2} Y_{1m} \quad \beta = 0.598 \text{ GeV}$$

This gives:

$$\begin{aligned} \langle \psi_{1S} | r | \psi_{1P} \rangle &= \frac{2}{\pi^{1/4}} \sqrt{\frac{8}{3}} \frac{1}{\pi^{1/4}} \beta_S^{3/2} \beta_P^{5/2} \int r^4 e^{-(\beta_S^2 + \beta_P^2)r^2 / 2} dr \\ &= \sqrt{\frac{8}{3}} 15 \frac{\beta_S^{3/2} \beta_P^{5/2}}{(\beta_S^2 + \beta_P^2)^{5/2}} \\ &= 5.2 \text{ GeV}^{-1} \end{aligned}$$

$$\Rightarrow \Gamma(^3P_2 \rightarrow ^3S_1 \gamma) = 0.59 \text{ MeV} \quad vs \quad \Gamma^{expt} = 0.351_{-0.14}^{+0.2} \text{ MeV}$$

$$\Gamma(^3P_1 \rightarrow ^3S_1 \gamma) = \text{MeV} \quad vs \quad \Gamma^{expt} < 0.355 \text{ MeV}$$



# 3. E1 transitions

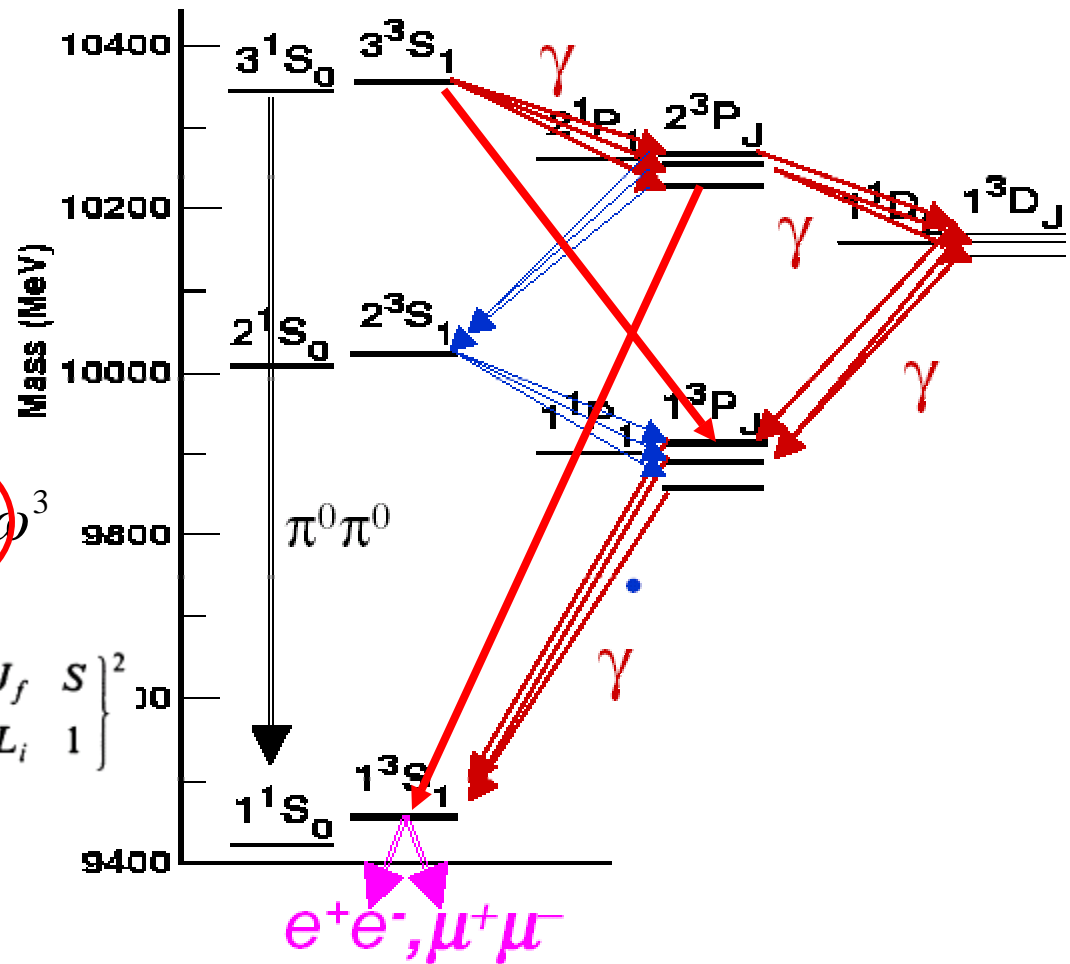
McClary and Byers, PR D28, 1692 (1983)

E1 decays sensitive to nodes in wavefunction

radiative transitions tests internal structure

$$\Gamma = \frac{4}{3} e_Q^2 \alpha C(J_i L_i J_f L_f S) |\langle P | r | S \rangle|^2 \omega^3$$

$$C(J_i L_i J_f L_f S) = \max(L_i, L_f) (2J_f + 1) \begin{Bmatrix} L_f & J_f & S \\ J_i & L_i & 1 \end{Bmatrix}^2$$



Including relativistic corrections corresponds to using eigenfunctions and eigenvalues of the Breit-Fermi Hamiltonian (Siegert's theorem)

	$ \langle 2P r 3S \rangle $		$ \langle 1P r 2S \rangle $		$ \langle 1P r 3S \rangle $		$ \langle 1S r 2P \rangle $ $ \langle 2S r 2P \rangle $	
	$GeV^{-1}$		$GeV^{-1}$		$GeV^{-1}$			
DATA	2.7±0.2		1.9±0.2		0.050±0.006		0.096±0.005	
	World Average				This measurement			
Model	NR	rel	NR	rel	NR	rel	NR	rel
Kwong,Rosner [13]	2.7		1.6		0.023		0.13	
Fulcher [14]	2.6		1.6		0.023		0.13	
Büchmüller et al.[15]	2.7		1.6		0.010		0.12	
Moxhay,Rosner [16]	2.7	2.7	1.6	1.6	0.024	0.044	0.13	0.15
Gupta et al.[17]	2.6		1.6		0.040		0.11	
Gupta et al.[18]	2.6		1.6		0.010		0.12	
Fulcher [19]	2.6		1.6		0.018		0.11	
Danghighian et al.[20]	2.8	2.5	1.7	1.3	0.024	0.037	0.13	0.10
McClary,Byers [21]	2.6	2.5	1.7	1.6			0.15	0.13
Eichten et al.[22]	2.6		1.7		0.110		0.15	
Grotch et al.[23]	2.7	2.5	1.7	1.5	0.011	0.061	0.13	0.19

Tomasz Skwarnicki, Syracuse U. ICHEP, Amsterdam July,2002



# Relativistic effects gives differences between E1 matrix elements:

$$\langle 2P|r|3S \rangle = 2.7 \pm 0.2 \text{ GeV}^{-1}$$

$$\langle 2^3P_2|r|3^3S_1 \rangle \approx -2.4 \text{ GeV}^{-1}$$

$$\langle 2^3P_1|r|3^3S_1 \rangle \approx -2.3 \text{ GeV}^{-1}$$

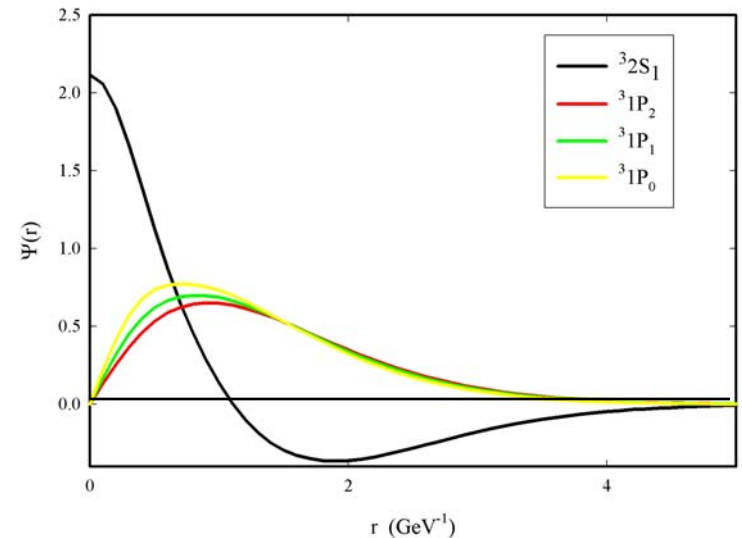
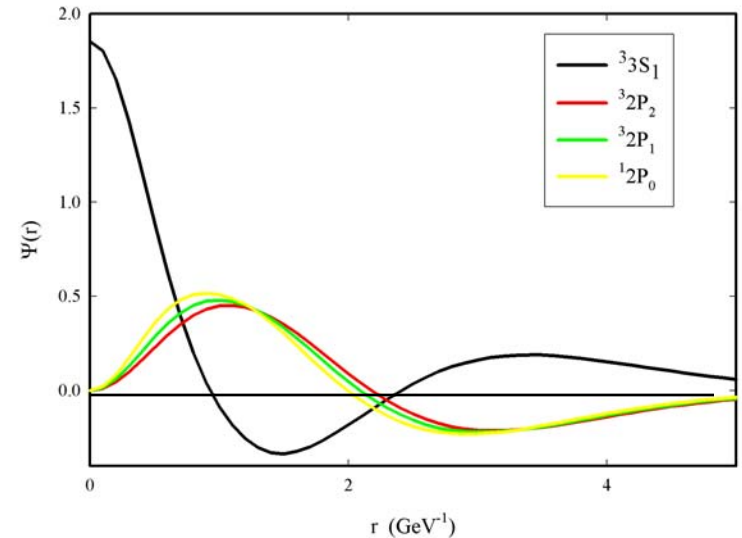
$$\langle 2^3P_0|r|3^3S_1 \rangle \approx -2.2 \text{ GeV}^{-1}$$

$$\langle 1P|r|2S \rangle \pm 1.9 \pm 0.2 \text{ GeV}^{-1}$$

$$\langle 1^3P_2|r|2^3S_1 \rangle \approx -1.5 \text{ GeV}^{-1}$$

$$\langle 1^3P_1|r|2^3S_1 \rangle \approx -1.4 \text{ GeV}^{-1}$$

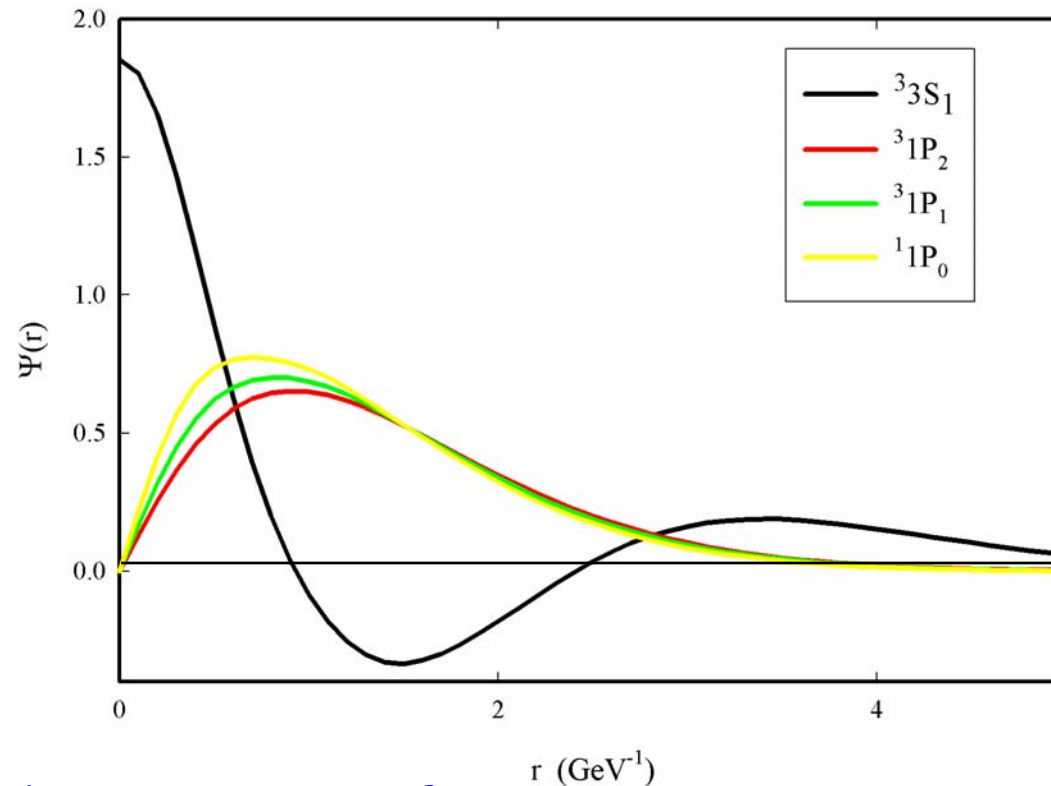
$$\langle 1^3P_0|r|2^3S_1 \rangle \approx -1.3 \text{ GeV}^{-1}$$



see also McClary and Byers, PR D28, 1692 (1983)



$\langle 1P r 3S \rangle$	
$\text{GeV}^{-1}$	
$0.050 \pm 0.006$	
This mea	
NR	rel
0.023	
0.023	
0.010	
0.024	0.044
0.040	
0.010	
0.018	
0.024	0.037
0.110	
0.011	0.061



Node in 3S wavefunction near maximum in 1P wavefunction so large cancellation very sensitive to details of the wavefunctions

$$\langle 1^3 P_2 | r | 3^3 S_1 \rangle \approx +0.096 \text{ GeV}^{-1}$$

$$\langle 1^3 P_1 | r | 3^3 S_1 \rangle \approx +0.040 \text{ GeV}^{-1}$$

$$\langle 1^3 P_0 | r | 3^3 S_1 \rangle \approx -0.026 \text{ GeV}^{-1}$$



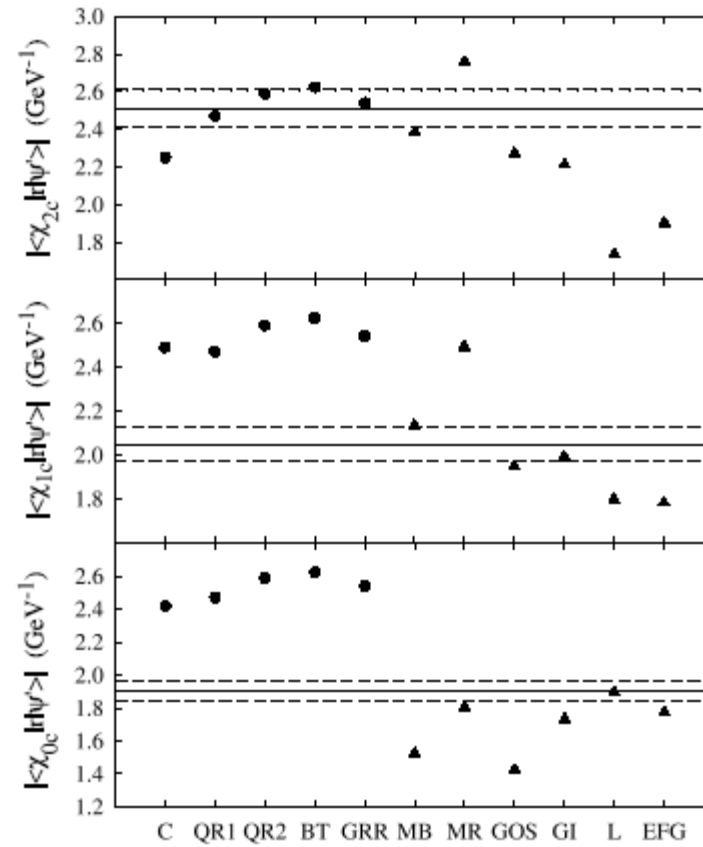


Table I: Properties of  $\psi(2S) \rightarrow \gamma\chi_{cJ}$  decays, using results from Refs. [54] and [66] as well as Eq. (6).

$J$	$k_\gamma$ (MeV)	$\mathcal{B}$ [66] (%)	$\Gamma[\psi(2S) \rightarrow \gamma\chi_{cJ}]$ (keV)	$ \langle 1P   r   2S \rangle $ (GeV $^{-1}$ )
2	$127.60 \pm 0.09$	$9.33 \pm 0.14 \pm 0.61$	$31.4 \pm 2.4$	$2.51 \pm 0.10$
1	$171.26 \pm 0.07$	$9.07 \pm 0.11 \pm 0.54$	$30.6 \pm 2.2$	$2.05 \pm 0.08$
0	$261.35 \pm 0.33$	$9.22 \pm 0.11 \pm 0.46$	$31.1 \pm 2.0$	$1.90 \pm 0.06$





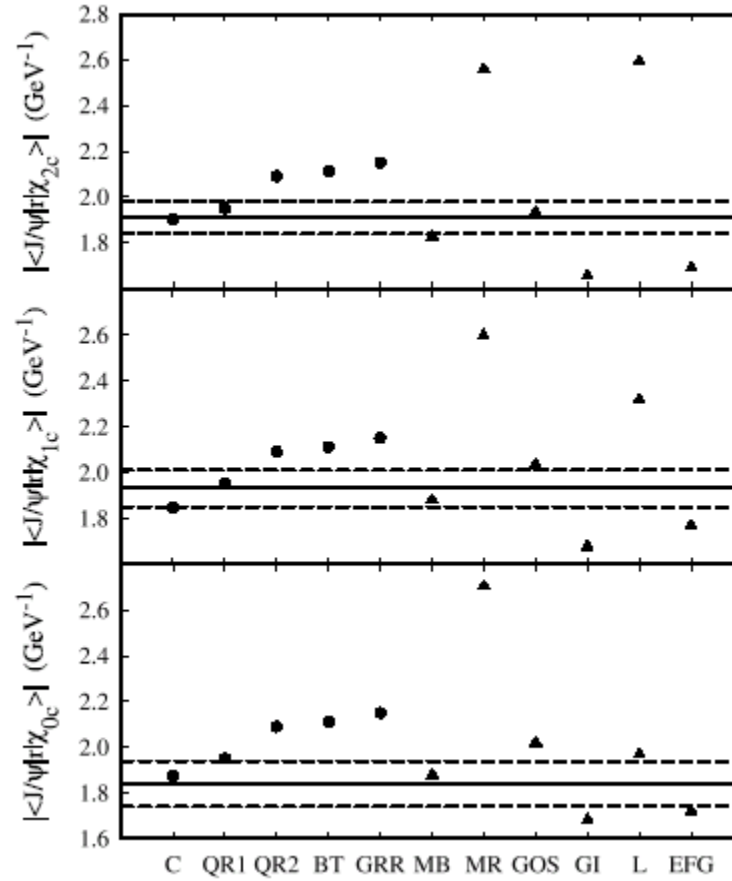
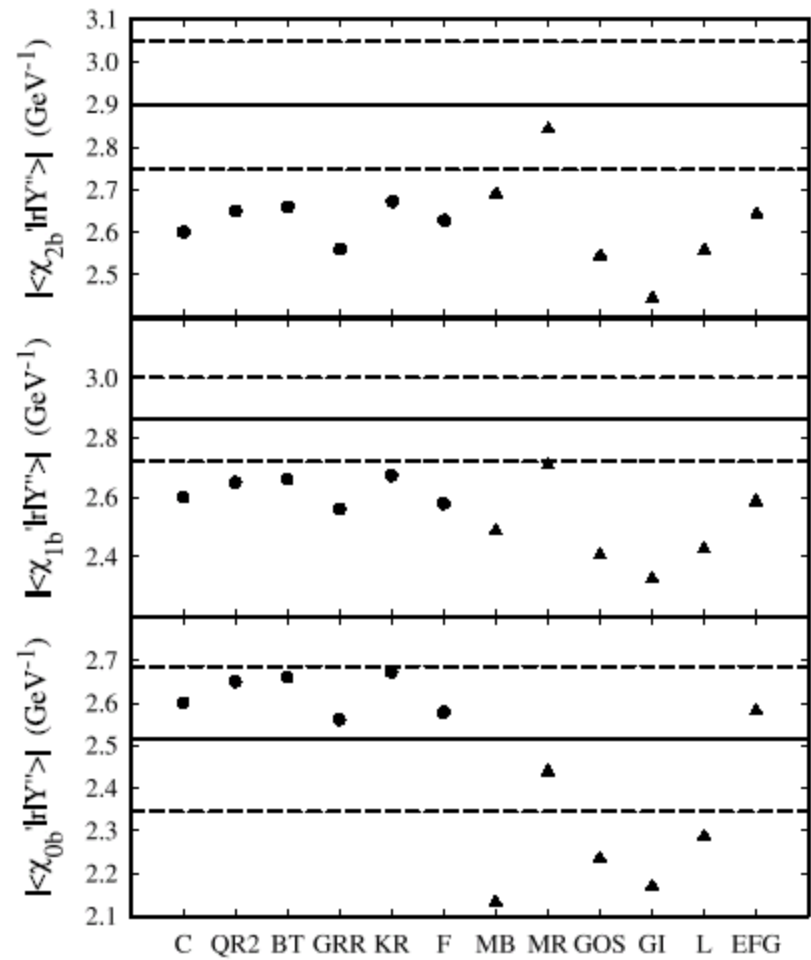
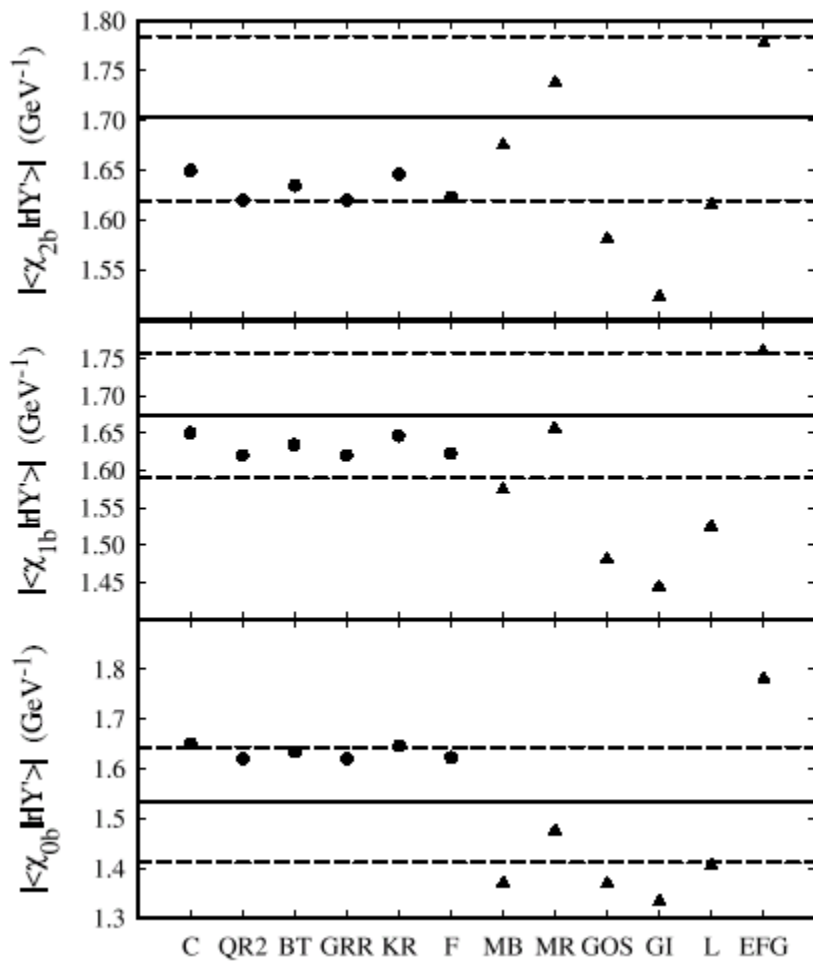


Table III: Properties of the transitions  $\chi_{cJ} \rightarrow \gamma J/\psi$ . (Ref. [54]; Eq. (6)).

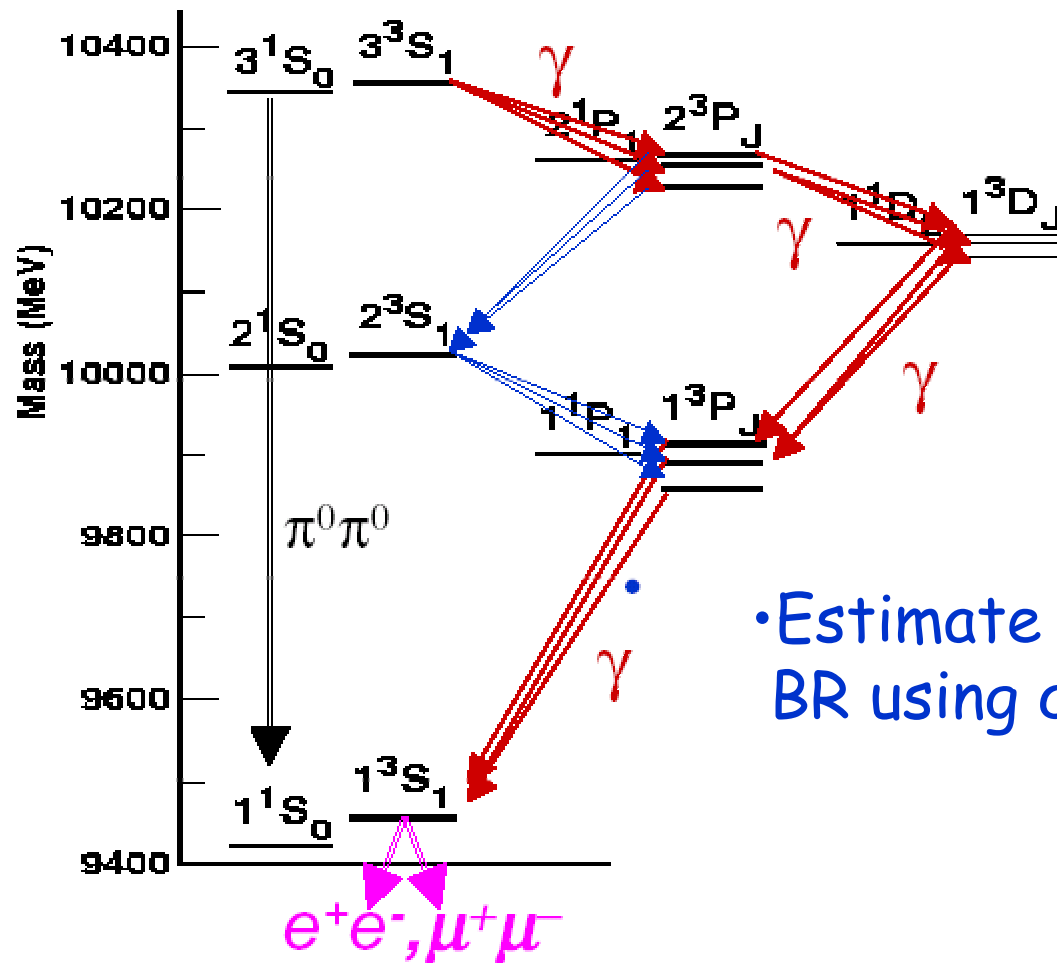
$J$	$k_\gamma$ (MeV)	$\Gamma(\chi_{cJ} \rightarrow \gamma J/\psi)$ (keV)	$ \langle 1S r 1P \rangle $ (GeV) $^{-1}$
2	$429.63 \pm 0.08$	$416 \pm 32$	$1.91 \pm 0.07$
1	$389.36 \pm 0.07$	$317 \pm 25$	$1.93 \pm 0.08$
0	$303.05 \pm 0.32$	$135 \pm 15$	$1.84 \pm 0.10$





# Production of the D-wave states

- By direct scans in  $e^+e^-$  to produce  $^3D_1$  ( $J^{PC} = 1^{--}$ )
- Use for  $4\gamma$  E1 cascade to search for  $\Upsilon(1^3D_J)$



• Estimate the radiative widths and BR using quark model



• In e.m. cascades:  $Y(3S) \rightarrow \gamma \chi'_b \rightarrow \gamma \gamma \ ^3D_J$

$$\Gamma = \frac{4}{3} e_Q^2 \alpha C(J_i L_i J_f L_f S) |\langle P|r|S \rangle| \omega^3 \quad C(J_i L_i J_f L_f S) = \max(L_i, L_f) (2J_f + 1) \begin{Bmatrix} L_f & J_f & S \\ J_i & L_i & 1 \end{Bmatrix}^2$$

• Some  $4\gamma$  cascades with observable # of events/ $10^6$   $Y(3S)$ 's:

Cascade	Events
$3^3S_1 \rightarrow 2^3P_2 \rightarrow 1^3D_3 \rightarrow 1^3P_2 \rightarrow 1^3S_1$	7.8
$3^3S_1 \rightarrow 2^3P_2 \rightarrow 1^3D_2 \rightarrow 1^3P_1 \rightarrow 1^3S_1$	2.7
$3^3S_1 \rightarrow 2^3P_1 \rightarrow 1^3D_2 \rightarrow 1^3P_1 \rightarrow 1^3S_1$	20
$3^3S_1 \rightarrow 2^3P_1 \rightarrow 1^3D_1 \rightarrow 1^3P_1 \rightarrow 1^3S_1$	3.3

S.G + J. Rosner, Phys Rev D64, 097501 (2001)

Expect  $\sim 38$  events / $10^6$   $Y(3S)$  via  $^3D_J$

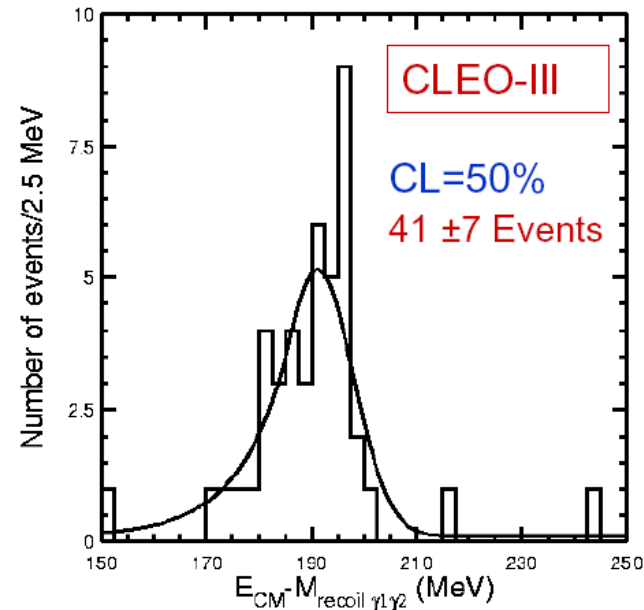
- The  $e^+e^-$  final states leads to less background
- $\mu^+\mu^-$  final states also contribute if  $\mu$ 's are identified



CLEO finds:

$$B(\Upsilon(3S) \rightarrow \gamma\Upsilon(1D) \rightarrow \gamma\gamma\Upsilon(1S) \rightarrow \gamma\gamma\gamma l^+l^-) = (3.3 \pm 0.6 \pm 0.5) 10^{-5}$$

(vs GR prediction of  $3.8 \times 10^{-5}$ )



- Mass averaged over different fits: **10162.2  $\pm$  1.6 MeV**
- Inconsistent with the  $\Upsilon(1D_3)$
- Could be the  $\Upsilon(1D_2)$  or  $\Upsilon(1D_1)$
- The theory predicts the rate ratio:  $\Upsilon(1D_2)/\Upsilon(1D_1)=6$
- **Thus, the  $\Upsilon(1D_2)$  is the most likely interpretation**



# M1 Transitions

Because quarks have spin they may emit a photon via a spin flip  
- The magnetic dipole transition

To obtain the interaction Hamiltonian we perform a non-relativistic reduction of

$$H_I = e \int dx j_{em}^\mu(x) A_\mu(x)$$

$$\text{where } j_{em}^\mu(x) = \bar{q}(x) Q \gamma^\mu q(x)$$

We expand the Dirac spinors to lowest order in  $p/m$   
Denoting the large and small components by  $q_1$  and  $q_2$

$$q_2(x) = -\frac{i\vec{\sigma} \cdot \vec{\nabla}}{2m} q_1(x)$$



$$\vec{j}_{em}(x) = \frac{-i}{2m} [q_1^\dagger Q(\nabla q_1) - (\nabla q_1^\dagger) Q q_1 + i\nabla \times q_1^\dagger Q \vec{\sigma} q_1]$$

So the interaction Hamiltonian is given by:

$$H_I = \frac{-eQ}{2m} [\vec{A}(\vec{r}) \cdot \vec{p} + \vec{p} \cdot \vec{A}(\vec{r}) + \vec{\sigma} \cdot [\vec{\nabla} \times \vec{A}(\vec{r})]]$$

So:

$$\langle 0 | H_I | \gamma(\vec{k}, \epsilon) \rangle = -\frac{1}{(2\pi)^{3/2}} \frac{1}{(2\omega)^{1/2}} eQ \frac{1}{2m} [e^{i\vec{k} \cdot \vec{r}} \vec{\epsilon} \cdot \vec{p} + \vec{\epsilon} \cdot \vec{p} e^{i\vec{k} \cdot \vec{r}} + i\vec{\sigma} \cdot (\vec{k} \times \vec{\epsilon}) e^{i\vec{k} \cdot \vec{r}}]$$

(For antiquarks change the sign of the charge)



$\mu = \frac{e}{2m_1}$  Is the magnetic dipole moment of the quark

For magnetic dipole transitions:

$$M_{if} = i\mu \langle f | \vec{\sigma} | i \rangle \cdot \vec{k} \times \vec{\epsilon}^*$$

$$\vec{\epsilon} = \frac{1}{\sqrt{2}}(1, \pm i, 0)$$

$$\begin{vmatrix} \sigma_x & \sigma_y & \sigma_z \\ k_x & k_y & k_z \\ 1 & i & 0 \end{vmatrix} = i\sigma_z(k_x + ik_y) - ik_z\sigma_x + k_z\sigma_y$$

Choosing z as the  $\gamma$  direction

$$M_{if} = -\frac{ie_q}{2m} k_\gamma \langle f | \sigma_x - i\sigma_y | i \rangle \text{ where } \sigma_x - i\sigma_y = \sigma_-$$

if instead take  $\vec{k} = k_y$

$$\begin{aligned} M_{if} &= -\frac{ie_q}{2m} k_\gamma \langle f | \sigma_z | i \rangle \\ &= k_\gamma \sqrt{2M_i} \sqrt{2M_f} \int d^3r \psi_f^*(r) \psi_i(r) \times \langle f | \sum \mu_i \sigma_{zi} | i \rangle \end{aligned}$$





e.g.  $J/\psi \rightarrow \eta_c \gamma (^3S_1 \rightarrow ^1S_0 \gamma)$

$$\begin{aligned}
 A(^3S_1 \rightarrow ^1S_0 \gamma) &= -ik_\gamma \sqrt{2M_i} \sqrt{2M_f} \langle f|i \rangle \\
 &\quad \times \left\langle \sqrt{\frac{1}{2}} (\uparrow\downarrow - \downarrow\uparrow) \left| \frac{e_q}{2m_q} \frac{(\sigma_x - i\sigma_y)_q}{\sqrt{2}} + \mu_{\bar{q}} \frac{(\sigma_x - i\sigma_y)_{\bar{q}}}{\sqrt{2}} \right| \uparrow\uparrow \right\rangle \\
 &= -ik_\gamma \sqrt{2M_i} \sqrt{2M_f} \langle f|i \rangle \left[ \frac{-e_q}{2m_q} + \frac{e_{\bar{q}}}{2m_{\bar{q}}} \right] \\
 &= -ik_\gamma \sqrt{2M_i} \sqrt{2M_f} \langle f|i \rangle \frac{ee_q}{m_c} \\
 \Rightarrow \frac{d\Gamma}{d\Omega} &= k_\gamma \frac{4\pi\alpha}{8\pi^2} k_\gamma^2 |\langle f|i \rangle|^2 \frac{e_c^2}{m_c^2}
 \end{aligned}$$

averaging over angles gives the total width

$$\Gamma = \frac{k_\gamma^3}{3\pi} |\langle f|i \rangle|^2 \frac{e_c^2}{m_c^2}$$

Take  $\langle f|i \rangle = 1$        $\omega = 115$

so  $\Gamma = 0.19$  MeV vs 0.88 keV (expt)

What about?  $2^3S_1 \rightarrow 1^1S_0$

$\langle f|i \rangle = 0$  since  $2S \perp 1S$



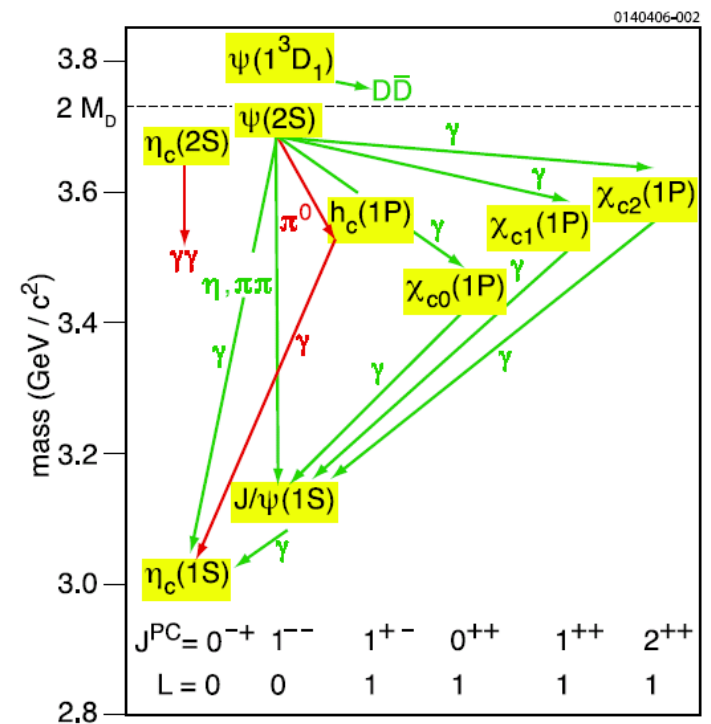
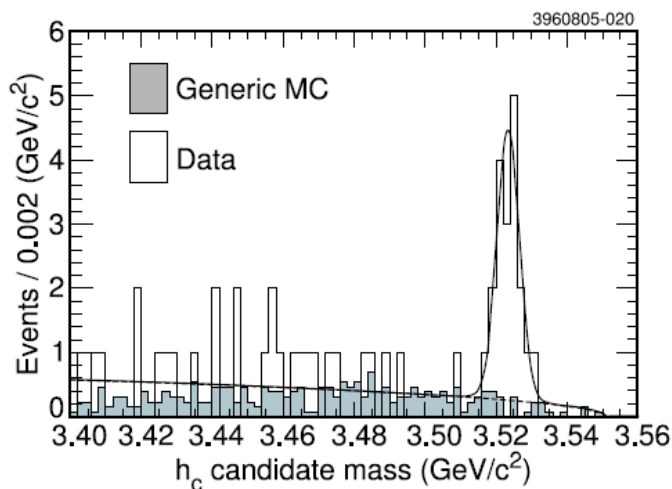
The decay  $\psi(2S) \rightarrow \gamma\eta_c(1S)$  is a forbidden magnetic dipole (M1) transition

The photon energy is 638 MeV, leading to a non-zero matrix element  $\langle 1S | j_0(kr/2) | 2S \rangle$ .

$$\Gamma[\psi(2S) \rightarrow \gamma\eta_c(1S)] = (1.00 \pm 0.16)$$

$$|\langle 1S | j_0(kr/2) | 2S \rangle| = 0.045 \pm 0.004.$$

The  $h_c(1^1P_1)$   $\Gamma[h_b(1^1P_1) \rightarrow \eta_b(1^1S_0) + \gamma] = \frac{4}{9} \alpha e_Q^2 \left| \langle 1S_0 | r | 1^1P_1 \rangle \right|^2 \omega^3 = 37 \text{ keV}$



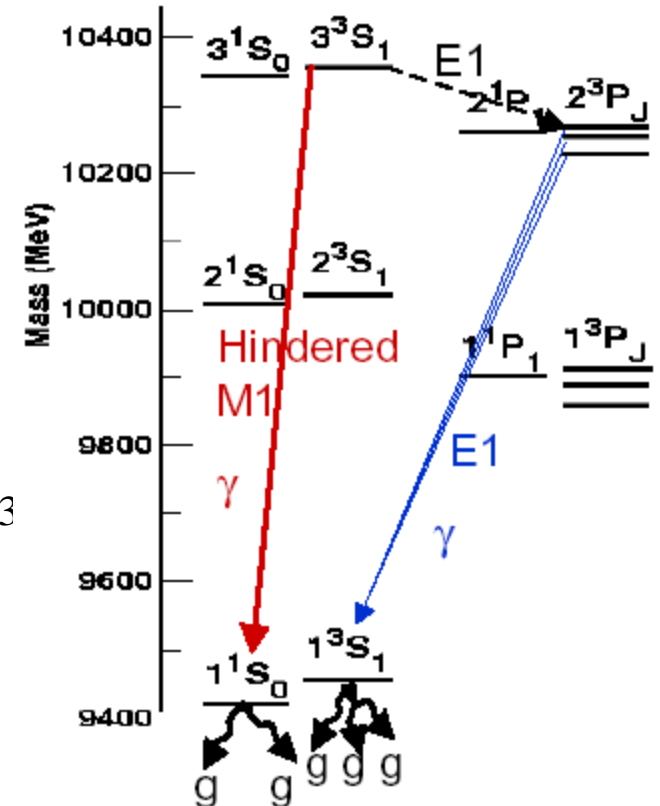
# M1 transitions: production of $\eta_b(nS)$ states

S.G + J. Rosner, Phys Rev D64, 074011 (2001)

Proceeds via magnetic dipole (M1) transitions:

$$Y(nS) \rightarrow \eta(n'S) + \gamma$$

$$\Gamma(^3S_1 \rightarrow ^1S_0 + \gamma) = \frac{4}{3} \alpha \frac{e_Q^2}{m_Q^2} \left| \langle f | j_0(kr/2) | i \rangle \right|^2 \omega^3$$



- Hindered transitions have large phase space
- Relativistic corrections resulting in differences in  $^3S_1$  and  $^1S_0$  wavefunctions due to hyperfine interaction



	Transition	BR ( $10^{-4}$ )
Y(3S)		
( $\Gamma_{\text{tot}}=52.5$ keV)	$\rightarrow 3^1S_0$	0.10
	$\rightarrow 2^1S_0$	4.7
	$\rightarrow 1^1S_0$	25
Y(2S)	$\rightarrow 2^1S_0$	0.21
( $\Gamma_{\text{tot}}=44$ keV)	$\rightarrow 1^1S_0$	13
Y(1S)	$\rightarrow 1^1S_0$	2.2
( $\Gamma_{\text{tot}}=26.3$ keV)		

- Expect substantial rate to produce  $\eta_b$ 's
- Also  $Y(3S) \rightarrow h_b(1P_1) \pi\pi \rightarrow \eta_b + \gamma + \pi\pi$   
BR=0.1-1%                      BR = 50%

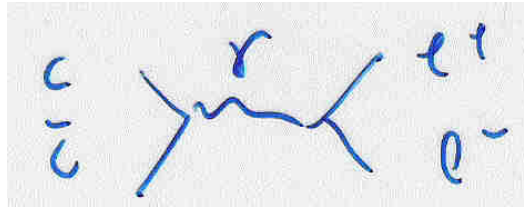
[Kuang & Yan PRD24, 2874 (1981); Voloshin Yad Fiz 43, 1571 (1986)]



# Decays:

$$J/\psi \rightarrow e^+ e^-$$

$$(^3S_1 \rightarrow e^+ e^-)$$



$$A(V_i \rightarrow e^+ e^-) \equiv \langle e^+ e^- | M | V_i \rangle$$

$$= \frac{4\pi\alpha e_q}{M^2} \langle e^+ e^- | j_k^{(em)} | 0 \rangle \langle 0 | j_k | V_i \rangle$$

$$= \frac{4\pi\alpha e_q}{M^2} \bar{U}_e(-p_+) \gamma_k U(p_-) \langle 0 | j_k | V_i \rangle$$

$$\langle 0 | j_k | V_i \rangle = \sqrt{3} \times 2M \int d^3 p \phi_s(p) Y_{00} \langle 0 | j_{em}^\mu | c\bar{c} \rangle$$

$$\text{where } \sum_{\text{colour}} \sqrt{\frac{1}{3}} (r\bar{r} + b\bar{b} + g\bar{g}) = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Typically express the matrix element in the form:

$$\langle 0 | j_{em}^\mu(0) | \psi(k, \lambda) \rangle = \frac{\epsilon^\mu(k, \lambda)}{(2\pi)^{3/2}} f_\psi$$



## In non-relativistic limit

$$\begin{aligned}\Rightarrow \langle 0 | j_{em}^\mu(0) | V(\uparrow) \rangle &= \sqrt{12M} \epsilon^\mu(\uparrow) \psi_S(0) \\ &\equiv \epsilon^\mu(k, \lambda) f_V \\ f_V &= \sqrt{12M} \psi_S(0)\end{aligned}$$



$$\begin{aligned}
\Gamma &= \frac{1}{2M} \int |M|^2 \frac{m_e}{E_{e+}} \frac{m_e}{E_{e-}} \frac{d^3 p_+}{(2\pi)^3} \frac{d^3 p_-}{(2\pi)^3} (2\pi)^4 \delta^4(P - p_+ - p_-) \\
&= \frac{e_Q^2 e^4}{12\pi M^3} (12M(2\pi)^3 |\psi(0)|^2) \\
&= \frac{16\pi^2 \alpha^2 e_Q^2}{\pi M^3} M |\psi(0)|^2 \\
&= \frac{16\pi \alpha^2 e_Q^2}{M^2} |\psi(0)|^2 \\
\psi_S(0) &= \frac{1}{\sqrt{4\pi}} R(0)
\end{aligned}$$



What about  $\psi''(3770)$ ?  $e^+ e^- \rightarrow \psi''(3770)$

${}^3D_1$  state so expect  $\Gamma=0$  since  $\psi_D(0)=0$  but not so

$$|V(\uparrow)\rangle = \sqrt{6M} \int d^3p \phi_D(p) \{ \sqrt{3/5} Y_{2+2}(\theta, \phi) |q(\downarrow) \bar{q}(\downarrow)\rangle \\ - \sqrt{3/10} Y_{2+1}(\theta, \phi) |q(\uparrow) \bar{q}(\downarrow)\rangle + \sqrt{1/10} Y_{20}(\theta, \phi) |q(\uparrow) \bar{q}(\uparrow)\rangle \}$$

After much work get:

$$\langle 0 | j_{em}^\mu(0) | V(\uparrow) \rangle = \frac{\sqrt{12M}}{(2\pi)^3} \epsilon^\mu(\uparrow) \int d^3p \frac{\phi_D(p)}{\sqrt{32\pi}} \frac{4}{3} \frac{p^2}{E(E+m)}$$

$$\lim_{x \rightarrow 0} \int d^3p \phi_D(p) \frac{p^2}{2m^2} \frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}} = -\frac{1}{2m^2} \lim_{x \rightarrow 0} \frac{\partial^2}{\partial x_i^2} \int d^3p \frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}} \phi_D(p)$$

$$= -\frac{1}{2m^2} \frac{\partial^2 R_D(0)}{\partial r^2} = -\frac{1}{2m^2} R_D''(0)$$

In general, for state of angular momentum L get  $R^{(L)}(0)$





More carefully get:

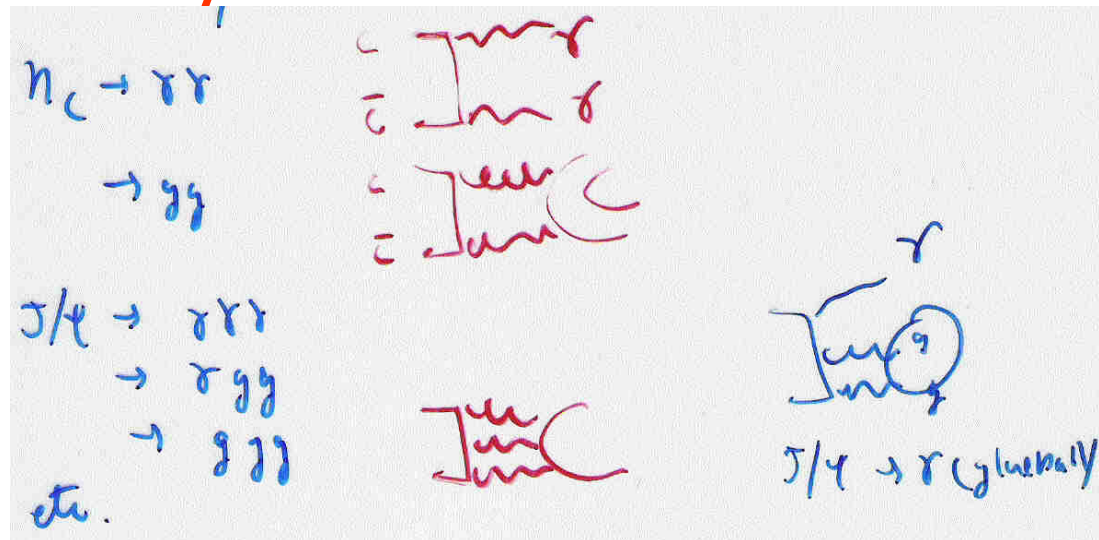
$$\langle 0 | j_{em}^\mu(0) | V(\uparrow) \rangle = \frac{\sqrt{12M}}{(2\pi)^{3/2}} \frac{5}{4} \frac{R''(0)}{m^2 \sqrt{2\pi}} \epsilon^\mu(\uparrow)$$

and

$$\Gamma = \frac{\alpha^2 (e_q/e)^2}{M_V^2} \frac{25}{2} \frac{|R_D''(0)|^2}{m_q^2}$$



Also have decays to hadronic final states:



Start with annihilation rates for positronium:

$$\Gamma(^1S_0 \rightarrow 2\gamma) = \frac{4\pi\alpha^2}{m^2} |\psi_S(0)|^2 = \frac{4\alpha^2}{m^2} |R_S(0)|^2$$

$$\Gamma(^3P_0 \rightarrow 2\gamma) = \frac{256}{3} \frac{\alpha^2}{m^4} |R'_P(0)|^2$$

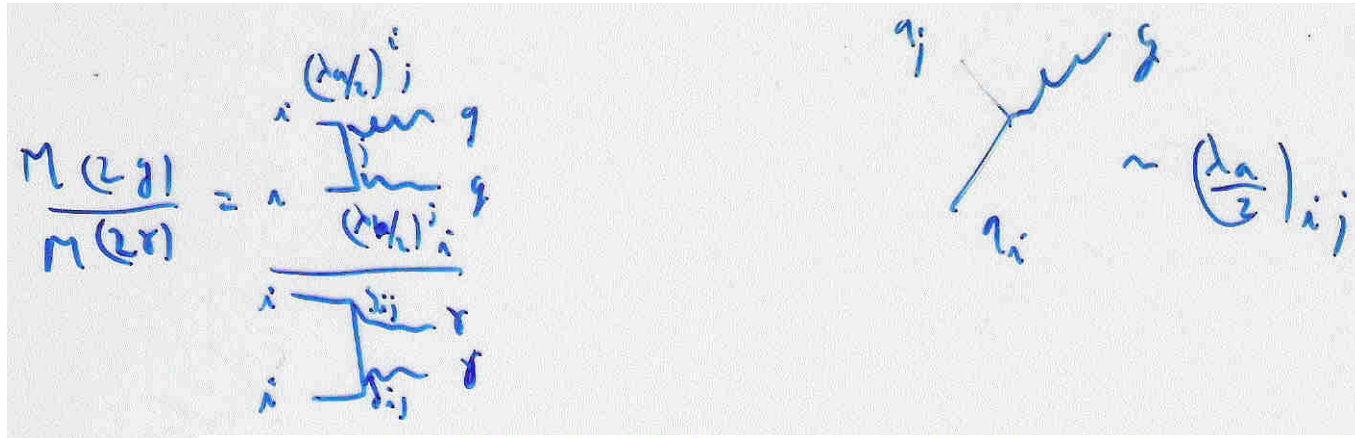
$$\Gamma(^3P_2 \rightarrow 2\gamma) = \frac{4}{15} \Gamma(^3P_0 \rightarrow \gamma\gamma) \left( \frac{M_0}{M_2} \right)$$

$$\Gamma(^3S_1 \rightarrow 3\gamma) = \frac{16}{9\pi} (\pi^2 - 9) \frac{\alpha^3}{m^2} |R_S(0)|^2$$

To relate to hadron decays include quark charges

For decays to gluons must include  $\alpha_s$  and  $\lambda$ 's for each gluon





$$= \frac{\alpha_s}{e_q^2 \alpha} \frac{(\lambda_a/2)_j^i (\lambda_b/2)_i^j}{\delta_j^i \delta_i^j} = \frac{\alpha_s}{\alpha e_q^2} \frac{\text{Tr}(\lambda_a/2 \lambda_b/2)}{3} = \frac{\alpha_s}{\alpha} \frac{1}{e_q^2} \frac{1}{3} \delta_{ab}$$

$$\text{where } \text{Tr}(\lambda_a/2 \lambda_b/2) = \frac{1}{2} \delta_{ab}$$

$$\Rightarrow \frac{\Gamma(2g)}{\Gamma(2\gamma)} = \frac{2}{9} \frac{\alpha_s^2}{\alpha^2 e_q^4}$$

For 3 gluons/photons:

$$\frac{M(3g)}{M(3\gamma)} = \frac{\alpha_s^{3/2}}{e_q^3 \alpha^{3/2}} \frac{(\lambda_a/2)_j^i (\lambda_b/2)_k^j (\lambda_c/2)_i^k}{\delta_j^i \delta_k^j \delta_i^k} = \frac{\alpha_s^{3/2}}{e_q^3 \alpha^{3/2}} \frac{1}{2} \frac{\text{Tr}(\{\lambda_a/2, \lambda_b/2\} \lambda_c/2)}{\delta_j^i \delta_k^j \delta_i^k}$$

$$\Rightarrow \frac{\Gamma(2g)}{\Gamma(2\gamma)} = \frac{5}{54} \frac{\alpha_s^3}{\alpha^3 e_q^6} \text{ where } \sum_{a,b,c} (d_{abc})^2 = 40/3$$



$$\Gamma(\eta_c \rightarrow 2\gamma) = 12\alpha^2 e_q^4 \frac{|R_S(0)|^2}{M^2}$$

$$\Gamma(\eta_c \rightarrow 2g) = \frac{8}{3}\alpha_s \frac{|R_S(0)|^2}{M^2}$$

$$\Gamma(J/\psi \rightarrow 3\gamma) = \frac{16(\pi^2 - 9)\alpha^3}{3} e_q^6 \frac{|R_S(0)|^2}{M^2}$$

$$\Gamma(J/\psi \rightarrow 3g) = \frac{40}{81\pi} (\pi^2 - 9)\alpha_s^3 \frac{|R_S(0)|^2}{M^2}$$

$$\Gamma(J/\psi \rightarrow 2g\gamma) = \frac{32}{9\pi} (\pi^2 - 9)\alpha_s^2 \alpha e_1^2 \frac{|R_S(0)|^2}{M^2}$$

For Completeness:

$$\Gamma(\chi_0 \rightarrow 2g) = 96\alpha_s^2 \frac{|R'_{\chi_0}(0)|^2}{M_{\chi_0}^4}$$

$$\Gamma(\chi_1 \rightarrow q\bar{q}g) = \frac{n_f}{3} \frac{128}{3\pi} \alpha_s^3 \frac{|R'_{\chi_1}(0)|^2}{M_{\chi_0}^4} \ln \left( \frac{4m_c^2}{4m_c^2 - M_{\chi}^2} \right)$$

$$\Gamma(\chi_2 \rightarrow 2g) = \frac{128}{5} \alpha_s^2 \frac{|R'_{\chi_2}(0)|^2}{M_{\chi_0}^4}$$

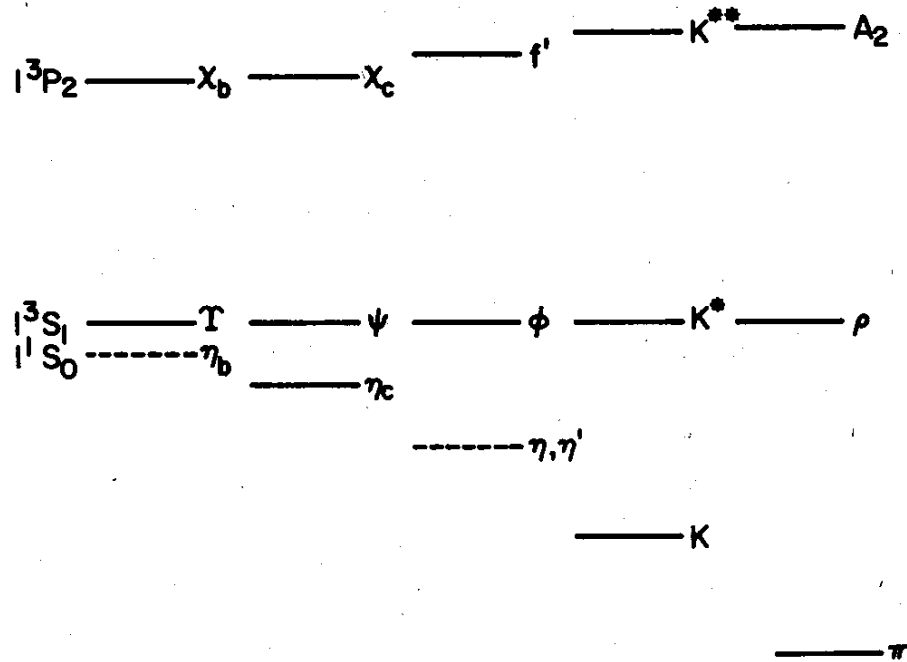
$$\Gamma(h_c \rightarrow q\bar{q}g) = \frac{320}{9\pi} \alpha_s^3 \frac{|R'_{h_c}(0)|^2}{M_{h_c}^4} \ln \left( \frac{4m_c^2}{4m_c^2 - M_{h_c}^2} \right)$$



## *4. What about mesons with light quarks?*

Historically, it was the successes of the quark model that led many physicists to believe that the quark model has something to do with reality





Essential features are the same, except:

- Relative importance of relativistic effects
- Hyperfine splittings are comparable in size to orbital splittings

Conclude

- potential models approximately valid



## Hadron masses in a gauge theory\*

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(Received 24 February 1975)

We explore the implications for hadron spectroscopy of the "standard" gauge model of weak, electromagnetic, and strong interactions. The model involves four types of fractionally charged quarks, each in three colors, coupling to massless gauge gluons. The quarks are confined within colorless hadrons by a long-range spin-independent force realizing infrared slavery. We use the asymptotic freedom of the model to argue that for the calculation of hadron masses, the short-range quark-quark interaction may be taken to be Coulomb-like. We rederive many successful quark-model mass relations for the low-lying hadrons. Because a specific interaction and symmetry-breaking mechanism are forced on us by the underlying renormalizable gauge field theory, we also obtain new mass relations. They are well satisfied. We develop a qualitative understanding of many features of the hadron mass spectrum, such as the origin and sign of the  $\Sigma$ - $\Lambda$  mass splitting. Interpreting the newly discovered narrow boson resonances as states of charmonium, we use the model to predict the masses of charmed mesons and baryons.



## Flavour content:

$$|\rho^+\rangle, |\pi^+\rangle = -|u\bar{d}\rangle$$

$$|\rho^0\rangle, |\pi^0\rangle = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle$$

$$|\omega\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$$

$$|\eta\rangle = \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle$$

$$|\eta'\rangle = \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle$$

$$|\phi\rangle = |s\bar{s}\rangle$$

$$|K^+\rangle = |u\bar{s}\rangle$$

$$|K^0\rangle = |d\bar{s}\rangle$$

$$|\bar{K}^0\rangle = -|s\bar{d}\rangle$$

$$|K^-\rangle = |s\bar{u}\rangle$$





In heavy quarkonium we used:

$$M = m_1 + m_2 + E_{nl}$$

$$\left[ \frac{p^2}{2\mu} + V(r) \right] \psi = E_{nl} \psi$$

$$H_{ij}^{conf} = -\frac{4}{3} \frac{\alpha_s(r)}{r} + br$$

This is a non-relativistic formula (v/c)=

$b\bar{b}$	0.26
$c\bar{c}$	0.45
$s\bar{s}$	0.78
$u\bar{u}$	0.9

*What do we do?*

- Use it anyway and see what happens. Taking this approach the general features are OK
- Try to relativize it.



# Spin dependent interactions:

$$\Delta[M(^3S_1) - M(^1S_0)] = \frac{3\pi\alpha_s}{9m_1m_2} |\psi(0)|^2$$

Approximate  $^3S_1$  and  $^1S_0$  masses by:

$$M(^3S_1) = M(S) + \frac{1}{4} \frac{a}{m_q m_{\bar{q}}}$$

$$M(^1S_0) = M(S) - \frac{3}{4} \frac{a}{m_q m_{\bar{q}}}$$

If  $a$  is approximately constant:

$$\frac{M(\rho) - M(\pi)}{M(K^*) - M(K)} \approx \frac{m_u m_s}{m_u m_u} \approx \frac{m_s}{m_u} \approx \frac{500}{300} \approx 1.7$$

$$\frac{770 - 140}{892 - 495} \approx \frac{630}{400} \approx 1.7$$

Similarly:

$$\frac{M(K^*) - M(K)}{M(D^*) - M(D)} \approx \frac{m_u m_c}{m_u m_s} \approx \frac{m_c}{m_s} \approx \frac{1.6}{0.55} \approx 2.9$$

$$\frac{892 - 494}{2010 - 1870} \approx \frac{400}{140} \approx 2.9$$

So splittings reasonably well described

Because  $^3P_{\text{cog}} - ^1P_1$  splitting is small supports short range contact interaction



# Electromagnetic transitions:

As before: 
$$\Gamma_{M1} = \frac{k_\gamma^3}{3\pi} |\langle f | i \rangle|^2 \left| \sum \mu_i \sigma_{zi} \right|^2$$

For example:

$$K^{*+} \rightarrow K^+ \gamma$$

$$\left\langle u\bar{s} \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \left| \frac{e_i}{2m_i} \sigma_z \right| u\bar{s} \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \right\rangle$$

$$= \frac{1}{2} \left\langle u\bar{s} \left| \frac{e_q}{2m_q} + \frac{e_q}{2m_q} - \frac{e_{\bar{q}}}{2m_{\bar{q}}} - \frac{e_{\bar{q}}}{2m_{\bar{q}}} \right| u\bar{s} \right\rangle$$

$$= \frac{1}{2} \left[ \frac{e_u}{m_u} - \frac{e_s}{m_s} \right] = \frac{1}{2} \left[ \frac{2}{3} \frac{1}{m_u} - \frac{1}{3} \frac{1}{m_s} \right]$$



# Strong (Zweig allowed) Decays:

A number of models to calculate strong decays.

Give good qualitative agreement with experiment with only 1 free parameter (using QM wavefunctions)

Important input to disentangle hadron spectrum



## Relativistic effects:

Clearly light quark hadrons are relativistic

Various attempts to "relativize" QM

Generally improves agreement

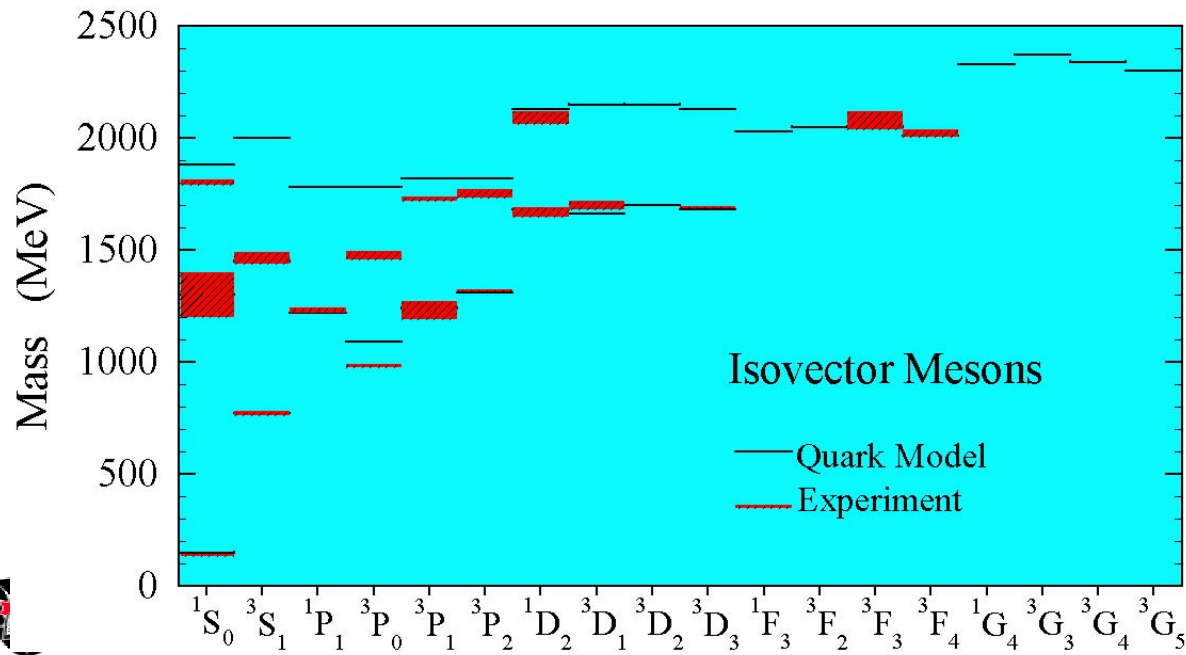
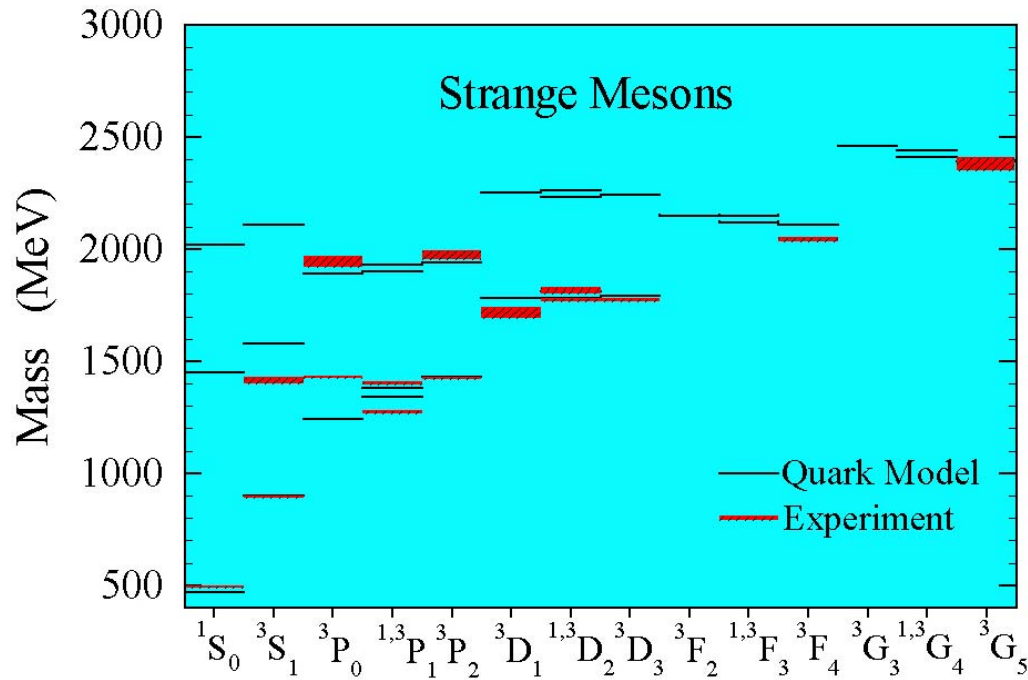
But much is missing. Major battles about what is correct approach.

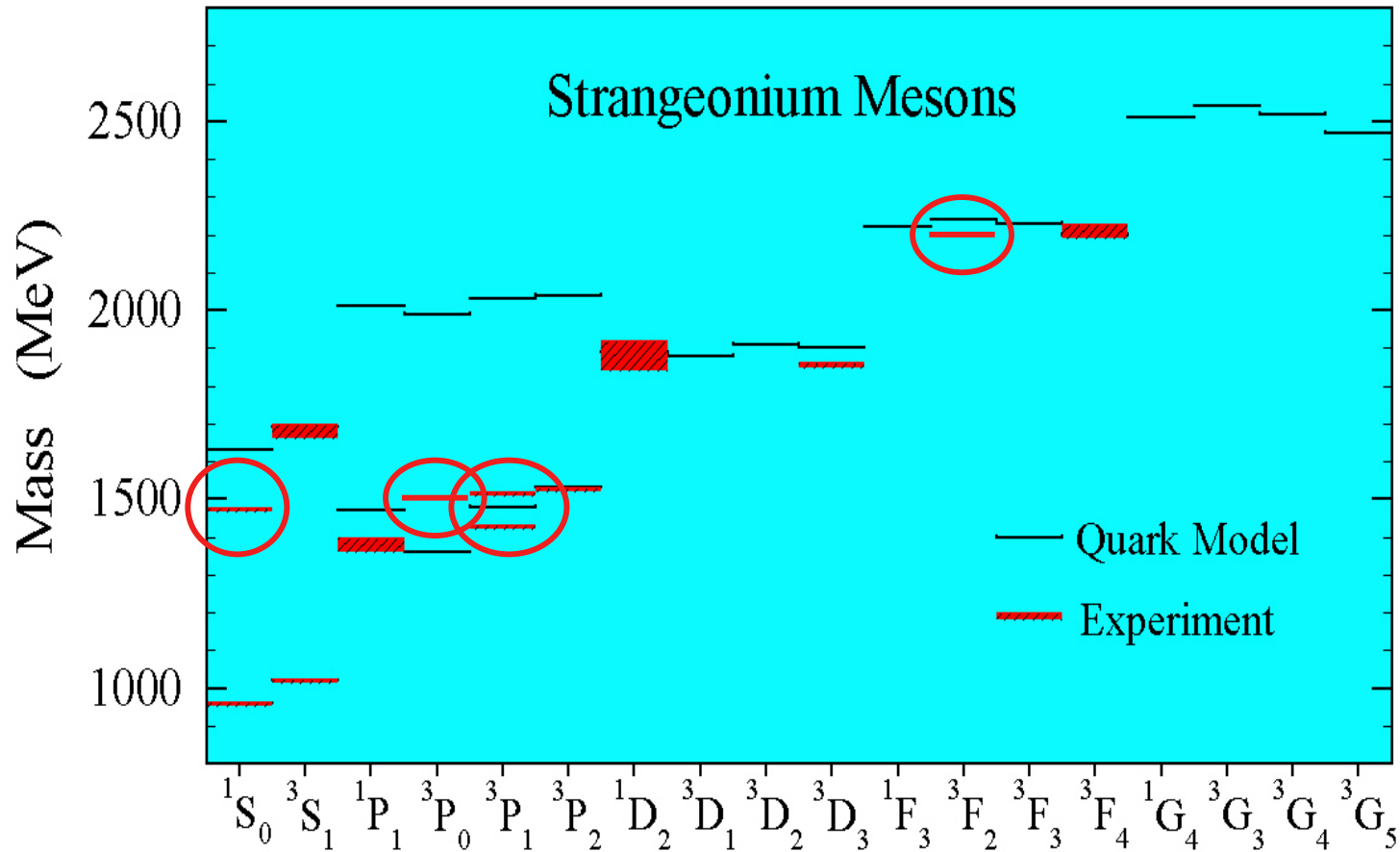
**BUT QM seems to get the physics right.**

**"Better to get the right degrees of freedom"**



Generally, good agreement for confirmed states





• Many unconfirmed states:

$f_1(1530)$ ,  $h_1(1380)$

• Many puzzles:

$\eta(1440)$ ,  $f_1(1420)$ ,  $f_0(1500)$   $f_J(1710)$ ,  $f_J(2200)$



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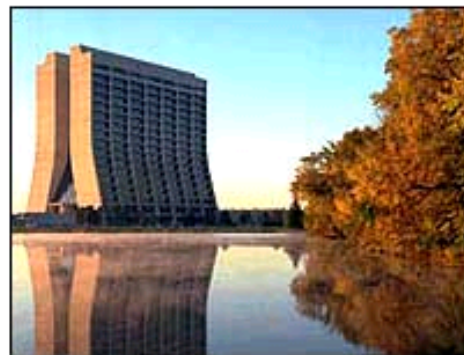
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## Scientists find mystery particle

By **Dr David Whitehouse**  
BBC News Online science editor

**Scientists have found a sub-atomic particle they cannot explain using current theories of energy and matter.**

The discovery was made by researchers based at the High Energy Accelerator Research Organisation in Tsukuba.



Fermilab confirmed the discovery

Classified as X(3872), the particle was seen fleetingly in an atom smasher and has been dubbed the "mystery meson".

The Japanese team says understanding its existence may require a change to the Standard Model, the accepted theory of the way the Universe is constructed.

### An eternity

X(3872) was found among the decay products of so-called beauty mesons - sub-atomic particles that are produced in large numbers at the Tsukuba "meson factory".

It weighs about the same as a single atom of helium and exists for only about one billionth of a trillionth of a second before it decays into other longer-lived, more familiar

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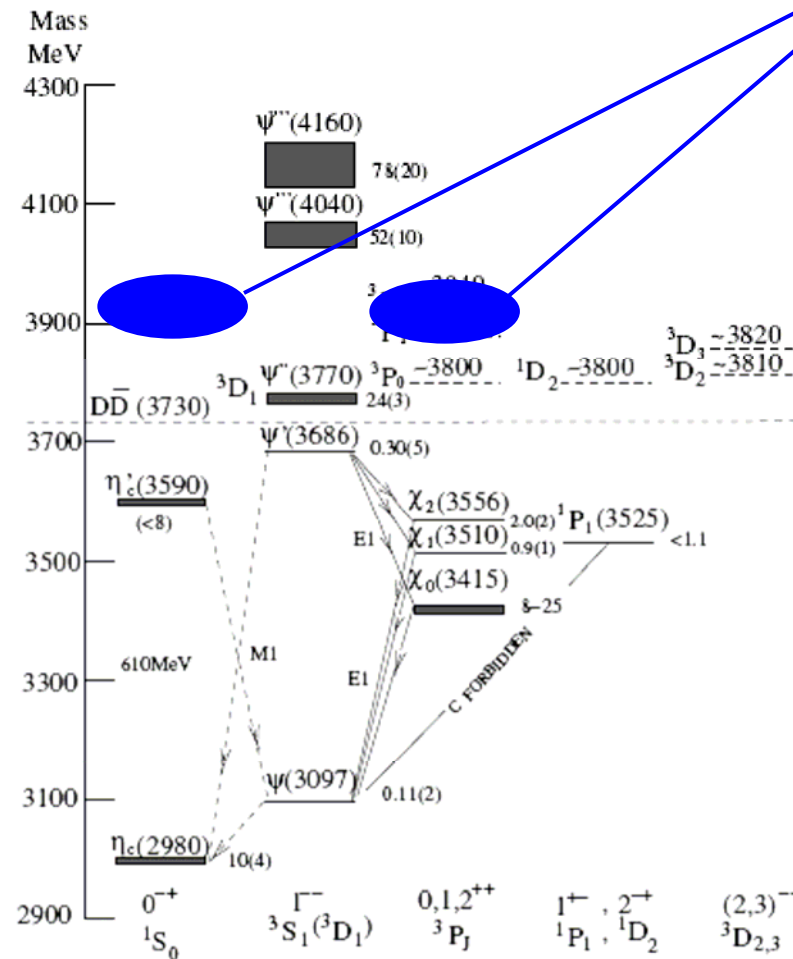
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# X(3943), Y(3943), and Z(3931)



Possible new  $C=+$   $c\bar{c}$  states at these masses!

## 2P or not 2P that is the question!



# X(3940)

Seen by Belle recoiling against  $J/\psi$  in  $e^+e^-$  collisions

$$M = 3943 \pm 6 \pm 6 \text{ MeV}$$

$$\Gamma < 52 \text{ MeV}$$

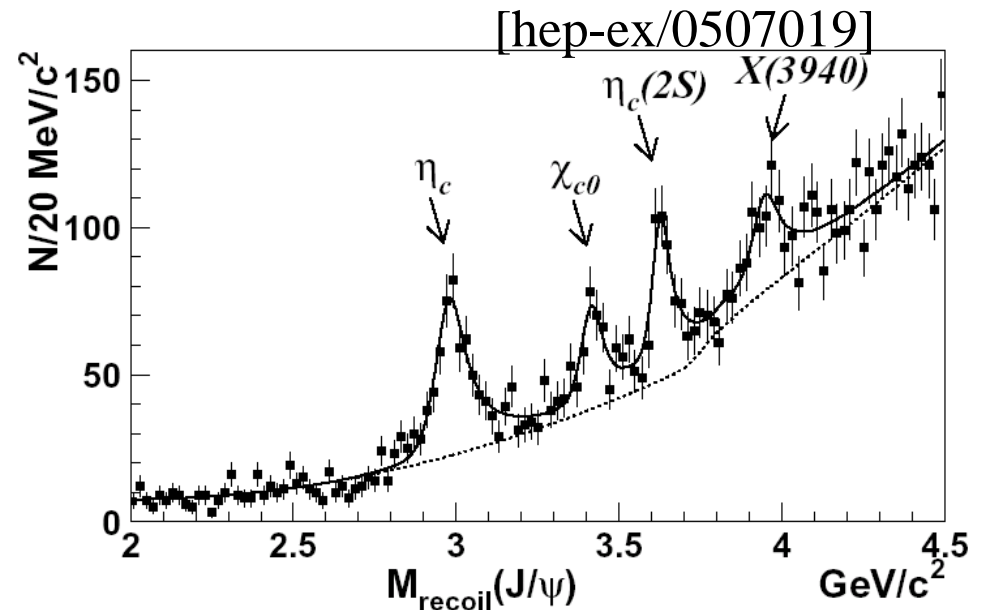
$$\text{BR}(X \rightarrow DD^*) = 96^{+45}_{-32} \pm 22\%$$

$$\text{BR}(X \rightarrow DD) < 41\% \text{ (90\% CL)}$$

Suggests unnatural parity state

$$\text{BR}(X \rightarrow \omega J/\psi) < 26\% \text{ (90\% CL)}$$

• Decay to  $DD^*$  but not  $DD$  suggests unnatural parity state



- Belle speculates that X is  $3^1S_0$  given the  $3^3S_1$   $\psi(4040)$ 
  - Mass is roughly correct
  - $\eta_c$  and  $\eta_c'$  are also produced in double charm production

See also Eichten Lane Quigg PRD73 014014(2006)
- Predicted width for  $3^1S_0$  with  $M=3943 \sim 50$  MeV close to  $\Gamma(X(3943))$  upper bound
- Identification of  $\psi(4040)$  as  $3^3S_1$  state implies hyperfine splitting 88 MeV with X(3943)
- Larger than the 2S splitting and larger than predicted in potential models
- Discrepancy could be due to:
  - Difficulty in fitting true pole position of  $3^3S_1$  state
  - Nearby thresholds with s-wave + p-wave charm mesons so possibly stronger threshold effects

**Test of  $3^1S_0$   $\eta_c$  assignment is search for this state in  $\gamma\gamma \rightarrow DD^*$**



# $\Upsilon(3940)$

See in  $\omega J/\psi$  subsystem of the decay  $B \rightarrow K \pi \pi J/\psi$

Belle: Phys. Rev. Lett. 94, 182002 (2005)

$$M = 3943 \pm 11 \pm 13 \text{ MeV}$$

$$\Gamma = 87 \pm 22 \pm 26 \text{ MeV}$$

Not seen in  $\Upsilon \rightarrow DD$  or  $DD^*$

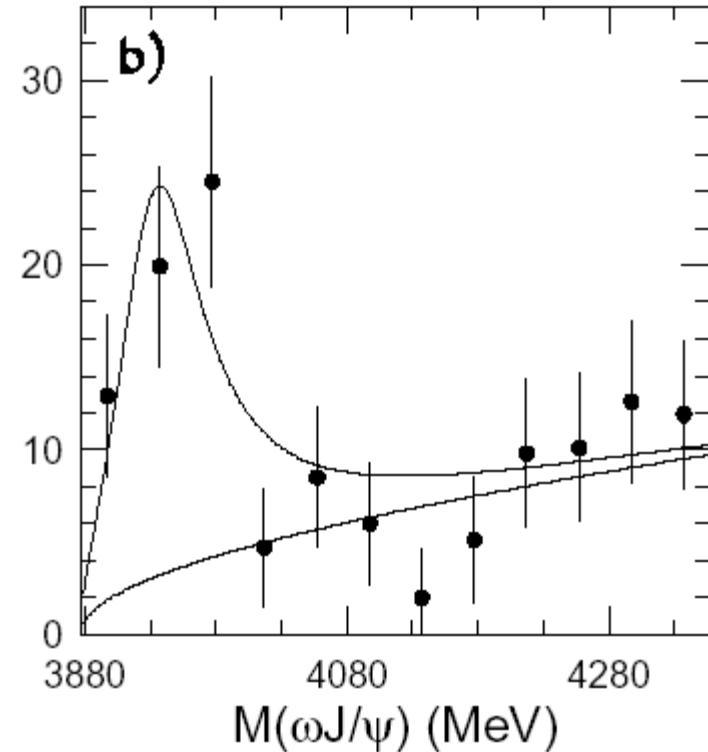
Mass and width suggest radially excited P-wave charmonium

**But  $\omega J/\psi$  decay mode is peculiar:**

$$BR(B \rightarrow KY) BR(\Upsilon \rightarrow \omega J/\psi) = 7.1 \pm 1.3 \pm 3.1 \cdot 10^{-5}$$

where one expects  $BR(B \rightarrow K\chi'_{cJ}) < BR(B \rightarrow K\chi_{cJ}) = 4 \cdot 10^{-4}$

Implies  $BR(\Upsilon \rightarrow \omega J/\psi) > 12\%$  which is unusual for state above open charm threshold



Possibility is  $2^3P_1$  cc state: identifies  $Y(3943)$  as  $2P \chi'_{c1}$

- $DD^*$  is the dominant decay mode
- Width consistent with  $Y(3943)$ :  $\Gamma=135$  MeV
- $\chi_{c1}$  is seen in B decays

•  $1^{++} \rightarrow \omega J/\psi$  is unusual

• but corresponding  $\chi'_{b1,2} \rightarrow \omega Y(1S)$  also seen

• Maybe rescattering:  $1^{++} \rightarrow DD^* \rightarrow \omega J/\psi$

• Maybe due to mixing with  $1^{++}$  molecular state  $X(3872)$ ?

• Important to - look for  $DD$  and  $DD^*$

- study angular distributions to  $DD$  and  $DD^*$



# Z(3930)

Belle: Phys Rev Lett 96, 082003(2006) [hep-ex/0512035]

- Observed by Belle in  $\gamma\gamma \rightarrow DD$

$$M = 3929 \pm 5 \pm 2 \text{ MeV}$$

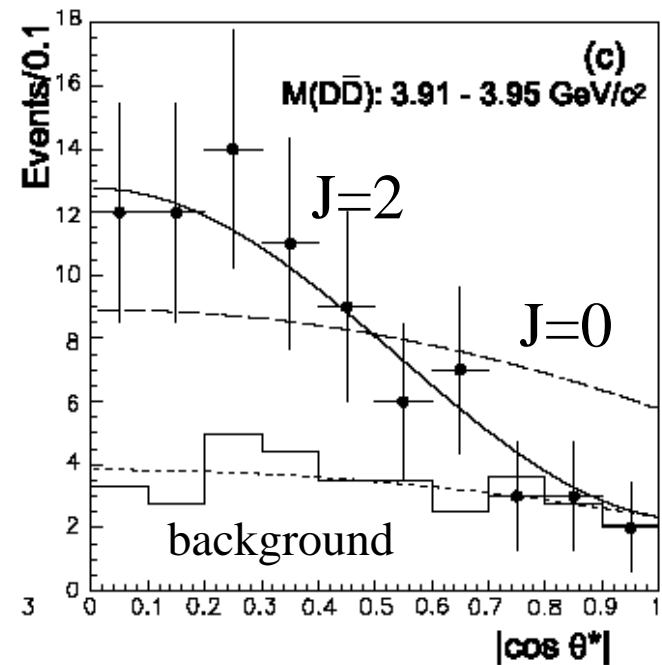
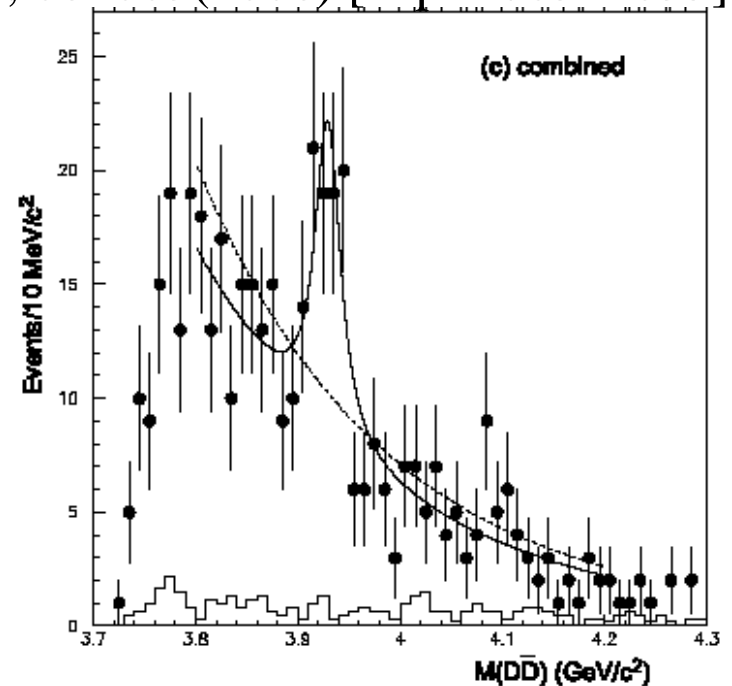
$$\Gamma = 29 \pm 10 \pm 2 \text{ MeV}$$

- Two photon width:

$$\Gamma_{\gamma\gamma} \cdot B_{DD} = 0.18 \pm 0.05 \pm 0.03 \text{ keV}$$

- DD angular distribution consistent with J=2

- Below  $D^* D^*$  threshold



• Obvious candidate for  $\chi'_{c2}$  (the  $\chi'_{c1}$  cannot decay to DD)

• Predicted  $\chi'_{c2}$  mass is 3972

$$\Gamma(\chi'_{c2} \rightarrow DD) = 21.5 \text{ MeV}$$

$$\Gamma(\chi'_{c2} \rightarrow DD^*) = 7.1 \text{ MeV}$$

$$\Gamma = 47 \text{ MeV assuming } M(\chi'_{c2}) = 3931$$

• In reasonable agreement with experiment

• Predicted  $\text{BR}(\chi'_{c2} \rightarrow DD) = 70\% \Rightarrow \Gamma_{\gamma\gamma} * B_{DD} = 0.47 \text{ keV}$   
( $\Gamma_{\gamma\gamma}$  from T. Barnes, IX<sup>th</sup> Intl. Conf. on  $\gamma\gamma$  Collisions, La Jolla, 1992.)

• Observed two-photon width about 1/2 predicted value for  $\chi'_{c2}$



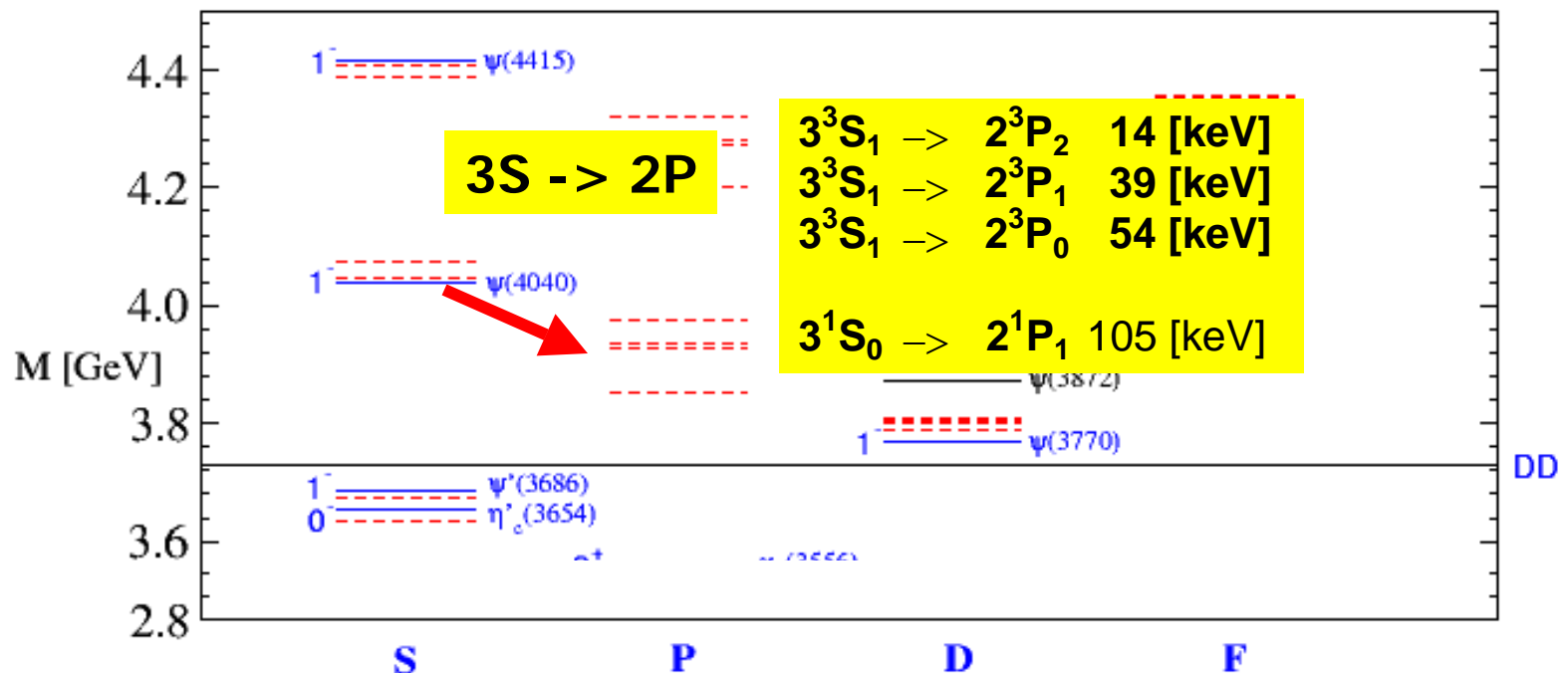
Could further study  $2^3P_J$  states via radiative transitions:

Can find all three  $^3 2P_J$  cc states using

$\psi(4040)$  and  $\psi(4160) \rightarrow \gamma DD, \gamma DD^*$

All three E1 rad BFs of the  $\psi(4040)$  are  $\sim 0.5 * 10^{-3}$ .

These would further test whether the  $Z, X, Y(3.9)$  are  $2P$  cc





# *X(3872)*

*New state 1<sup>st</sup> observed by Belle: X(3871)*

Phys Rev. Lett. 91, 2622001 (2003) [hep-ex/0309032]

Confirmed by: **CDF** Phys Rev. Lett. 93, 072001 (2004)

**DO** Phys Rev. Lett. 93, 162002 (2004)

**BABAR** Phys Rev. D71, 071103 (2005)

$M = 3872.0 \pm 0.6 \pm 0.5 \text{ MeV}$        $\Gamma < 2.3 \text{ MeV}$  at 90% C.L.  
width consistent with detector resolution.

1.  $D^0 D^{*0}$  molecule
2. A charmonium hybrid
3.  $2^3P_J$   $1^3D_2$  state?
4. Glueball?



# Charmonium Options for the X(3872)

T.Barnes,S.Godfrey, Phys Rev D69, 050400 (2004) [hep-ph/0311162]

Eichten, Lane & Quigg, Phys Rev D69, 094019 (2004) [hep-ph/0401210]

Barnes, Godfrey & Swanson, in preparation

*New state 1<sup>st</sup> observed by Belle: X(3871)* hep-ex/0309032

Observation of a new narrow charmonium state in exclusive

$B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$  decays

- $M = 3872.0 \pm 0.6 \pm 0.5$  MeV       $\Gamma < 2.3$  MeV at 90% C.L.  
width consistent with detector resolution.

1.  $D^0 D^{*0}$  molecule
2. A charmonium hybrid
3.  $1^3D_2$  state?



# $D^0D^{*0}$ molecule

Quantity	MeV	$M_X - M_{\text{threshold}}$
$M_X$	$3871.8 \pm 0.7 \pm 0.4$	
$M_{D^0} + M_{D^{*0}}$	$3871.5 \pm 0.7$	$+0.3 \pm 1.1$
$M_{D^+} + M_{D^{*+}}$	$3879.5 \pm 0.7$	$-7.7 \pm 1.1$

- . The mass of the state is right at the  $D^0D^{*0}$  threshold!  
This suggests a loosely bound  $D^0D^{*0}$  molecule, right below the dissociation energy
- “Molecular Charmonium” discussed in literature since 1975



# Charmonium Options for the X(3872)

- Consider all 1D and 2P cc possibilities
- Assume  $M=3872$  MeV
  - calculate radiative widths and
  - strong decay widths



# Strong Decays:

## 1. Zweig-allowed open-charm decays (DD)

expect  $1^3D_2$  and  $1^1D_2$  but  $1^3D_3$  also narrow because of angular momentum barrier

## 2. Annihilation type decays

summarized in Ref.[50]. Expressions for decay widths relevant to the 1D and 2P  $c\bar{c}$  states in particular are:

$$\Gamma(^3D_J \rightarrow ggg) = \frac{10\alpha_s^3}{9\pi} C_J \frac{|R_D''(0)|^2}{m_Q^6} \ln(4m_Q\langle r \rangle) \quad (7)$$

$$\Gamma(^1D_2 \rightarrow gg) = \frac{2\alpha_s^2}{3} \frac{|R_D''(0)|^2}{m_Q^6} \quad (8)$$

$$\Gamma(^3P_2 \rightarrow gg) = \frac{8\alpha_s^2}{5} \frac{|R_P'(0)|^2}{m_Q^4} \quad (9)$$

$$\Gamma(^3P_1 \rightarrow q\bar{q}g) = \frac{8n_f\alpha_s^3}{9\pi} \frac{|R_P'(0)|^2}{m_Q^4} \ln(m_Q\langle r \rangle) \quad (10)$$

$$\Gamma(^1P_1 \rightarrow ggg) = \frac{20\alpha_s^3}{9\pi} \frac{|R_P'(0)|^2}{m_Q^4} \ln(m_Q\langle r \rangle) \quad (11)$$

$$\Gamma(^1P_1 \rightarrow gg\gamma) = \frac{36}{5} e_q^2 \frac{\alpha}{\alpha_s} \Gamma(^1P_1 \rightarrow ggg) \quad (12)$$

$$\Gamma(^3P_0 \rightarrow gg) = 6\alpha_s^2 \frac{|R_P'(0)|^2}{m_Q^4} \quad (13)$$

## 3. Hadronic transitions



# Radiative transitions:

$$\Gamma(n^{2S+1}L_J \rightarrow n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3} e_c^2 \alpha \omega^3 C_{fi} \delta_{SS'} |\langle n'^{2S'+1}L'_{J'} | r | n^{2S+1}L_J \rangle|^2,$$

$$C_{fi} = \max(L, L')(2J' + 1) \left\{ \begin{matrix} L' & J' & S \\ J & L & 1 \end{matrix} \right\}^2.$$

TABLE II: Radiative transitions in scenario 1: Predictions for the E1 transitions 1D→1P, 2P→2S, 2P→1S and 2P→1D, assuming in all cases that the initial  $c\bar{c}$  state has a mass of 3872 MeV. The matrix elements were obtained using the wavefunctions of the Godfrey-Isgur model, Ref.[17]. Unless otherwise stated, the widths are given in keV and the final  $c\bar{c}$  masses are PDG values [38].

Initial state X(3872)	Final state	$M_f$ (MeV)	$\omega$ (MeV)	$\langle f r i \rangle$ (GeV <sup>-1</sup> )	$C_{fi}$	Width (keV)
1 <sup>3</sup> D <sub>3</sub>	$\chi_{c2}(1^3P_2) \gamma$	3556.2	303	2.762	$\frac{2}{5}$	367
1 <sup>3</sup> D <sub>2</sub>	$\chi_{c2}(1^3P_2) \gamma$	3556.2	303	2.769	$\frac{1}{10}$	92
	$\chi_{c1}(1^3P_1) \gamma$	3510.5	345	2.588	$\frac{3}{10}$	356
1 <sup>3</sup> D <sub>1</sub>	$\chi_{c2}(1^3P_2) \gamma$	3556.2	303	2.769	$\frac{1}{90}$	10.2
	$\chi_{c1}(1^3P_1) \gamma$	3510.5	345	2.598	$\frac{1}{6}$	199
	$\chi_{c0}(1^3P_0) \gamma$	3415	430	2.390	$\frac{2}{9}$	437
1 <sup>1</sup> D <sub>2</sub>	$h_c(1^1P_1) \gamma$	3517 <sup>a</sup>	339	2.627	$\frac{2}{5}$	464



TABLE IV: Partial widths and branching fractions for strong and electromagnetic transitions in scenario 1: We assume in all cases that the initial  $c\bar{c}$  state has a mass of 3872 MeV. Details of the calculations are given in the text.

Initial state	Final state	Width (MeV)	B.F. (%)
$1^3D_3$	DD	4.04	84.2
	$ggg$	0.18	3.8
	$J/\psi\pi\pi$	$0.21 \pm 0.11$	4.4
	$\chi_{c2}(1^3P_2)\gamma$	0.37	7.7
	Total	4.80	100
$1^3D_2$	$ggg$	0.08	10.8
	$J/\psi\pi\pi$	$0.21 \pm 0.11$	28.4
	$\chi_{c2}(1^3P_2)\gamma$	0.09	12.2
	Total	0.74	100
$1^3D_1$	DD	184	98.9
	$ggg$	1.15	0.6
	$J/\psi\pi\pi$	$0.21 \pm 0.11$	0.1
	$\chi_{c1}(1^3P_1)\gamma$	0.20	0.1
	Total	186	100
$1^1D_2$	$gg$	0.19	22.1
	$\eta_c\pi\pi$	$0.21 \pm 0.11$	24.4
	$h_c(1^1P_1)\gamma$	0.46	53.5
	Total	0.86	100

$2^3P_2$	DD	21.1	82.4
	$gg$	4.4	17.2
	$\psi'(2^3S_1)\gamma$	0.06	0.2
	$J/\psi(1^3S_1)\gamma$	0.04	0.2
	Total	25.6	100
$2^3P_1$	$q\bar{q}g$	1.65	95.9
	$\psi'(2^3S_1)\gamma$	0.06	3.5
	$J/\psi(1^3S_1)\gamma$	0.01	0.6
	Total	1.72	100
$2^3P_0$	DD	13.7 (see text)	24.6
	$gg$	42.	75.3
	$\psi'(2^3S_1)\gamma$	0.07	0.1
	$\psi'(1^3D_1)\gamma$	0.02	$4 \times 10^{-2}$
	Total	55.8	100
$2^1P_1$	$ggg$	1.29	81.6
	$gg\gamma$	0.13	8.2
	$\eta'_c(2^1S_0)\gamma$	0.09	5.7
	$\eta_c(1^1S_0)\gamma$	0.07	4.4
	Total	1.58	100

too wide

too wide

too wide



## $1^3D_2$ and $1^1D_2$ and $1^3D_3$

$1^3D_2$	$ggg$	0.08	10.8
	$J/\psi\pi\pi$	$0.21 \pm 0.11$	28.4
	$\chi_{c2}(1^3P_2)\gamma$	0.09	12.2
	$\chi_{c1}(1^3P_1)\gamma$	0.36	48.6
	Total	0.74	100

$1^1D_2$	$gg$	0.19	22.1
	$\eta_c\pi\pi$	$0.21 \pm 0.11$	24.4
	$h_c(1^1P_1)\gamma$	0.46	53.5
	Total	0.86	100

$1^3D_3$	DD	4.04	84.2
	$ggg$	0.18	3.8
	$J/\psi\pi\pi$	$0.21 \pm 0.11$	4.4
	$\chi_{c2}(1^3P_2)\gamma$	0.37	7.7
	Total	4.80	100

## $2^3P_1$ and $2^1P_1$

$2^3P_1$	$q\bar{q}g$	1.65	95.9
	$\psi(2^3S_1)\gamma$	0.06	3.5
	$J/\psi(1^3S_1)\gamma$	0.01	0.6
	Total	1.72	100

$2^1P_1$	$ggg$	1.29	81.6
	$gg\gamma$	0.13	8.2
	$\eta'_c(2^1S_0)\gamma$	0.09	5.7
	$\eta_c(1^1S_0)\gamma$	0.07	4.4
	Total	1.58	100

The problem here is that the BR to  $\gamma$  and  $\pi\pi$  is quite small and not the final states being looked for





## Consider the charmonium possibilities:

T.Barnes,S.Godfrey, PR D69, 050400 (2004)  
Eichten, Lane, Quigg, PR D69, 094019 (2004)  
Barnes, Godfrey, Swanson, PR D 054026 (2005)

1D and 2P multiplets only states nearby in mass

$1^1D_2$   $2^3P_0$   $2^3P_1$   $2^3P_2$  have  $C=+$

But  $X(3872) \rightarrow \gamma J/\psi$  implies  $C=+$  Belle [hep-ex/0505037]

Babar Gowdy Moriond talk

Angular distributions favour  $J^{PC}=1^{++}$  Belle [hep-ex/0505038]

The unique surviving charmonium candidate is  $2^3P_1$

**BUT** identification of  $Z(3931)$  with  $2^3P_2$   
implies 2P mass  $\sim 3940$  MeV

$D^0\bar{D}^{*0}$  molecule or "tetraquark"

is a popular/likely explanation: see Voloshin



# $\Upsilon(4260)$

Discovered by Babar as enhancement in  $\pi\pi J/\psi$  subsystem

in  $e^+e^- \rightarrow \gamma_{\text{ISR}} \Upsilon\pi\pi$

PRL 95, 142001(2005) [hep-ex/0506081]

$M = 4259 \pm 8 \pm 4 \text{ MeV}$

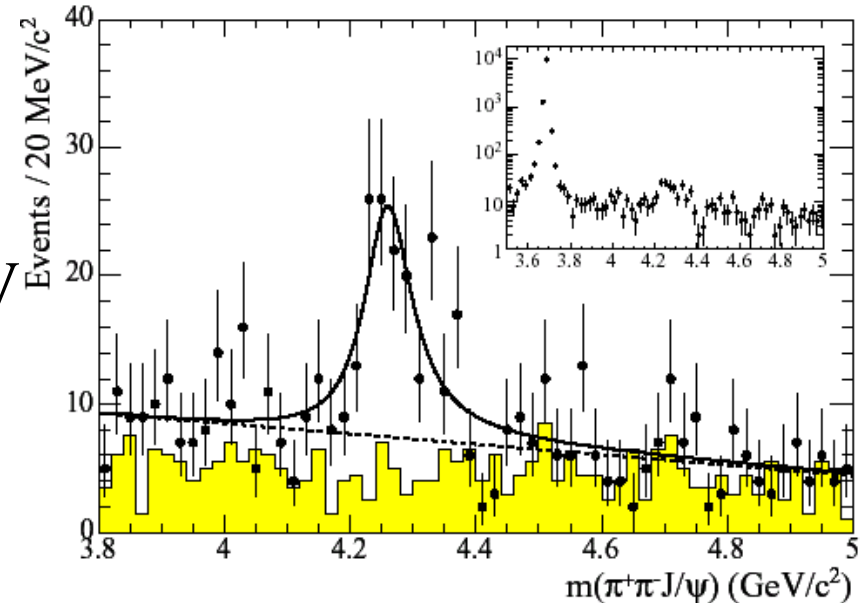
$\Gamma = 88 \pm 23 \pm 5 \text{ MeV}$

$\Gamma_{ee} \times \text{BR}(\Upsilon \rightarrow \pi^+\pi^- J/\psi) = 5.5 \pm 1.0 \pm 0.8 \text{ eV}$

ISR production tells us  $J^{PC} = 1^{--}$

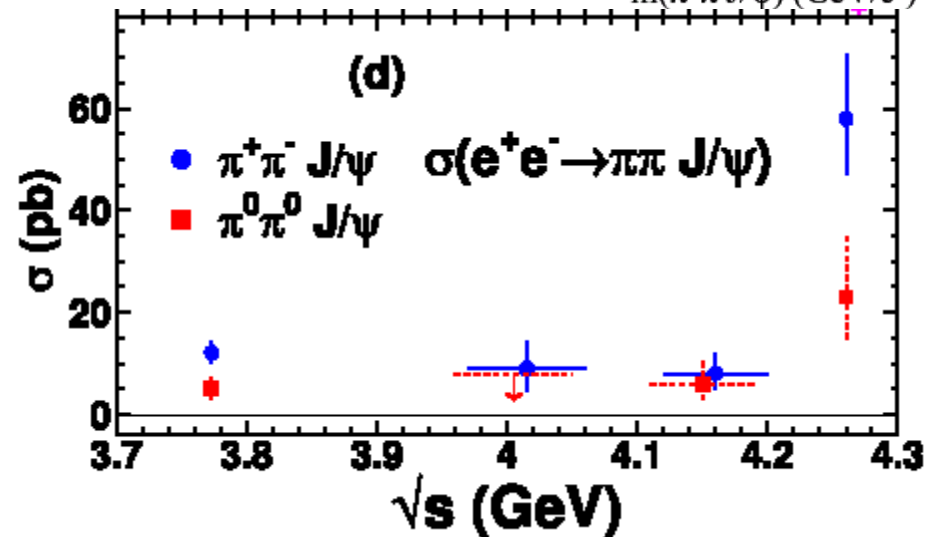
Further evidence in

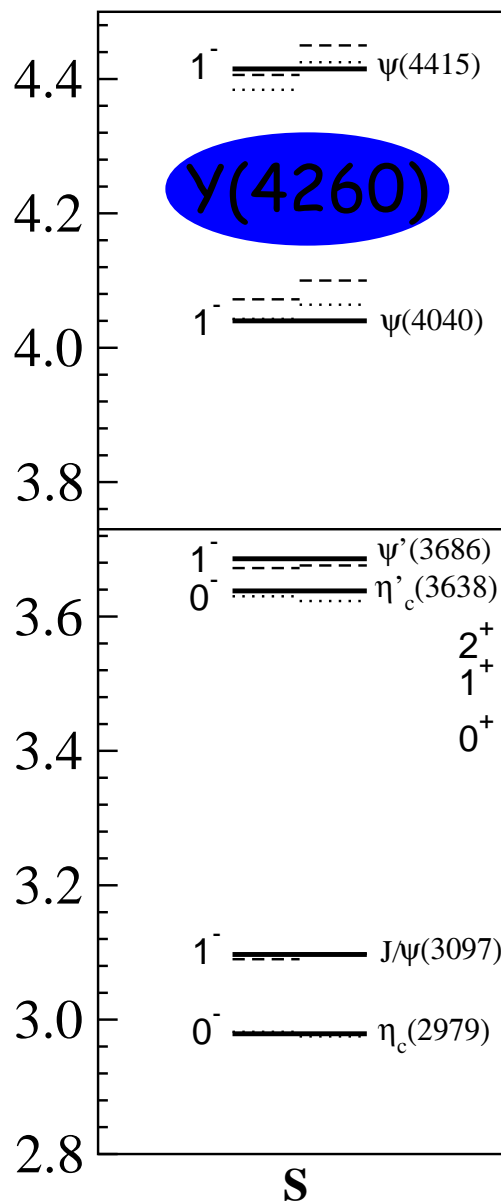
$B \rightarrow K(\pi^+\pi^- J/\psi)$  PR D73, 011101(2006)



Confirmed by CLEO

hep-ex/0602034





- The first unaccounted  $1^-$  state is the  $\psi(3D)$
- Quark models estimate  $M(\psi(3D)) \sim 4500$  MeV much too heavy for the  $Y(4260)$

$Y(4260)$  represents an overpopulation of expected  $1^-$  states

Absence of open charm production also against conventional  $cc$  state

Other explanations are:

- $\psi(4S)$  Phys Rev D72, 031503 (2005)
- Tetraquark Phys Rev D72, 031502 (2005)
- $cc$  hybrid Phys Lett B625, 212 (2005);  
Phys Lett B628, 215 (2005)  
Phys Lett B631, 164 (2005)



## *Y(4260): Hybrid?*

- Flux tube model predicts lowest cc hybrid at 4200 MeV
- LGT expects lowest cc hybrid at 4200 MeV [Phys Lett B401, 308 (1997)]
- Models of hybrids say  $\Psi(0)=0$  so would have small  $e^+e^-$  width
- LGT found bb hybrids have large couplings to closed flavour modes
  - Similar to BaBar observation of  $Y \rightarrow \pi^+\pi^-J/\psi$ :  
$$\text{BR}(Y \rightarrow \pi^+\pi^-J/\psi) > 8.8\%$$
$$\Gamma(Y \rightarrow \pi^+\pi^-J/\psi) > 7.7 \pm 2.1 \text{ MeV}$$
- Much larger than typical charmonium transitions:  
$$\Gamma(\psi(3770) \rightarrow \pi^+\pi^-J/\psi) \sim 80 \text{ keV}$$
- Y is seen while  $\psi(4040)$ ,  $\psi(4160)$   $\psi(4415)$  are not



# How to test $Y(4260)$ hybrid assignment:

## Decays:

- LGT study suggest searching for other closed charm modes with  $J^{PC}=1^{--}$   $J/\psi\eta$ ,  $J/\psi\eta'$ ,  $\chi_{J\omega}$  . . .
- Models predict the dominant hybrid charmonium open-charm decay modes will be a meson pair with S-wave ( $D$ ,  $D^*$ ,  $D_s$ ,  $D_s^*$ ) + P-wave ( $D_J$ ,  $D_{sJ}$ )
- The dominant decay mode expected to be  $D+D_1(2430)$   
 $D_1(2420)$  has width  $\sim 300$  MeV and decays to  $D^*\pi$
- **Suggests search for  $Y(4260)$  in  $DD^*\pi$**
- Evidence of large  $DD_1(2430)$  signal would be strong evidence for hybrid
- But models of hybrids are untested so to be cautious
- If seen in other modes like  $DD^*$ ,  $D_sD_s^*$  comparable to  $\pi^+\pi^-J/\psi$  maybe still hybrid but decay model not accurate



## Search for Partner States: (fill in the multiplet)

- Mass *ca.* 4.0 - 4.5 GeV, with LGT preferring the higher range.  
(e.g.: X.Liao and T.Manke, hep-lat/0210030)
- Confirm that no  $c\bar{c}$  states with the same  $J^{PC}$  are expected at this mass.
- Identify  $J^{PC}$  partners of the hybrid candidate nearby in mass.
- The most convincing evidence:
  - partners, especially  $J^{PC}$  exotics.
- The f-t model expects:  
 $0^{+-}, 1^{-+}, 2^{+-}, 0^{-+}, 1^{+-}, 2^{-+}, 1^{++}, 1^{--}$



# Summary

Many new results, considerable progress!

$D_{sJ}(2317)$	Most likely $0^+(c\bar{s})$
$D_{sJ}(2460)$	Most likely $1^+(c\bar{s})$
$D_{sJ}(2632)$	Needs confirmation
$X(3872)$	Molecule? - see Voloshin
$X(3943)$	$\eta''_c (3^1S_0)$ -look for $\gamma\gamma \rightarrow DD^*$
$Y(3943)$	$\chi'_{c1} (2^3P_1)$ -look for $DD$ & $DD^*$
$Z(3930)$	$\chi'_{c2} (2^3P_2)$ -confirm by $DD^*$
$Y(4260)$	Hybrid?

• Much more to learn; ie search for  $1^3D_3$   $1^3D_2$   $1^1D_2$   $1^3F_2$   $1^3F_4$

