The Standard Model

OUTLINE

Electroweak Unification

- Chiral fermion states
- Electroweak reactions: $e^+e^- \rightarrow f \bar{f}$
- PETRA and LEP1 data

Higgs Field

- Spontaneous symmetry breaking
- LEP2 and LHC

Weak Mixing in the Quark Sector

- CKM matrix
- CP violation in the B^0 system

Weak Mixing in the Neutrino Sector

- SNO
- Neutrino masses and Oscillations

Conclusion

Elementary Particles



Standard Model



The SM provides a general description of the physics physics currently accessible with modern particle accelerators. The minimal SM postulates that matter is composed of fundamental spin- $\frac{1}{2}$ quarks and spin- $\frac{1}{2}$ leptons interacting via spin one gauge bosons.

Electroweak Lagrangian:

 $\mathcal{L} = \mathcal{L}(\text{weak CC}) + \mathcal{L}(\text{weak NC}) + \mathcal{L}(\text{em NC})$ $\mathcal{L}(\text{weak CC}) = \frac{g}{\sqrt{2}} \left(J_{\mu}^{-} W^{\mu +} + J_{\mu}^{+} W^{\mu -} \right)$ $\mathcal{L}(\text{weak NC}) = \frac{g}{\cos \theta_{W}} \left(J_{\mu}^{3} - \sin^{2} \theta_{W} J_{\mu}^{\text{em}} \right) Z^{\mu}$ $\mathcal{L}(\text{em NC}) = e J_{\mu}^{\text{em}} A^{\mu}$

QCD: The gluon couples to the color charge of the quark. The strong potential for short interquark distances $(r \lesssim R_{\rm hadron} \simeq 1/\Lambda_{\rm QCD} \simeq 1 \text{ fm})$ is:

$$V_{\rm QCD} \simeq -\frac{4\,\alpha_s}{3\,r}\,,$$

where α_s is the strong coupling constants between quarks and gluons. At large distances (r > 1 fm), a confining term must be added to confine quarks inside hadrons.

Chiral Fermion States

Unification of the E&M and weak interactions: the former has a purely vectorial coupling $[\gamma_{\mu}]$ while the latter as a V - A character $[\gamma_{\mu}(1 - \gamma_5)]$. Let's absorb the $(1 - \gamma_5)$ in the definition of the spinors:

$$u_L(p) = \frac{(1 - \gamma_5)}{2} u(p)$$
 and $v_R(p) = \frac{(1 - \gamma_5)}{2} v(p)$

$$u_R(p) = \frac{(1+\gamma_5)}{2} u(p)$$
 and $v_L(p) = \frac{(1+\gamma_5)}{2} v(p)$

Here L =left-handed and R =right-handed. Thus:



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Pure vectorial weak vertex ____

Conseqences:

$$u = \frac{(1 - \gamma_5)}{2}u + \frac{(1 + \gamma_5)}{2}u = u_L + u_R$$

and

$$\bar{u} = \bar{u}\frac{(1+\gamma_5)}{2} + \bar{u}\frac{(1-\gamma_5)}{2} = \bar{u}_L + \bar{u}_R$$

Plus,

$$\bar{u}_L \gamma_\mu u_R = \bar{u}_R \gamma_\mu u_L = 0$$

Thus the electromagntic current can be written as:

$$j^{em}_{\mu} = -\bar{e}\gamma_{\mu}e = -\bar{e}_L\gamma_{\mu}e_L - \bar{e}_R\gamma_{\mu}e_R$$

Define:
$$j^{\pm}_{\mu} = \bar{\chi}_L \gamma_{\mu} \sigma^{\pm} \chi_L$$
 with $\sigma^{\pm} = \frac{1}{2} (\sigma^1 \pm i \sigma^2)$:

$$ec{j}_{\mu} = rac{1}{2} ar{\chi}_L \gamma_\mu \, ec{\sigma} \, \chi_L \;\; ext{and} \;\; J^Y_\mu = 2 j^{em}_\mu - 2 j^3_\mu$$

with

$$\chi_{L}$$

$$\begin{pmatrix} \nu_{e} \\ e \end{pmatrix}_{L} \begin{pmatrix} \nu_{\mu} \\ \mu \end{pmatrix}_{L} \begin{pmatrix} \nu_{\tau} \\ \tau \end{pmatrix}_{L}$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_{L} \begin{pmatrix} c \\ s' \end{pmatrix}_{L} \begin{pmatrix} t \\ b' \end{pmatrix}_{L}$$

$$SU(2)_{L} \otimes U(1)_{Y}$$
Unified electro-weak vertex:

$$-i \left[g_{w} \vec{j}_{\mu} \cdot \vec{W}^{\mu} + \frac{g'}{2} J_{\mu}^{Y} B^{\mu} \right]$$
with $W_{\mu}^{\pm} \equiv \frac{1}{\sqrt{2}} (W_{\mu}^{1} \mp i W_{\mu}^{2})$
 $\vec{j}_{\mu} \cdot \vec{W}^{\mu} = j_{\mu}^{1} W^{\mu 1} + j_{\mu}^{2} W^{\mu 2} + j_{\mu}^{3} W^{\mu 3}$
 $\vec{j}_{\mu} \cdot \vec{W}^{\mu} = \frac{1}{\sqrt{2}} j_{\mu}^{+} W^{\mu +} + \frac{1}{\sqrt{2}} j_{\mu}^{-} W^{\mu -} + j_{\mu}^{3} W^{\mu 3}$

such that

$$-\frac{ig_w}{\sqrt{2}}j_{\mu}^{\pm} = -\frac{ig_w}{2\sqrt{2}}[\bar{u}\gamma_{\mu}(1-\gamma_5)u]W^{\mu\pm}$$

The neutral underlying $SU(2)_L \otimes U(1)_Y$ allow the W^3 and the B to mix to the physical states called photon and the Z:

 $A_{\mu} = B_{\mu} \cos \theta_w + W_{\mu}^3 \sin \theta_w$ $Z_{\mu} = -B_{\mu} \sin \theta_w + W_{\mu}^3 \cos \theta_w$

and with $g_w \sin \theta_w = g' \cos \theta_w = g_e$ then

$$-i\left[g_w\,j_\mu^3\cdot W^{\mu3}+rac{g'}{2}J^Y_\mu B^\mu
ight]
onumber\ -ig_ej^{em}_\mu A^\mu-ig_z(j^3_\mu-\sin^2 heta_wj^{em}_\mu)Z^\mu$$

Weak Neutral Current

Knowing that $j_{\mu}^{em} = j_{\mu}^3 + \frac{1}{2}j_{\mu}^Y$ and $g_Z = \frac{g_e}{\sin \theta_w \cos \theta_w}$.

 $-ig_z \, (j^3_\mu - \sin^2 heta_w j^{em}_\mu) Z^\mu \qquad [Z^0 \, \, {
m weak \, current}]$

where within a particle doublet:

$$j_{\mu}^{em} = \sum_{i=1}^{2} Q_i (\bar{u}_{iL} \gamma_{\mu} u_{iL} + \bar{u}_{iR} \gamma_{\mu} u_{iR})$$

Then the $Z^0 \rightarrow f \bar{f}$ vertex factor depends on the particular quark and lepton (*i.e.* f) involved:

$$\frac{-ig_Z}{2}\gamma^{\mu}(c_V^f - c_A^f\gamma_5) \qquad [Z^0 \text{ vertex factor}]$$

with

f	c_V	c_A
$ u_\ell$	$\frac{1}{2}$	$\frac{1}{2}$
ℓ	$-\frac{1}{2} + 2\sin^2\theta_w$	$-\frac{1}{2}$
q	$\frac{1}{2} - \frac{4}{3}\sin^2 heta$	$\frac{1}{2}$
q'	$-\frac{1}{2}+\frac{2}{3}\sin^2 heta_w$	$-\frac{1}{2}$

Weinberg Angle

Reaction like the *pure* neutral reaction $u_{\mu} + e^- \rightarrow \nu_e + \mu^-$ were used to make the first measurement in 1973 of c_V^{ℓ} and c_A^{ℓ} and thus:

 $\sin^2 \theta_w = 0.22 \pm 0.03$

At SLC/LEP Z^0 bosons were produced copiously and it allowed a very precise determination of the properties Z. Using all experimental data:

 $\sin^2 \theta_w = 0.23147 \pm 0.00016$



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Mass & Width of the Z Boson



Total Width (final LEP average June 2001) $\Gamma_{\rm Z} \equiv \Gamma({\rm Z} \rightarrow {X}) = 2.4952 \pm 0.0023 \text{ GeV}$

Hadronic Width

 $\Gamma(Z \rightarrow q\overline{q}) = 1.7442 \pm 0.0020 \text{ GeV}$

Leptonic Width

 $\Gamma(Z \to \ell^+ \ell^-) = 0.083991 \pm 0.000087 \text{ GeV}$

Such that

 $\Gamma_{\rm Z} = \Gamma({\rm Z} \to {\rm q}{\rm \overline{q}}) + 3\,\Gamma({\rm Z} \to \ell^+\ell^-) + {\rm invisible}$

Number of Neutrino Types

Invisible Width (calculated) $\Gamma(Z \rightarrow \nu_{\ell} \bar{\nu}_{\ell}) = 0.1666 \text{ GeV}$

$$\label{eq:Gamma-Lambda} \begin{split} \mathbf{Thus} \\ \Gamma_{\rm Z} &= \Gamma_{\rm q\overline{q}} + 3\,\Gamma_{\ell\bar{\ell}} + N_{\nu}\,\Gamma_{\nu\bar{\nu}} \end{split}$$

 $\label{eq:nonlinear} \begin{array}{l} \mbox{Implies} \\ N_{\nu} = 2.9841 \pm 0.0083 \end{array}$



Higgs Field

Physicists have theorized the existence of the so-called Higgs field, which in theory interacts with other particles to give them **mass**. The Higgs field requires a particle, the Higgs boson. The Higgs boson has not been observed, but physicists are looking for it with great enthusiasm.

Limit on the SM Higgs mass with data collected at LEP2 $\sqrt{s} = 161 - 210$ GeV:

$m_H > 113.5$ GeV (95% CL)

Spontaneous symmetry breaking:

The form of the Lagrangian that couples the Higgs field and the quarks is constrained by $SU(2)_L$ gauge invariance:

$$\mathcal{L} = \sum_{j,k} [Y_{jk}(\bar{u}_L^j, \bar{d}_L^j) \begin{pmatrix} \phi^0 \\ -\phi^+ \end{pmatrix}^* u_R^k + Y'_{jk}(\bar{u}_L^j, \bar{d}_L^j) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R^k]$$

where j and k run over quarks generations, L and R denotes left- and right-handed component, and Y_{jk} and Y'_{jk} are the Yukawa couplings. The complex Higgs doublet undergoes spontaneous symmetry breaking:

$$\left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right) \to \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + H(x) \end{array}\right)$$

where v is the Higgs vacuum expectation and H(x) is the Higgs field corresponding to the Higgs boson.

$$\mathcal{L} = \sum_{j,k} [Y_{jk} \bar{u}_L^j u_R^k + Y'_{jk} \bar{d}_L^j d_R^k] \frac{1}{\sqrt{2}} [v + H(x)]$$

The term proportional to v generate the quark mass $m_{jk} \equiv \frac{-v}{\sqrt{2}} Y_{jk}$ and $m'_{jk} \equiv \frac{-v}{\sqrt{2}} Y'_{jk}$.



LHC: Large Hadron Collider ___

The LHC, in construction at CERN, is a proton-proton collider with $\sqrt{s} = 14 \text{ TeV}$. The SPS collider which discovered the W - Z bosons had $\sqrt{s} = 0.45 \text{ TeV}$ and the Tevatron collider at FermiLab has $\sqrt{s} = 1.8 \text{ TeV}$.

Higgs events at the LHC



LHC will take data in 2007 !!!

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NLC: Next linear Collider

The Next Linear Collider (NLC) is proposed as the future generation of accelerator to probe matter. The design of the NLC is a 0.5 TeV e^+e^- collider to investigate the properties of the W - Z bosons, the top quark, and their couplings; and search for super-symmetric particles (SUSY).

Superconducting Acceleration Cavity



CKM Matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
 [CKM Matrix]

such that

$$\left(\begin{array}{c}d'\\s'\\b'\end{array}\right) = \left(\begin{array}{ccc}V_{ud} & V_{us} & V_{ub}\\V_{cd} & V_{cs} & V_{cb}\\V_{td} & V_{ts} & V_{tb}\end{array}\right) \left(\begin{array}{c}d\\s\\b\end{array}\right)$$

The CKM matrix can be decomposed as:

	1 0 0	$0 \\ c_{23} \\ -s_{23}$	$\begin{array}{c} 0 \\ s_{23} \\ c_{23} \end{array}$	$\left(\begin{array}{c}c_{13}\\0\\-s_{13}\gamma\end{array}\right)$	$\begin{array}{c} 0 \\ 1 \\ 0 \end{array}$	$s_{13} egin{smallmed} & eta \\ & 0 \\ & c_{13} \end{array}$	$\begin{smallmatrix}c_{12}\\-s_{12}\\0\end{smallmatrix}$	${}^{s_{12}}_{{}^{c_{12}}_0}$)
×		20	20 /	(157		10			

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, and i, j denote the quark generations. The middle matrix has incorporate the complex phase δ such that $\beta = e^{-i\delta}$ and $\gamma = e^{i\delta}$ to describe a rotation between quarks that are two generations apart. Multiplying these matrices:

1	c ₁₂ c ₁₃	^s 12 ^c 13	$s_{13}e^{-i\delta}$
	$-s_{12}c_{23} - c_{12}s_{23}s_{13}e_{15}^{io}$	$c_{12}c_{23} - s_{12}s_{23}s_{13}e^{io}$	$s_{23}c_{13}$
	$s_{12}s_{23} - c_{12}c_{23}s_{13}e^{io}$	$-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{io}$	$c_{23}c_{13}$

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Wolfenstein Parameterization

Based on hierarchical we can expand in powers of the Cabibbo angle $\lambda = s_{12} = 0.22$, with $s_{23} = A\lambda^2$ and $s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Complex phase allow CP violation in the framework of Standard Model for a 3×3 CKM matrix

Or be neglecting the CP phase and the small $b \rightarrow u$ and $t \rightarrow d$ transitions:

$$V \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & 0 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ 0 & -A\lambda^2 & 1 \end{pmatrix}$$

By neglecting the s_{23} we have the simple Cabbibo description $\theta_C \equiv \lambda$:

$$V \sim \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & 0\\ -\lambda & 1 - \frac{1}{2}\lambda^2 & 0\\ 0 & 0 & 1 \end{pmatrix} \simeq \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0\\ -\sin\theta_C & \cos\theta_C & 0\\ 0 & 0 & 1 \end{pmatrix}$$



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B Factories

Time evolution of the B^0 system since the integration over time gives simply the mass difference and NOT the CP phase: Asymmetric B-factories [BaBar & Belle] operating at the $\Upsilon(4S)$ with luminosity $\sim 10^{34} cm^{-2} s^{-1}$ (PETRA in the 1980's had $\mathcal{L} \sim 10^{31} cm^{-2} s^{-1}$!!!).

Measure the angles of the unitary triangle



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Sudbury

Neutrino

Observatory





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Construction Phase





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Solar Neutrino Event



Cherenkov Light

When a particle travels through a medium such that its velocity v is greater than the velocity of light in the medium c/n, radiation is emitted. The radiation is confined to a **cone** around the direction of the incident particle.

SNO = Heavy Water Cherenkov Detector



Solar Neutrino Problem

The Solar Neutrino Problem

	BP SSM	Expt	Expt/BPSSM
Homestake	$9.3^{+1.2}_{-1.4}$ a)	$2.55 \pm 0.14 \pm 0.14$ a)	0.273 ± 0.021
Kamiokande Super-Kamiokande Combined	$6.62^{+0.93}_{-1.12}$ b)	$\begin{array}{c} 2.80 \pm 0.19 \pm 0.33 \\ 2.51 \substack{+0.14 \\ -0.13} \pm 0.18 \\ 2.586 \pm 0.195 \\ \mathrm{b} \end{array}$	0.423 ± 0.058 0.379 ± 0.029 0.391 ± 0.029
SAGE GALLEX Combined	137^{+8}_{-7} a)		$0.504 \pm 0.089 \\ 0.509 \pm 0.059 \\ 0.507 \pm 0.049$

Units a) SNU $(10^{-36}/s/tgt atom)$ b) $10^{6}/cm^{2}/s$



From Hata and Langacker, preprint 1997.

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Neutrino Oscillations

Neutrino Oscillations

For simplicity consider only two neutrino flavours, ν_e , ν_{μ}

• Suppose the flavour eigenstates, $|\nu_e\rangle$, $|\nu_{\mu}\rangle$ are not the mass eigenstates, $|\nu_1\rangle$, $|\nu_2\rangle$ Then the flavour eigenstates can be represented as a superposition of the mass eigenstates,

$$\left(\begin{array}{c}\nu_e\\\nu_\mu\end{array}\right) = \left(\begin{array}{c}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{array}\right) \left(\begin{array}{c}\nu_1\\\nu_2\end{array}\right)$$

where θ is the mixing angle between the mass states.



The time evolution of the flavour states becomes,

$$\begin{aligned} |\nu_e\rangle_t &= \cos\theta e^{-iE_1t} |\nu_1\rangle + \sin\theta e^{-iE_2t} |\nu_2\rangle \\ |\nu_\mu\rangle_t &= -\sin\theta e^{-iE_1t} |\nu_1\rangle + \cos\theta e^{-iE_2t} |\nu_2\rangle \end{aligned}$$

Writing the time evolution in terms of the mass matrix gives,

$$i\frac{d}{dt}\left(\begin{array}{c}\nu_{e}\\\nu_{\mu}\end{array}\right) = \frac{1}{2}\left(\begin{array}{c}-\frac{\Delta m^{2}}{2E}\cos 2\theta & \frac{\Delta m^{2}}{2E}\sin^{2}\theta\\\frac{\Delta m^{2}}{2E}\sin^{2}\theta & \frac{\Delta m^{2}}{2E}\cos 2\theta\end{array}\right)\left(\begin{array}{c}\nu_{e}\\\nu_{\mu}\end{array}\right)$$

where E is the energy of the electron neutrino in MeV and

$$\Delta m^2 \equiv |m_2^2 - m_1^2|$$

• The survival probability of an electron neutrino after travelling a distance, L, is

$$P_e = 1 - \sin^2 2\theta \sin^2 \left[\pm \frac{1.27\Delta m^2 L}{E} \right]$$

• Furthermore, you can get an enhancement of flavour conversion in the sun due to the Mikheyev Smirnov Wofenstein (MSW) Effect

Deuterium Reactions

Detecting Neutrinos with Deuterium

Charged Current







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neutrino

electron

SNO: First Results



$$\begin{split} \Phi_{CC}^{\rm SNO}: & \text{Sensitive to } \nu_e \text{ only!} \\ \Phi_{ES}^{\rm SK}: & \text{Sensitive to } \nu_e, \ \nu_\mu, \text{ and } \nu_\tau \\ & \text{Here } \Phi_{ES}^{\rm SK} = \Phi(\nu_e) + 0.154 \Phi(\nu_{\mu\tau}) \end{split}$$

 $\Phi(\nu_{\mu\tau}) \neq 0$ at the 3.3 standard deviation. \rightarrow First evidence of solar neutrino oscillation !!!

Next: measure CC/NC will provide an unambiguous statement on whether neutrinos oscillate on their way to the earth from the core of the sun.

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Summary ____

The open questions of particle physics:

- Weak flavor mixing in the quark and neutrino sectors.
- Search of the Higgs boson and new physics beyond the SM.

Elementary Particles 75-462 & 562:

- Constituents of matter
- Fundamental forces
- Conservation Laws
- Invariance Principles and Symmetries
- Relativistic Kinematics
- Quark Model
- QED and QCD
- Feynman Rules
- Electroweak interactions
- Open questions!

URL: http://www.physics.carleton.ca/~alainb/